Determination of Probability of no Disruption and Freeway Volume Threshold for Ramp Metering Based on Gap Acceptance Theory

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Total Text:	5000
Total Tables:	0
Total Figures	9
Total Words:	7,250

Submitted for Presentation at the 84th TRB Annual Meeting

January 2005

(Preprint No. 05-0157)

ABSTRACT

Freeway ramp metering is one of the major freeway management techniques used worldwide to improve safety and operations. However, studies have shown that ramp metering is effective only when freeway traffic flow reaches a certain threshold level. When freeway traffic is low, there will be enough gaps in the freeway flow to accommodate the ramp flow, even when ramp traffic enters the freeway in platoons. In practice, ramp metering threshold values are typically determined based on empirical studies. The purpose of this paper is to develop theoretical models based on gap-acceptance theory that can be used to determine ramp metering threshold values. The models take into account the effect of platoon size resulted from various ramp controls, including random arrivals from uncontrolled ramp, platoon arrivals from an upstream signal control, and uniform arrivals from a ramp metering control. Volume-based ramp metering threshold values are derived using the models under different ramp control situations. The study results clearly indicate that more significant disturbance on freeway operations exist due to large platoon arrivals resulting from an upstream traffic signal, compared to when traffic arrives randomly or uniformly. The models are also applied to provide quantitative assessments from the perspectives of freeway capacity and safety, indicating that ramp metering results in increased freeway capacity and safety.

Keywords: ramp-metering threshold, gap acceptance, freeway operations

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INTRODUCTION

Freeway ramp metering is one of the major freeway management techniques used worldwide to improve safety and operations. However, general field studies have indicated that ramp metering is effective only when freeway traffic flow reaches a certain threshold level. When freeway traffic is low, there will be enough gaps in the freeway flow to accommodate the ramp flow, even when ramp traffic enters the freeway in platoons. In practice, ramp-metering threshold values are typically determined based on empirical studies. To our best of knowledge, no studies have been conducted to address the ramp-metering threshold issue from the theoretical point of view. This paper develops a new theoretical model based on gap-acceptance theory that can be used to determine ramp-metering threshold values.

The traffic from a freeway on-ramp enters the freeway in different patterns, depending on the type of ramp control. For example, when a freeway ramp is located near an upstream signal, vehicles tend to arrive in large platoons, which would be difficult to accommodate on the freeway without disturbing the mainline traffic. When the ramp is controlled by a ramp meter, traffic would tend to enter the freeway more uniformly, where the ramp traffic can be accommodated more easily with the available gaps on the freeway flow. On the other hand, the drivers from the ramp usually adjust their speeds while seeking for a suitable gap in the mainline traffic, and the mainline traffic may also try to accommodate the ramp traffic by either slowing down (without affecting the following vehicle) or shifting to the adjacent lanes. Figure 1 illustrates some of the merging situations of how the ramp traffic can be accommodated without significant disturbance on the mainline traffic. These situations involve no more than one vehicle making a lane change or slowing down, which are defined as no disturbance events later in this paper. The traffic volumes on both the freeway mainline and the ramp that would result in no disturbance of freeway operations are defined as the volume thresholds for ramp metering. Traffic volumes below the threshold values are not necessary for initiating ramp metering operations because normal freeway operations would maintain. Gap-acceptance based models to determine the volume threshold values are presented in the following sections of this paper.

GAP DISTRIBUTION AND LANE VOLUME DISTRIBUTION

In general, the gaps in a traffic stream follow a distribution function $f(t) = f(t,q)$, where t is the length of the gap and q is the traffic volume. For example, the probability density function for partially bunched traffic conditions can be given by the Cowan's M3 model (*1*) shown in Equation (1) in the cumulative form:

$$F(t) = \begin{cases} 1-\alpha\cdot\exp(-\lambda\cdot(t-\Delta)) & for\ \ t \ge \Delta \\ 0 & for\ t < \Delta \end{cases} \qquad \text{(-)} \qquad (1)$$

where:
- t is the sample gap (s)
- Δ is the minimum gap within bunches (s)
- α is the proportion of non-bunched vehicles (-)

$$\lambda = \frac{\alpha\cdot q}{1-\Delta\cdot q} \qquad \text{(veh/s)} \quad (2)$$

- q is the stream flow rate (veh/s)
- λ is the flow rate within the bunched vehicles (veh/s)

Usually we can use $\alpha = 1 - q\cdot\Delta$ and $\lambda = q$ for normal traffic conditions without impedance of traffic signals (*2*). Thus, we should also use these parameters for freeway traffic flow. In this case Equation (1) yields

$$F(t)=\begin{cases}1-(1-q\cdot\Delta)\cdot\exp(-q\cdot(t-\Delta)) & for\ \ t\geq\Delta\\ 0 & for\ \ t<\Delta\end{cases}\qquad(-)\qquad(3)$$

Given the total flow rate of a freeway q_{sum}, the proportion of traffic flow, p_1, p_2,..., on different traffic lanes 1, 2... can be calculated based on Equation (4):

$$\begin{cases}p_1=q_1/q_{sum}\\ p_2=q_2/q_{sum}\\ \ldots\\ p_1+p_2+\ldots=1\end{cases}\quad ..\qquad(-)\qquad(4)$$

Of course the lane volume distribution is highly variable from section to section and different from country to country because of different traffic behavior and regulations. For standard motorways, the country-related lane volume distribution should be used for further calculations.

In the U.S. and other North American countries, the regulation of "Keep in Lane" is common. Based on a sample data set collected in Canada and the U.S., Equations (5) and (6) can be established as sample regression functions to represent the lane flow distribution on a 2-lane (each direction) freeway and on a 3-lane (each direction) freeway.

$$\begin{cases}p_1=0.332+(1-0.332)\cdot\exp(-1.440\cdot q_{sum}^{0.5})\\ p_2=1-p_1\end{cases}\qquad(-)\qquad(5)$$

$$\begin{cases}p_1=0.235+(1-0.235)\cdot\exp(-4.758\cdot q_{sum}^{0.5})\\ p_3=0.420\cdot(1-\exp(-2.340\cdot q_{sum}))\\ p_2=1-p_1-p_3\end{cases}\qquad(-)\qquad(6)$$

The lane flow distribution from Equations (5) and (6) are illustrated in Figure 2. If the traffic flow rates of different lanes, q_1, q_2, ..., can be obtained directly from field measurements, they should be used for further calculations.

