

# Tightening Network Calculus Delay Bounds by Predicting Flow Prolongations in the FIFO Analysis

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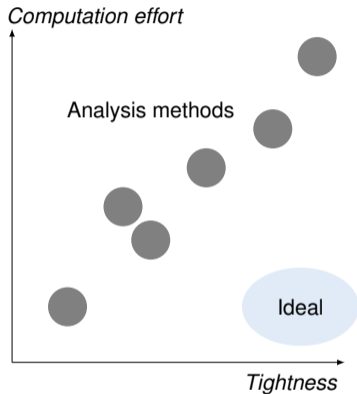
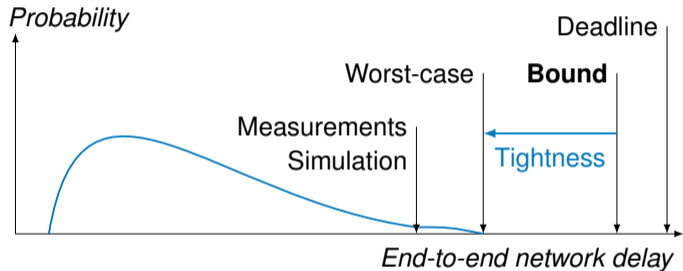
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Munich, Germany

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Ruhr University Bochum, Germany



## Motivation

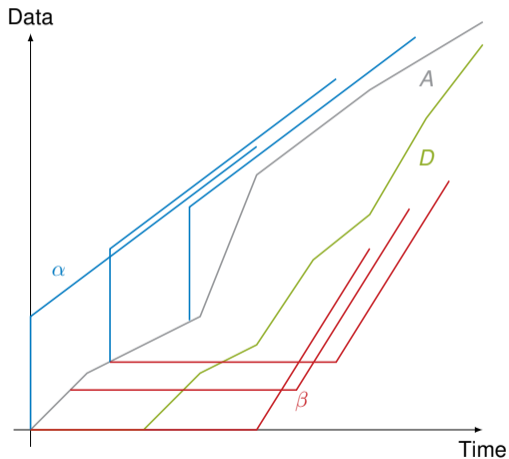
### Worst-Case End-to-End Performance Analysis



- Trade-off between computational effort and tightness
- **This talk: network analysis method with good tightness and fast execution**

# Background

## Network Calculus – Basics



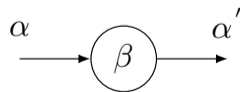
**Basis:** Cumulative arrivals and services [Cruz, 1991]



**Arrival curve**  $\alpha$ :  $A(t) - A(t - s) \leq \alpha(s), \forall t \leq s$

**Service curve**  $\beta$ : If the service by system  $S$  for a given input  $A$  results in an output  $D$ , then  $S$  offers a service curve  $\beta \in \mathcal{F}_0$  iff

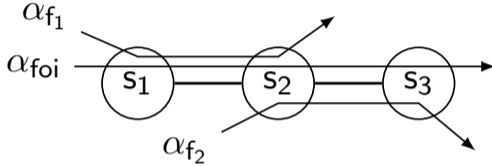
$$\forall t : D(t) \geq \inf_{0 \leq d \leq t} \{A(t - d) + \beta(d)\}.$$



# Background

## Network Calculus – FIFO Analysis

How to derive an end-to-end delay bound?



### LUDB – Least Upper Delay Bound

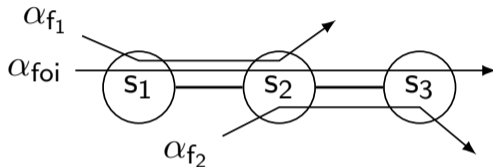
[Bisti et al., 2008, Bisti et al., 2012]

- Step 1: Compute the nesting tree
- Step 2: Compute an end-to-end service curve by removing cross-flows step by step
- Step 3: Compute the end-to-end delay bound

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### Network Calculus – FIFO Analysis

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### LUDB – Least Upper Delay Bound