Once the proportion of traffic flow, p_1, p_2,..., are given, the traffic flow of different lanes, q_1, q_2, ..., can be calculated by:

$$\begin{cases}q_1=p_1\cdot q_{sum}\\ q_2=p_2\cdot q_{sum}\\ \ldots\\ \ldots\end{cases}\qquad(\text{veh/s})\qquad(7)$$

THE CAPACITY OF THE ON-RAMP LANE

The capacity for a freeway on-ramp lane is usually calculated using gap-acceptance models (*3, 4, 5, 6, 7, 8*). For example, the capacity, C_{ramp} can be expressed by

$$C_{ramp}=\frac{\alpha_1}{t_{f,ramp}}\cdot\exp\left(-\lambda_1\cdot(t_{c,ramp}-\frac{t_{f,ramp}}{2}-\Delta_1)\right)\qquad(\text{veh/h})\text{ or }(\text{veh/s})\ (8)$$

where $t_{c,\text{ramp}}$ is the critical gap for merging from the on-ramp lane to lane 1, $t_{f,\text{ramp}}$ is the follow-up time.

To take into account the effect of the length of the acceleration lane, a correction factor must be applied to the critical gap, $t_{c,\text{ramp}}$. In general, the value of $t_{c,\text{ramp}}$ can be modified according to the length of the acceleration lane and to the possible accelerating rate of the vehicle. The value of $t_{c,\text{ramp}}$ decreases with increasing length of the acceleration lane.

In HCM (*9*) the capacity of the on-ramp is expressed by a linear function. The HCM-formula is given by

$$C_{ramp}=4600-(q_1+q_2)\qquad(\text{veh/h})\qquad(9)$$

PROBABILITY OF NO DISRUPTION IN MAJOR FLOW DUE TO ON-RAMP TRAFFIC

We first define a parameter, *B*, the required mainline gap to accommodate a particular size of traffic platoon from the ramp. For example, for a platoon of size *N*, $B = (N+1)\cdot h$, where *h* is the minimum headway in the on-ramp stream. Considering the merge process as a floating queuing system, the average queue length in the system is $L = (N+x)$ and $B = L\cdot h = (N+x)\cdot h$, where *N* is the average queue length of the queuing system excluding the vehicle in the counter (= average size of platoon), *x* is the degree of saturation for the on-ramp. Note, for $q = 0$ is $B = 0$.

The probabilities of no disruption to freeway mainline traffic are calculated for the three cases illustrated in Figure 1.

Case a - Gap in the merge lane, h_i, is no less than $B = (N+x)h$

In this case, the probability of no disruption $P_{ND,a}$ is equal to the probability that the length of a gap in the major stream is larger than *B,* i.e., the probability of no disruption is simply equal to the probability for $t \geq B$. Thus,

$$P_{ND,a} = \Pr(t \geq B) \qquad (-) \qquad (10)$$

From Equation (1) we have

$$P_{ND,a} = 1 - F_t(t = B) = \alpha_1 \cdot \exp(-\lambda_1 \cdot (B - \Delta_1)) \qquad (-) \qquad (11)$$

where $F_t(t{=}B)$ is the probability distribution function (cumulative) of gap *t* with a length *B*.

Case b - Gap in the first lane is Δ*t* shorter than required. The freeway vehicle can slow down by Δ*t* without affecting the following vehicle

In this case we are looking for the probability of $t^1+t^2>B+\Delta_1$ under the condition of $t^1 \leq B$, where t^1 and t^2 are two consecutive gaps in the freeway mainline stream in lane 1. Generally, the probability of $t^1+t^2>B+\Delta_1$ and the condition $t^1 \leq B$ are independent of each other for freeway headway distributions, thus, we have

$$\begin{aligned} P_{ND,b} &= \Pr(t^1 + t^2 > B + \Delta_1 \mid t^1 \leq B) \\ &= \Pr(t^1 + t^2 > B + \Delta_1) \cdot \Pr(t^1 \leq B) \end{aligned} \qquad (-) \qquad (12)$$

Since the probability of t^1+t^2 obeys the shifted Erlang distribution, we have (*10*)

$$F_{t^1+t^2}(t) = 1 - \alpha_1 \cdot \exp(-2 \cdot \lambda_1 \cdot (t - \Delta_1)) \cdot [1 + 2 \cdot \lambda_1 \cdot (t - \Delta_1)] \qquad (-) \qquad (13)$$

Combining with Equation (1) yields

$$\begin{aligned} P_{ND,b} &= \Pr(t^1 + t^2 > B + \Delta_1) \cdot \Pr(t^1 \leq B) \\ &= [1 - F_{t^1+t^2}(B + \Delta_1)] \cdot F_{t^1}(B) \\ &= \alpha_1 \cdot \exp(-2 \cdot \lambda_1 \cdot (B - \Delta_1 + \Delta_1)) \cdot [1 + 2 \cdot \lambda_1 \cdot (B - \Delta_1 + \Delta_1)] \\ &\quad \cdot [1 - \alpha_1 \cdot \exp(-\lambda_1 \cdot (B - \Delta_1))] \\ &= \alpha_1 \cdot \exp(-2 \cdot \lambda_1 \cdot B) \cdot (1 + 2 \cdot \lambda_1 \cdot B) \cdot [1 - \alpha_1 \cdot \exp(-\lambda_1 \cdot (B - \Delta_1))] \end{aligned}$$

$$(-) \qquad (14)$$

Case c - Gap in the first lane is smaller than required and the freeway vehicle cannot slow down without affecting the following vehicle. But the freeway vehicle can make a lane change to the adjacent lane

In this case, the probability of no disruption is for $t_1<B$, $t^1+t^2<B+\Delta_1$, and $t_2>t_{c,2}$,

$$P_{ND,c} = \Pr(t_1 < B) \cdot \Pr(t^1 + t^2 < B + \Delta_1) \cdot \Pr(t_2 > t_{c,2}) \qquad (-) \qquad (15)$$

Using Equation (1) and Equation (13) yields

$$\begin{aligned} P_{ND,c} &= \Pr(t_1 < B) \cdot \Pr(t^1 + t^2 < B) \cdot \Pr(t_2 > t_{c,2}) \\ &= F_{t_1}(B) \cdot F_{t^1+t^2}(B) \cdot [1 - F_{t_2}(t_{c,2})] \\ &= [1 - \alpha_1 \cdot \exp(-\lambda_1 \cdot (B - \Delta_1))] \cdot [1 - \alpha_1 \cdot \exp(-2 \cdot \lambda_1 \cdot B) \cdot (1 + 2 \cdot \lambda_1 \cdot B)] \\ &\quad \cdot \alpha_2 \cdot \exp(-\lambda_2 \cdot (t_{c,2} - \Delta_2)) \end{aligned}$$

$$(-) \qquad (16)$$

That total probability of no disruption is then

$$P_{ND} = P_{ND,a} + P_{ND,b} + P_{ND,c} \qquad (-) \qquad (17)$$

Where $t_{c,1}$ is the critical gap for changing from lane 2 to lane 1, Δ_1 is the minimum headway within bunches on lane 1, $t_{c,2}$ is the critical gap for changing from lane 3 to lane 2, Δ_2 is the minimum headway within bunches on lane 2. The lanes are numbered from the shoulder to the median (cf. Figure 1). For further calculation, parameter $t_{c,1}$=4s, Δ_1=1.2s, $t_{c,2}$ =4s, and Δ_2=1s are used.

In Figure 3, the total probability of no disruption ($P_{sum} = P_{ND}$) as a function of the average length (B) of time for accommodating the platoon is illustrated both for a two-lane freeway and a three-lane freeway under the sample traffic conditions in North America. It can be recognized, that the total probability of no disruption decreases with increasing values of B.

CALCULATIONS ON PARAMETER *B* VALUES

Using Equations (11), (14), (16) and (17), the probability of no disruption can be obtained if the required length of time (B) for accommodating the platoon is known. Because B is a function of platoon size, which is related to the type of ramp controls and ramp traffic arrival patterns, we need first to determine the platoon size based on ramp conditions. In general, the size of the platoon can be achieved using a suitable queuing theory.

Uncontrolled Ramp – Random Arrival

When a ramp is uncontrolled and is far from upstream signals, traffic enters the freeway randomly. The system can be represented by an M/M/1 - queuing system for simplification. Here, the average queuing length in the on-ramp stream is given by

$$N_{M/M/1} + x = \frac{x^2}{1-x} + x = \frac{x}{1-x} \qquad (-) \qquad (18)$$

Thus, we have the average length of time for accommodating the platoon

$$B_2 = \frac{x \cdot h}{1-x} \qquad (s) \qquad (19)$$

Ramp with Metering – Uniform Arrival

When a ramp has ramp metering in operation, the output of the metering is the input of the queuing system (ramp). This input is in this case a uniform input. The system can be then classified as a D/M/1-queuing system for simplification. Despite of the metering, there still will be platoon before the merge point if the capacity of the merge point is relatively low. For a D/M/1-queuing system, the average queuing length in the on-ramp stream (before the merge point) is given by

$$N_{D/M/1} + x = \frac{2x^2 - x}{2(1-x)} + x = \frac{x}{2(1-x)} \qquad (-) \qquad (20)$$

where x is the degree of saturation of the on-ramp. Thus, we have the average length of time for accommodating the platoon (the headway in the queuing platoon remains h)

$$B_1 = \frac{x \cdot h}{2(1-x)} \qquad (s) \qquad (21)$$

We can recognize that B_2 is only the half of B_1. That is, the average length of platoon can be cut in half by ramp metering.

Ramp with Upstream Signal – Large Platoon Arrival

When a ramp is located close to an upstream traffic signal, traffic enters the freeway in bunches (i.e., large platoons). In this case, the average length of time for accommodating the platoon can be calculated by (cf. Figure 4)

$$B_3 \approx [m \cdot P_{\text{bunch}} + N \cdot P_{\text{free}} + x] \cdot h \qquad (s) \qquad (22)$$

where P_{bunch} is the proportion of bunched time, m is the number of bunched traffic volume, and P_{free} is the proportion of time for free traffic. N can be calculated from an M/M/1 queuing system by

$$N_{M/M/1} = \frac{x^2}{1-x} \qquad (-) \qquad (23)$$

For normal signalized, not coordinated intersections we have

$$P_{free} = 1 - \frac{r/c}{1 - q/q_s} \qquad (-) \qquad (24)$$

$$P_{bunch} = \frac{r/c}{1 - q/q_s} - \frac{r}{c} = \frac{r}{c}\left(\frac{q/q_s}{1 - q/q_s}\right) \qquad (-) \qquad (25)$$

and

$$m = \frac{r/c}{1/q - 1/q_s} \qquad (-) \qquad (26)$$

where q_s is the saturation flow, c is the cycle length, and r is the red time.

Here, only the case for one single movement is considered. However, it can be extended to model a more general case with more than one feeding traffic movement based on the same principle.

MODEL APPLICATIONS

For all the three cases mentioned in the previous section, the freeway volume threshold can be obtained implicitly from Equation (17) based on certain P_{ND} values. It is noted that these threshold values are determined based on the conditions of no disruption to freeway traffic as defined early in this paper. They are not exactly the volume threshold for ramp metering applications. In order to derive the threshold for ramp metering applications, the relationship between freeway breakdown and the probability of no disruption must be established. Such a relationship needs to be verified based on field studies.

The freeway volume threshold is a function of the on-ramp volume. The threshold can be easily obtained from the graphs developed below. The graphs are developed based on the sample traffic characteristics in North America. In General, the value of the threshold depends on the major flow volume on the freeway, the volume of the on-ramp and the predefined probability P_{ND} for no disruption. To obtain the practical threshold values for ramp metering, these parameters must be calibrated based on field conditions.

Two-lane freeway

Using the following parameter values and based on the sample traffic characteristics in North America: q_{ramp}=700 veh/h, c=60s and r=30s, p_1 and p_2 from Equation (5) and C_{ramp} from Equation (9), the average length (B) of time for accommodating the platoon and the probability of no-disruption (P_{ND}) for different on-ramp traffic conditions are depicted in Figure 5.