[Bisti et al., 2008, Bisti et al., 2012]

*Step 1:* Compute the nesting tree

*Step 2:* Compute an end-to-end service curve  
by removing cross-flows step by step

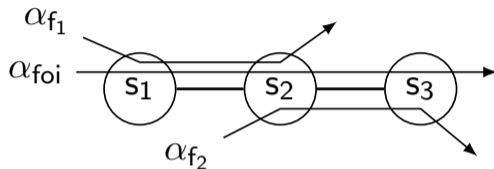
*Step 3:* Compute the end-to-end delay bound

**Nesting:** A sequence of servers ("tandem") is called nested if any two flows have disjunct paths or one flow is completely included in the path of the other flow.

## Background

### Network Calculus – FIFO Analysis

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### LUDB – Least Upper Delay Bound

[Bisti et al., 2008, Bisti et al., 2012]

**Step 1: Derive all cuts creating nested subtandems**

Step 2: Compute the nesting trees

Step 3: Compute an end-to-end service curves

Step 3a: by removing cross-flows step by step and

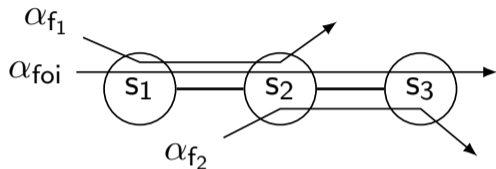
**Step 3b: by concatenating the intermediate service curves**

Step 4: Compute the end-to-end delay bound

## Background

### Network Calculus – FIFO Analysis

How to derive an end-to-end delay bound?



### LUDB – Least Upper Delay Bound

[Bisti et al., 2008, Bisti et al., 2012]

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Step 3: Compute an end-to-end service curves

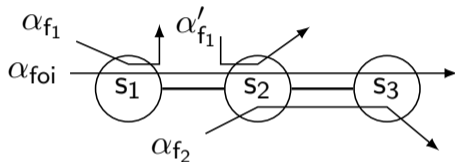
Step 3a: by removing cross-flows step by step and

**Step 3b: by concatenating the intermediate service curves**

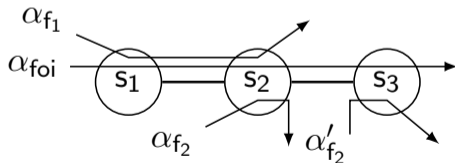
Step 4: Compute the end-to-end delay bound

### Where to cut?

Cutting alternative 1:



Cutting alternative 2:

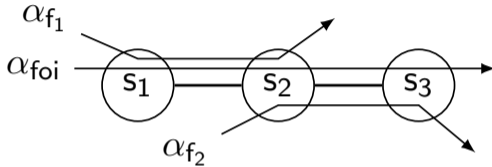


Neither alternative is strictly better than the other

# Background

## Network Calculus – FIFO Analysis

How to derive an end-to-end delay bound?



### LUDB – Least Upper Delay Bound

[Bisti et al., 2008, Bisti et al., 2012]

**Step 1: Derive all cuts creating nested subtandems**

Step 2: Compute the nesting trees

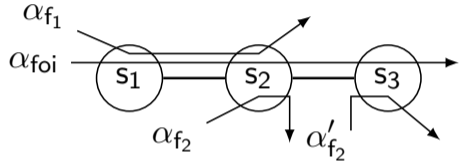
Step 3: Compute an end-to-end service curves

Step 3a: by removing cross-flows step by step and

**Step 3b: by concatenating the intermediate service curves**

Step 4: Compute the end-to-end delay bound

**What's the problem with cutting alternative 2?**



$$h(\alpha_{foi}, ((\beta_1 \otimes (\beta_2 \ominus \alpha_2)) \ominus \alpha_1) \otimes (\beta_3 \ominus (\alpha_2 \otimes (\beta_2 \ominus ((\alpha_{foi} + \alpha_1) \otimes \beta_1))))))$$

with

$$\text{convolution: } (f \otimes g)(d) = \inf_{0 \leq u \leq d} \{f(d-u) + g(u)\}$$