If a probability of no disruption P_{ND}=0.8 is predefined, we have a freeway volume threshold of 2648 (q_{ramp}+q_{sum}=3348) veh/h for randomly arriving ramp traffic (Figure 5, a), 3274 (q_{ramp}+q_{sum}=3974) veh/h for uniformly (equivalent to ramp metering) arriving ramp traffic (Figure 5, b), and only 1082 (q_{ramp}+q_{sum}=1782) veh/h for ramp traffic arriving in platoons due to upstream traffic signals (Figure 5, c).

Three-lane freeway

Using the same parameter values as in the case of two-lane freeway (except for p_1, p_2, and p_3 from Equation (6)), the average length (B) of time for accommodating the platoon and the probability of no-disruption (P_{ND}) for different on-ramp traffic conditions are depicted in Figure 6.

Again, if a probability of no disruption P_{ND}=0.8 is predefined, we have a freeway volume threshold of 4875 (q_{ramp}+q_{sum}=5575) veh/h for randomly arriving ramp traffic (Figure 6, a) 5932 (q_{ramp}+q_{sum}=6632) veh/h for uniformly (equivalent to ramp metering) arriving ramp traffic (Figure 6, b), and only 1780 (q_{ramp}+q_{sum}=2480) veh/h for ramp traffic arriving in platoons due to upstream traffic signals (Figure 6, c).

DISCUSSIONS

Effect of Ramp Control Conditions

The modeling process presented previously can be directly applied to analyze the effect of different ramp control conditions. Figure 7 illustrates the total probabilities of no-disruption (P_{ND}=P_{sum}) for different ramp traffic arriving conditions for the case with a ramp flow rate q_{ramp}=700 veh/h. It can be clearly seen that ramp metering results in the highest P_{ND} values, while ramp with upstream signals results in the lowest P_{ND}.

The effect of different ramp control conditions could also be analyzed from the freeway capacity perspective. The capacity of a freeway is a stochastically varying value, which depends on the actual traffic conditions (*11, 12, 13, 14, 15*). Particularly the capacity in no disrupted state, C_{ND} (corresponding to the capacity before breakdown), has a higher value than the capacity in the disrupted state, C_D (similar to the capacity after breakdown, because the disrupted vehicles have to decelerate and re-accelerate in order to follow the vehicle in the front. This behavior causes larger headway between two consecutive vehicles similar to a congested traffic flow). The difference is called "capacity drop", $C_{\mathrm{drop}} = C_{ND} - C_D$. The actual capacity of a freeway is then a function of the probability of no disruption, P_{ND}. That is

$$\begin{aligned} C_{real} &= C_{ND} \cdot P_{ND} + C_D \cdot (1 - P_{ND}) \\ &= C_{ND} - C_{drop} \cdot (1 - P_{ND}) \end{aligned} \qquad \text{(veh/h)} \quad (27)$$

If the probability of no disruption can be increased by ΔP_{ND}, the capacity can be increased by

$$\Delta C_{real} = C_{drop} \cdot \Delta P_{ND} \qquad \text{(veh/h)} \quad (28)$$

For example, a "capacity drop" of C_{drop} = ca. 20% can be observed on certain two-lane freeways (i.e. we use the capacity drop from free flow capacity down to queue discharge capacity for simplification). Thus, for a total flow of 3200 veh/h (q_{sum}=2500, q_{ramp}=700) we can obtain an increase of the probability of no disruption, ΔP_{ND}=$P_{ND,metering}$-$P_{ND,no\ metering}$ = 0.92-0.83 = 0.09 (cf. Figure 7). Therefore, we can obtain a capacity enhancement of 20%*0.09≈2% using ramp metering in this case. For a total flow of 3700 veh/h (q_{sum}=3000, q_{ramp}=700) we can obtain an increase of the probability of no disruption, ΔP_{ND}=0.20 (cf. Figure 7). The capacity enhancement is than 20%*0.2=4%. These increases of capacity could be crucial for reducing the probability of breakdowns.

Safety Measures

The probability of disruption, $P_D = 1-P_{ND}$, can be considered as a safety measure. If the probability of disruption P_D, is high, the drivers are likely to be forced slowing down and changing lanes. This can directly raise the chance of accidents. The absolute increase of this type of safety due to the metering is

$$\Delta_{safety} = \Delta P_D = \Delta P_{ND} = P_{ND,mertering} - P_{ND,no\ \ mertering} \qquad (29)$$

The relative enhancement of safety is

$$E_{safety} = \frac{\Delta P_D}{P_{D,no\ \ mertering}} = \frac{\Delta P_{ND}}{1 - P_{ND,no\ \ mertering}} \qquad (30)$$

It is to be pointed out, that different parts of the probability of disruption represent different dangers in the ramp area. From Equation (17), the total probability of disruption can be obtained. However, some vehicles still need braking or changing lane in order to avoid a collision. Thus, for evaluating different dangers caused by the merge process, different probabilities of disruption must be used.

Danger caused by breakdown, by forced breaking, and by forced lane changing

This danger can be expressed by the probability that the gap in the major stream is larger than the length of time for accommodating the platoon. This corresponds exactly to case (a). Thus, the danger can be expressed by $P_{D,a}$ = 1-$P_{ND,a}$, where the value of $P_{ND,a}$ is calculated by Equation (11).

Danger caused by breakdown and by forced lane changing

This danger correspondents exactly case (a) + case (b). Thus, the danger can be expressed by $P_{D,a+b}$ = 1-$P_{ND,a}$-$P_{ND,b}$, where the values of $P_{ND,a}$ and $P_{ND,b}$ are calculated by Equation (11) and Equation (14). Here the danger caused by forced breaking is excluded.

Danger caused by breakdown

The danger caused by breakdown can be expressed by the total probability of disruption, P_D = 1-P_{ND}, where the value of P_{ND} is calculated by Equation (17). Here the dangers caused by forced lane changing and by forced breaking are excluded.