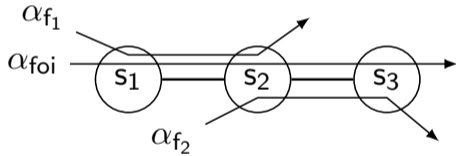
$$\text{deconvolution: } (f \otimes g)(d) = \sup_{u \geq 0} \{f(d+u) - g(u)\}$$



# Contribution

## Network Calculus – LUDB and Flow Prolongation

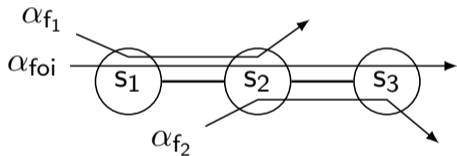
What can we do about it?



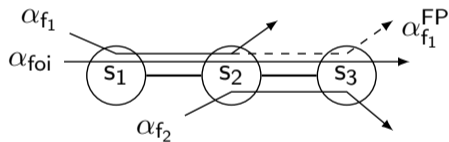
## Contribution

### Network Calculus – LUDB and Flow Prolongation

What can we do about it?



Create a nested tandem in a different way,  
before we face the cutting-problem!



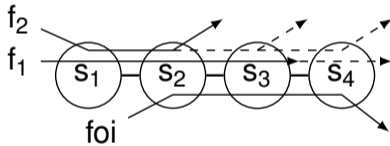
$$h(\alpha_{foi} + \alpha_1, \beta_1 \otimes ((\beta_2 \otimes \beta_3) \ominus \alpha_2))$$

# Contribution

## Network Calculus – LUDB and Flow Prolongation

### Does it Scale?

Flow prolongation in general does not [Bondorf, 2017],  
e.g., see:

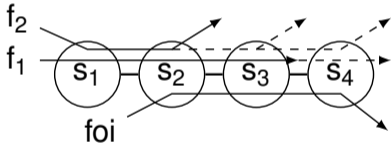


# Contribution

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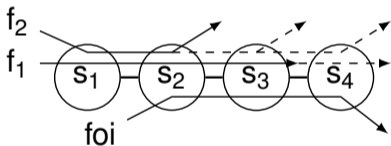
Not if you try to search exhaustively,  
not even when considering the objective to convert  
non-nested tandems to nested tandems.

## Contribution

### Network Calculus – LUDB and Flow Prolongation

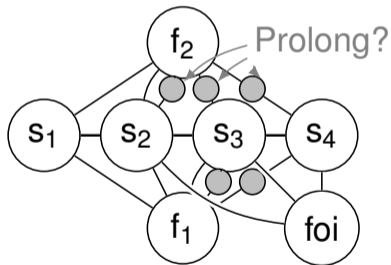
#### Does it Scale?

Flow prolongation in general does not [Bondorf, 2017], e.g., see:



Not if you try to search exhaustively, not even when considering the objective to convert non-nested tandems to nested tandems.

Thus, we converted the tandem into a Graph Neural Network:



We call the new analysis *DeepFP*.

# Heuristic based on Graph Neural Networks

## Graph Neural Networks – Introduction

**Graph Neural Networks** [Scarselli et al., 2009] and related architectures are able to process general graphs and predict feature of nodes  $\mathbf{o}_v$

### Principle

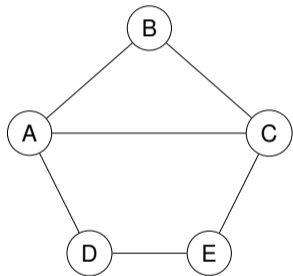
- Each node has a *hidden* vector  $\mathbf{h}_v \in \mathbb{R}^k$
- ... computed according to the vector of its neighbors
- ... and are propagated through the graph

### Algorithm

- Initialize  $\mathbf{h}_v^{(0)}$  according to features of nodes
- for  $t = 1, \dots, T$  do
  - $\mathbf{a}_v^{(t)} = \text{AGGREGATE} \left( \left\{ \mathbf{h}_u^{(t-1)} \mid u \in \text{Nbr}(v) \right\} \right)$
  - $\mathbf{h}_v^{(t)} = \text{COMBINE} \left( \mathbf{h}_v^{(t-1)}, \mathbf{a}_v^{(t)} \right)$
- return  $\text{READOUT} \left( \mathbf{h}_v^{(T)} \right)$

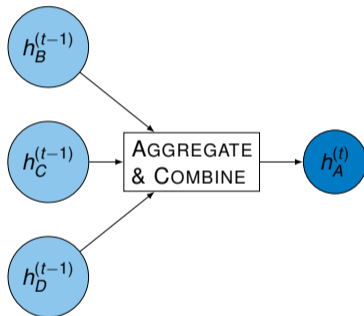
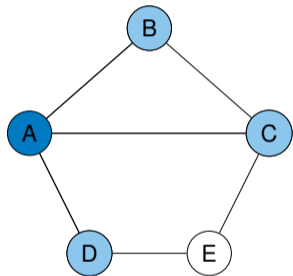
## Heuristic based on Graph Neural Networks

### Graph Neural Networks – Illustration



# Heuristic based on Graph Neural Networks

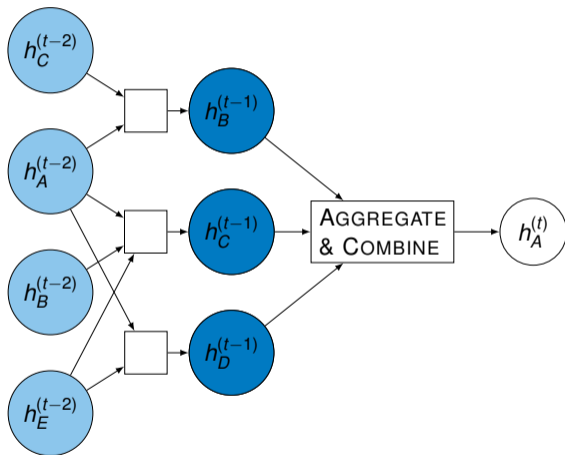
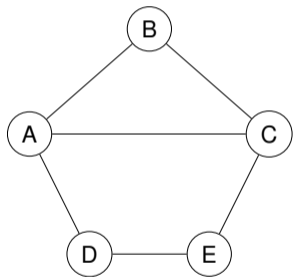
## Graph Neural Networks – Illustration





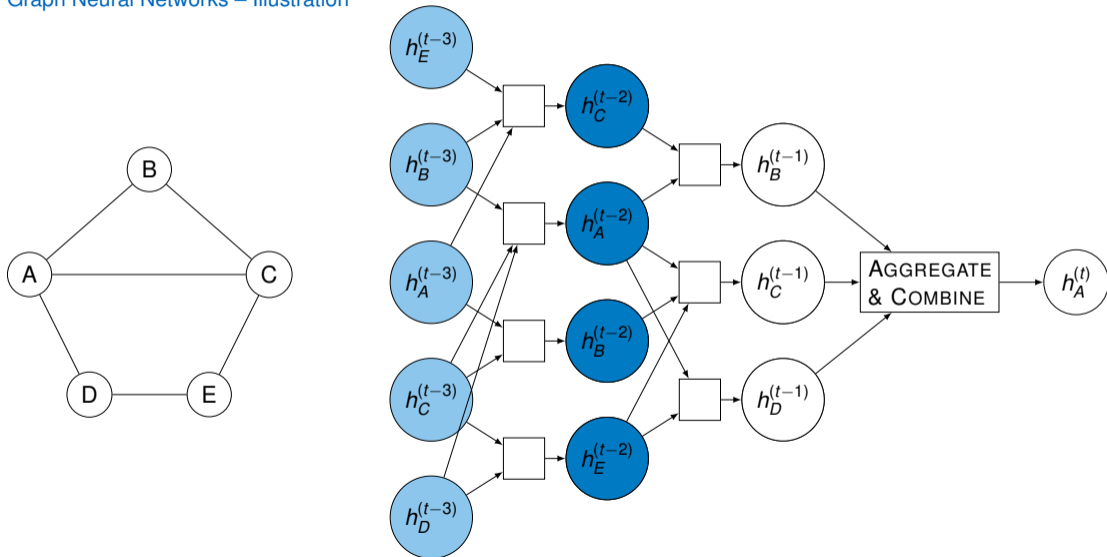
# Heuristic based on Graph Neural Networks