For the same case used in analyzing capacity enhancement, we have for $q_{sum}+q_{ramp}$=3200 veh/h an increase of absolute increase of safety (danger caused by breakdown), $\Delta P_{ND} = P_{ND,metering} - P_{ND,no\ metering}$ = 0.92-0.83 = 0.09. That is, the dangers due to disruption caused by the on-ramp traffic is reduced by 0.09/(1-0.83)=53%. Here, the dangers caused by forced breaking and by forced lane changing are not taken into account. If these dangers have to be considered, we have $\Delta P_{ND,a} = P_{ND,a,metering} - P_{NDa,,no\ metering}$ = 0.79-0.60 = 0.19 (cf. Figure 5). That is, the total danger due to disruption (by breakdown, by forced lane changing, and by forced breaking) caused by the on-ramp traffic is reduced by 0.19/(1-0.60)=48%. For $q_{sum}+q_{ramp}$=3700 veh/h an increase of absolute increase of safety (danger caused by breakdown), ΔP_{ND} = 0.20, can be obtained. The dangers due to disruption caused by the on-ramp traffic can be reduced by 59%. The total danger due to disruption (by breakdown, by forced lane changing, and by forced breaking) caused by the on-ramp traffic can be reduced by 64% (cf. Figure 5).

The capacity enhancement and the reduction of disruption probability confirm the empirical results from Trupat (*16*) very well. In a comprehensive investigation, he observed for a German freeway metering system a capacity increase of ca. 3-5% and a disruption decrease of ca. 50%. The frequency of heavy accidents was reduced by 30-40%.

METERING THRESHOLD

Metering threshold for random on-ramp flow

In the United States, different thresholds have been used for ramp meriting on freeways. For example, Wisconsin uses the volume-to-capacity (*q/C)* ratio 0.7 for urban and 0.6 - 0.65 for rural areas. The capacity *C* is based on HCM, the flow rate *q* includes both the on-ramp flow and the mainline flow. Illinois uses occupancy of 11.7% (upstream of meter). Denver uses the volume, occupancy, and speed, whichever is controlling, and the values are measured downstream of meter and must last for 3 consecutive minutes. The threshold for volume is 1900 veh/h/ln (average of all lanes), for occupancy is 20%, and for speed is 35 mph. In general, the downstream volume or the *q/C* ratio is considered to be suitable parameters for metering control. The occupancy can be considered as a function of the flow rate *q*.

From the proposed method in this paper, the probability of no-disruption P_{ND} as a function of the total downstream volume $q_{sum}+q_{ramp}$ can be estimated. In Figure 8, this functional relationship is illustrated. In case of a two-lane freeway (cf. Figure 8, a), the relationship is nearly independent of the individual on-ramp volume q_{ramp}. Thus, for any on-ramp volume, the downstream volume threshold can be simply defined as a function of the probability of no-disruption. For example, using a predefined probability of no-disruption P_{ND}=0.8, the downstream volume threshold is 3400 veh/h. This correspondents to a lane volume of 1700 veh/h/ln and a *q/C* ratio of 0.74 (for *C*=2300 veh/h/ln). Unfortunately, for a three-lane freeway, the independence between P_{ND} and q_{ramp} does not exist (cf. Figure 8, b). Thus, for different on-ramp volumes, different downstream volume thresholds must be used if the constant value of P_{ND} is predefined. For example, with P_{ND}=0.8, the downstream volume threshold varies from 5100 through 5900 veh/h or 1700 through 1967 veh/h/ln (q_{ramp} from 1100 to 500 veh/h). This correspondents to *q/C* ratios ranging between 0.74 and 0.85 (for *C*=2300 veh/h/ln). On the other hand, if a downstream volume threshold of 1700 veh/h/ln is used, the value of the probability of no-disruption P_{ND} would vary from 0.8 through 0.91 (cf. Figure 8, b). For simplification, a downstream volume threshold of 1700 veh/h/ln can be used both for two-lane and three-lane freeway. In this case, the probability of no-disruption is always equal to or greater than 0.8.

Metering threshold for ramp traffic arriving in platoons (upstream signal)

The probability of no-disruption P_{ND} is dependent on the cycle time *c* and the red time *r* of the upstream signal. As an example for the sample North America conditions, the probability of no-disruption P_{ND} as a function of the total downstream volume $q_{sum}+q_{ramp}$ is illustrated in Figure 9 for *c*=60s and *r*=30s. It can be seen, that the independence between P_{ND} and q_{ramp} does not exist both for the two-lane and the three-lane freeway. With P_{ND}=0.8, the downstream volume threshold varies from 1700 through 2300 veh/h/ln or 750 through 1150 veh/h/ln (q_{ramp} from 800 to 500 veh/h) for a two-lane freeway. For a three-lane freeway, the downstream volume threshold varies from 1600 through 3900 veh/h or 533 through 1300 veh/h/ln (q_{ramp} from 1100 to 500 veh/h). Because the thresholds depend strongly on the upstream signal parameters, no general recommendations can be made for the metering control. For practical applications, Figure 3 (for sample North America conditions) can be used for estimating the downstream volume threshold if the size of platoon *B* can be measured or estimated prior to the calculation.

CONCLUSIONS AND OUTLOOK

Using the gap-acceptance theory, the probability of no disruption caused by on-ramp traffic can be evaluated in details. Equations (11), (14), (16), (21), (19), and (22) are the most critical elements for the developed models. From these equations, the probability of no disruption for three different on-ramp traffic conditions of a) randomly arriving, b) uniformly arriving (equivalent to ramp metering), and c) arriving in platoons (e.g., different size of platoons resulted from an upstream signal) can be obtained. From these probabilities of no disruption, the control thresholds of the freeway volume can be determined. Furthermore, these probabilities of no disruption can also be associated to the bottleneck capacity and safety measures in the merge area. The modelings results indicate that ramp metering significantly enhances the probability of no disruption and thus improve the operations and safety for freeways. Another important finding is that the effect of upstream signals on the merge operation can be positive or negative, depending on the traffic volume level on the freeway. For normal traffic conditions, thresholds of freeway volume for the case with ramp metering control are lower than those with upstream signals and uncontrolled random arrivals.

Most importantly, the study delivers a theoretical framework for estimating the probability of no disruption in the merge area on freeways, thus providing a theoretical basis for determining ramp metering threshold values in practice. It is recommended that the models be calibrated based on country-specific conditions before being applied in the practice. Furthermore, for every particular traffic regulation, the capacity formula (cf. eq. (8)) and the formulae for calculation of the lane volume distribution must be carefully calibrated against field more measurements.

As a reference, Figure 8 is useful for estimating downstream volume thresholds for freeways in North America with random on-ramp flows. For on-ramp flow under other conditions (e.g. upstream signals), Figure 3 can be used in case that the size of the platoon can be measured or estimated prior to the calculation.

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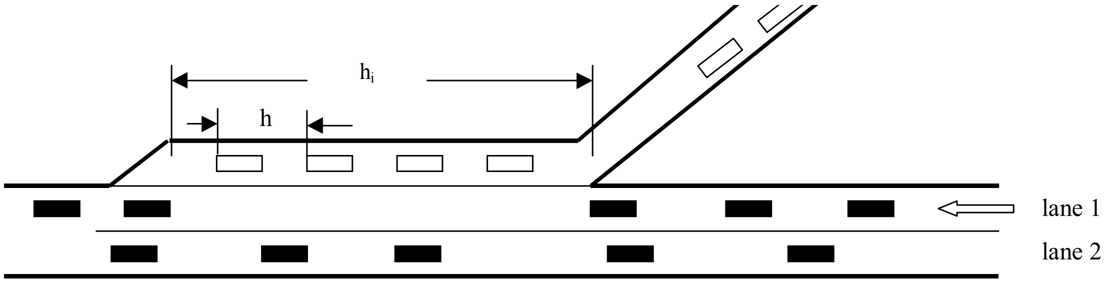
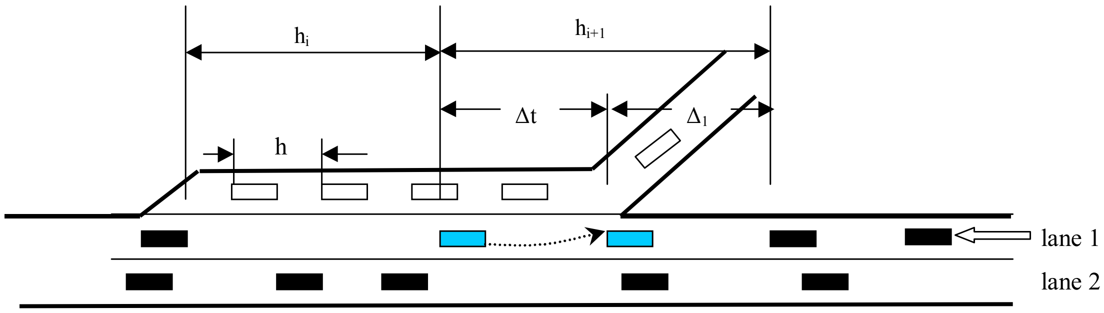
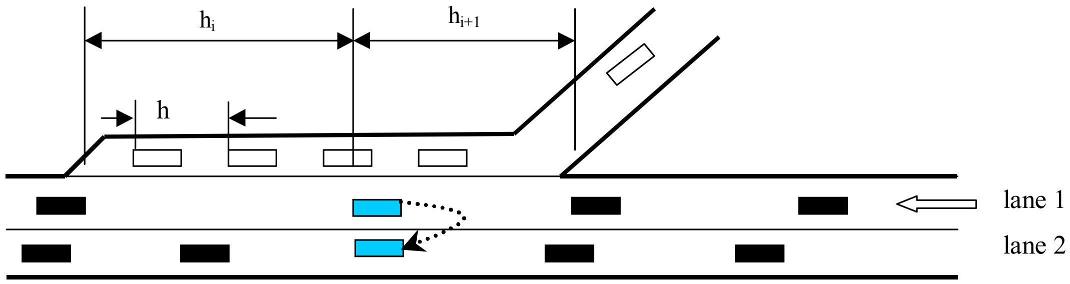


a) Platoon size n = 4; Gap in the merge lane, h_i is no less than (n + 1) h

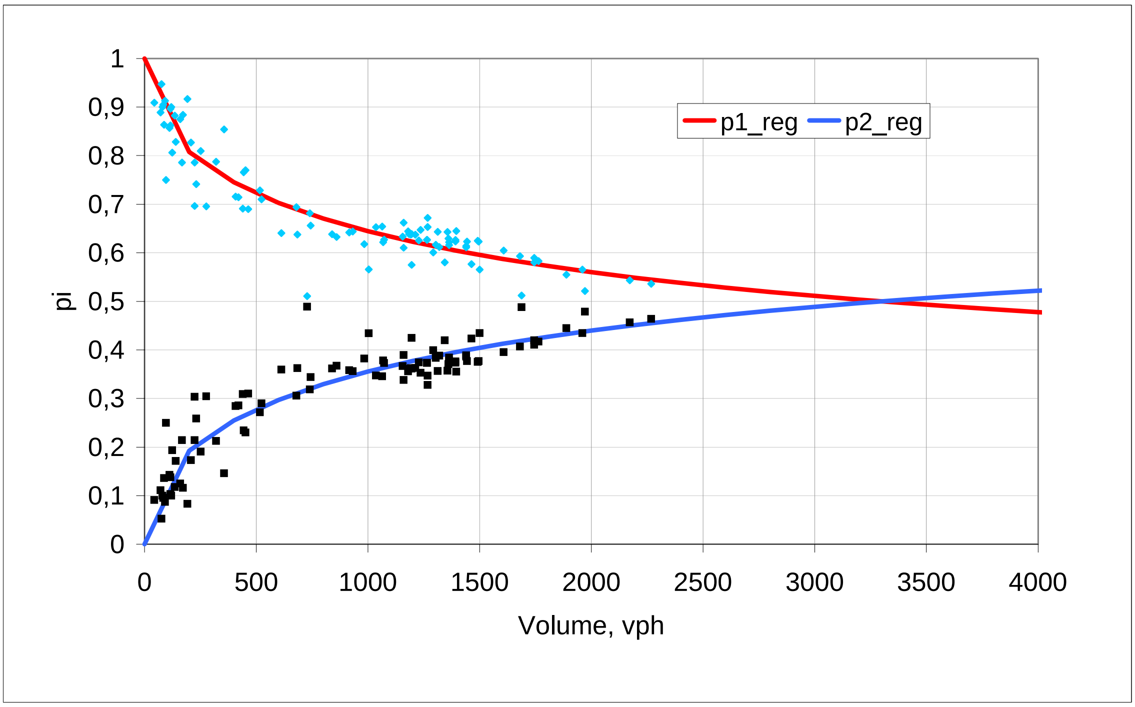


b) Gap in the first lane is Δt shorter than required. The freeway vehicle can slow down by Δt without affecting the following vehicle



c) Gap in the first lane is smaller than required. The freeway vehicle can make lane change to the adjacent lane.