## Graph Neural Networks – Illustration



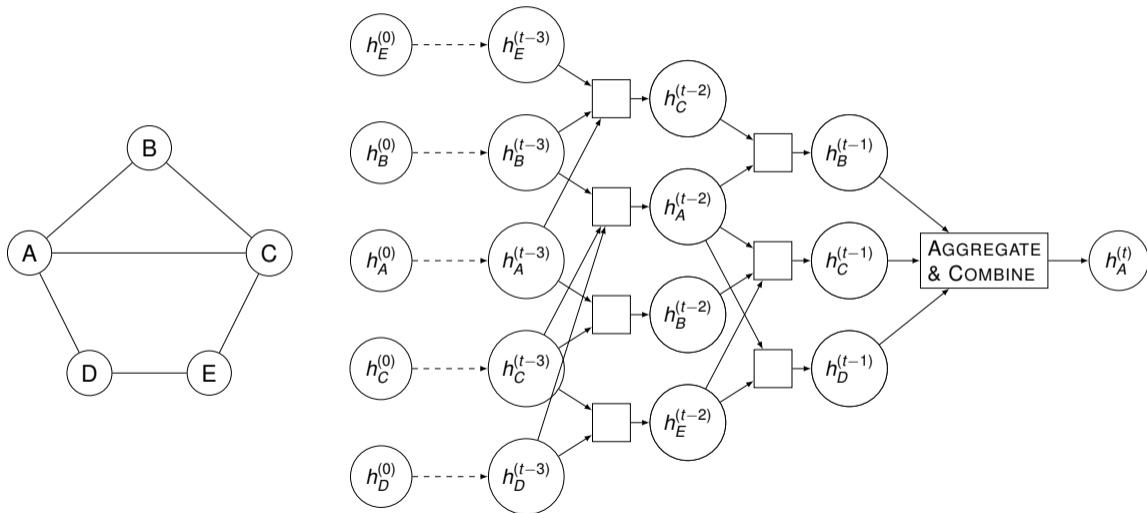
# Heuristic based on Graph Neural Networks

## Graph Neural Networks – Illustration



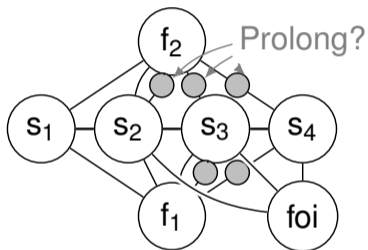
# Heuristic based on Graph Neural Networks

## Graph Neural Networks – Illustration



## Contribution

### Network Calculus – LUDB and Flow Prolongation and Predictions

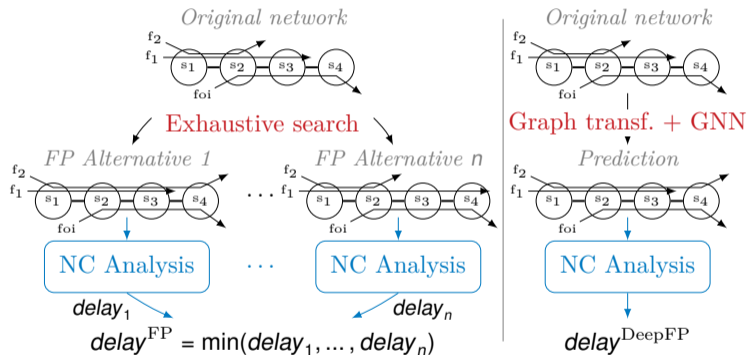


DeepFP<sub>n</sub>

- Converts the Network Calculus graph into a GNN network
- Predicts a score for each prolongation node, ranking the top prolongation choices
- Let's Network Calculus pick the top  $n \geq 1$  combinations of prolongations, to compute  $n$  valid delay bounds