Figure 1 - Three different cases of disruption in major stream (here we use *h*=gap=headway)



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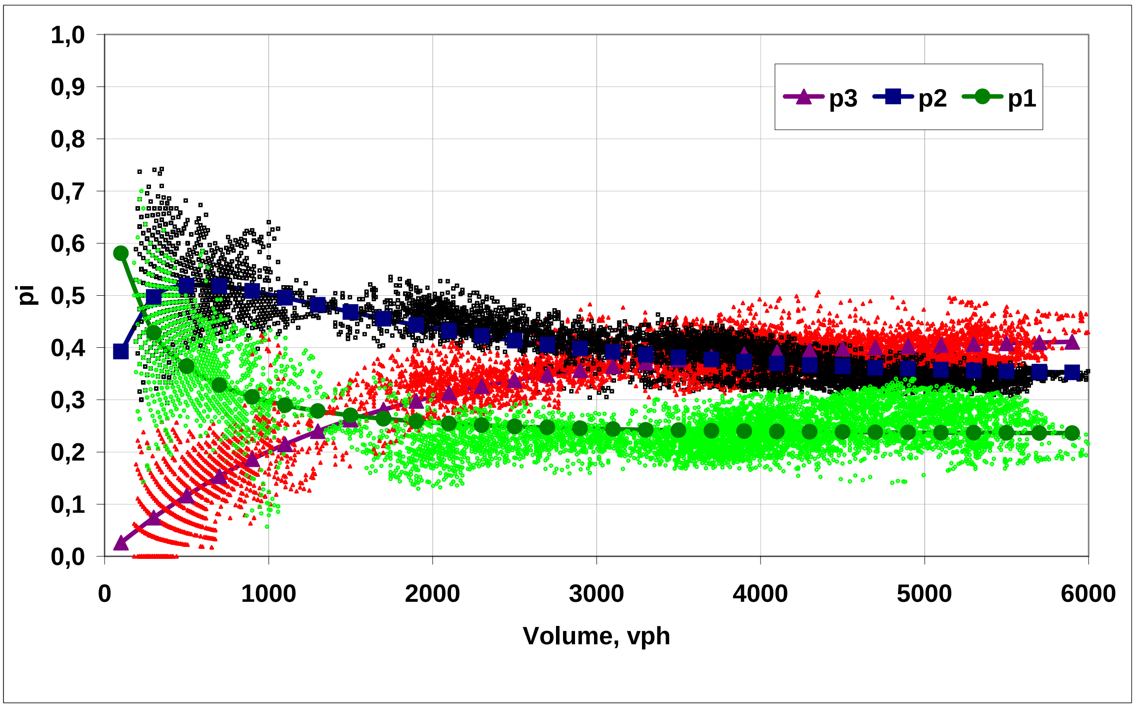
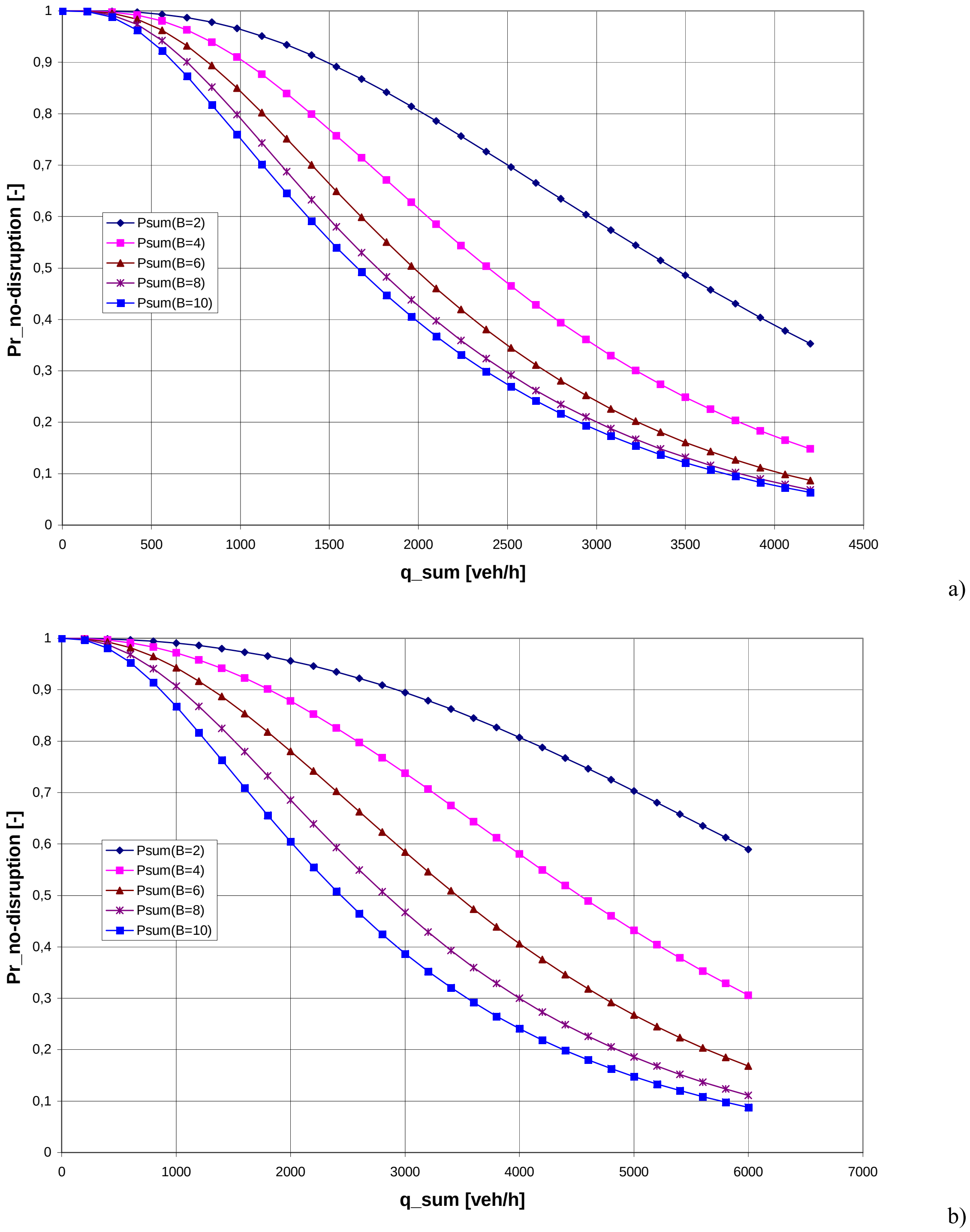


b)