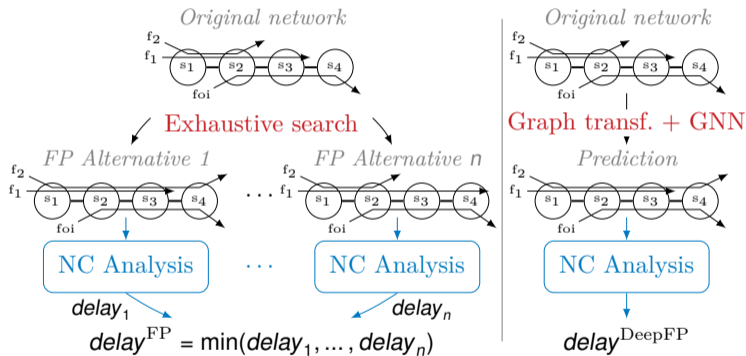
# DeepFP Overview

## ... and Related Work



# DeepFP Overview

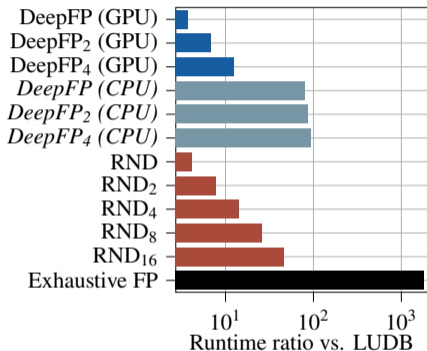
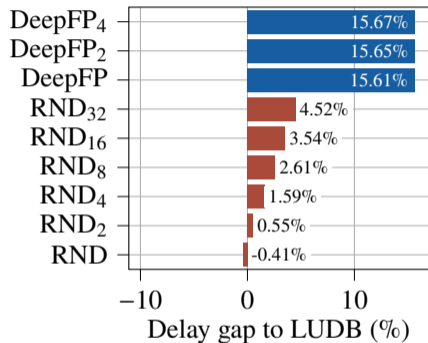
## ... and Related Work



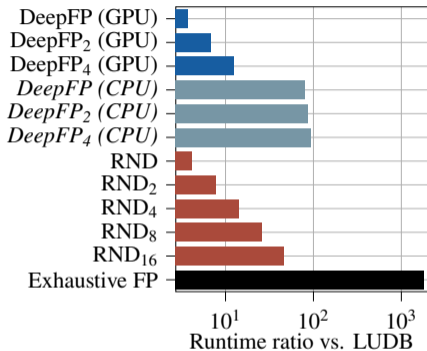
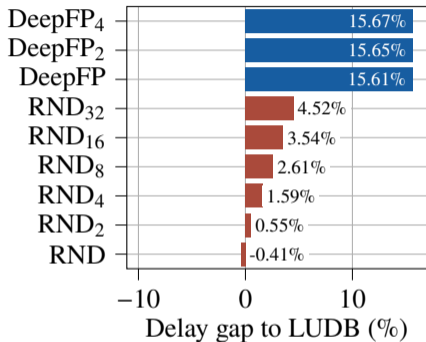
Related Work on NC + GNN:  
[Geyer and Carle, 2018,  
Geyer and Bondorf, 2019,  
Geyer and Bondorf, 2020],  
all of which focuses on the  
complexities in FIFO systems.

## Evaluation

### Benchmark to LUDB and Random Heuristic



## Evaluation



**That's it, thank you for your attention!**



## Bibliography

- [Bisti et al., 2008] Bisti, L., Lenzini, L., Mingozi, E., and Stea, G. (2008). Estimating the worst-case delay in FIFO tandems using network calculus. In *Proc. of ICST ValueTools*.
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