Figure 2 - Lane volume distribution for typical freeways in North America: a) two-lane freeway (Regression: eq.(5), 15-min-Intervals), b) three-lane freeway (Regression: eq.(6), 20-sec-Intervals)



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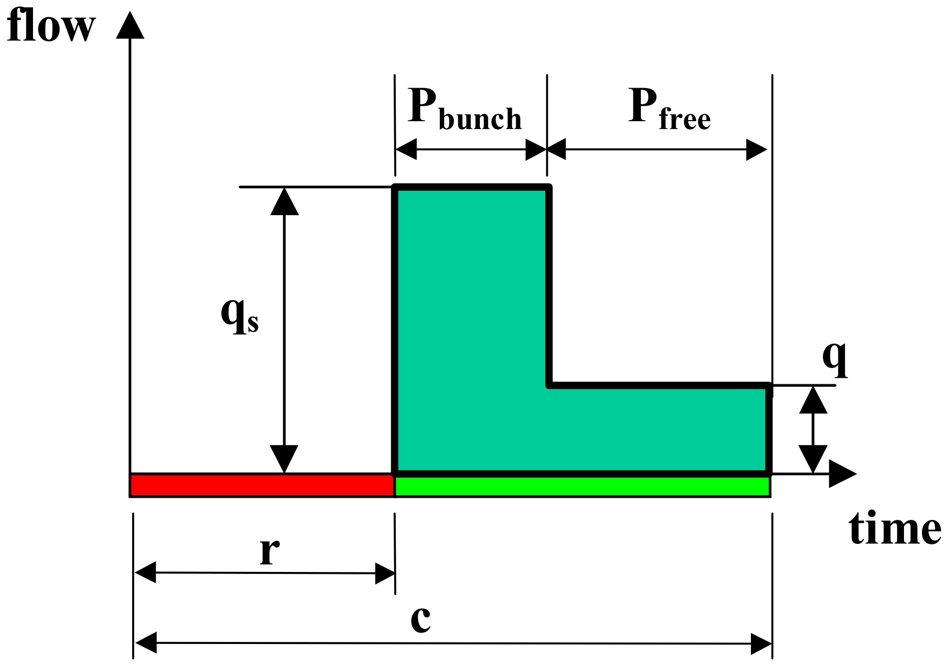

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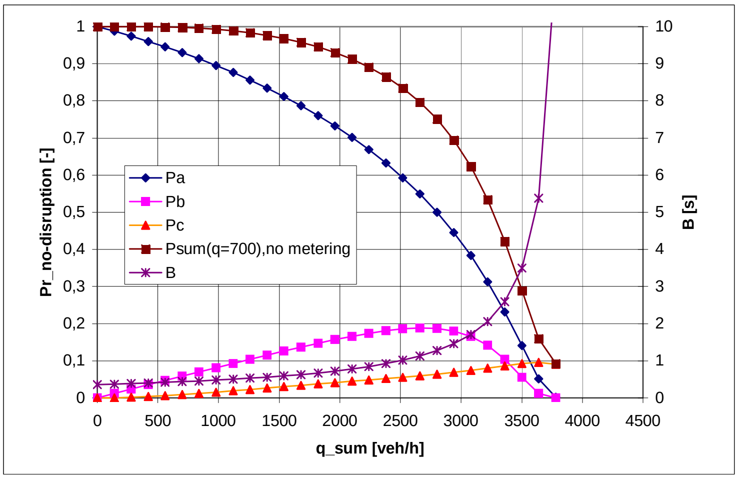

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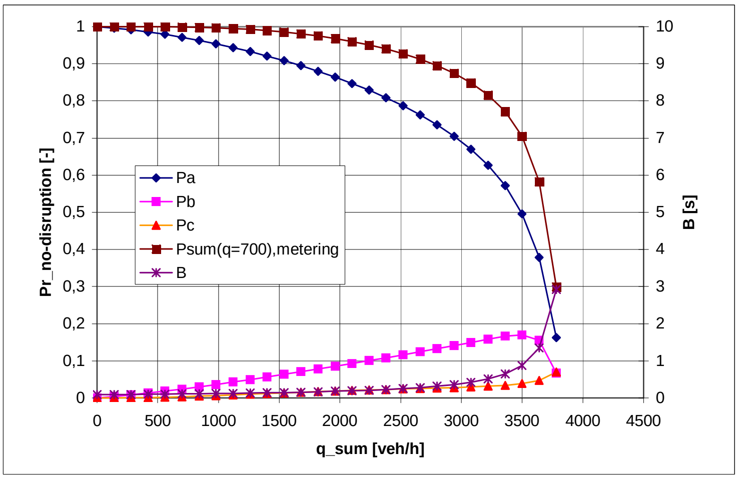

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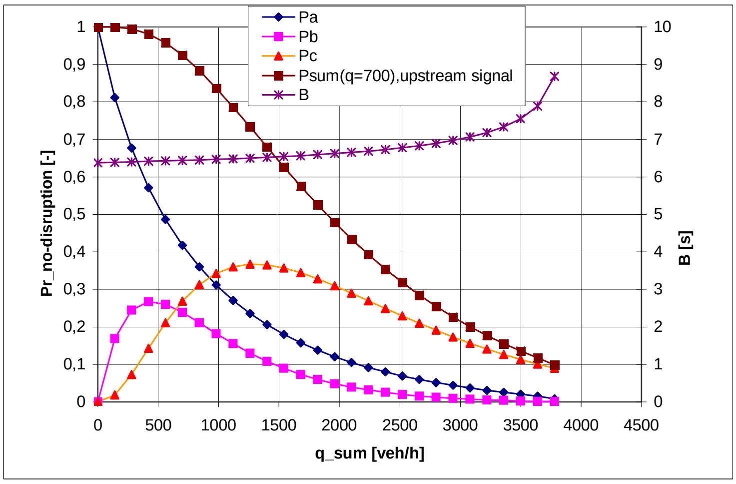
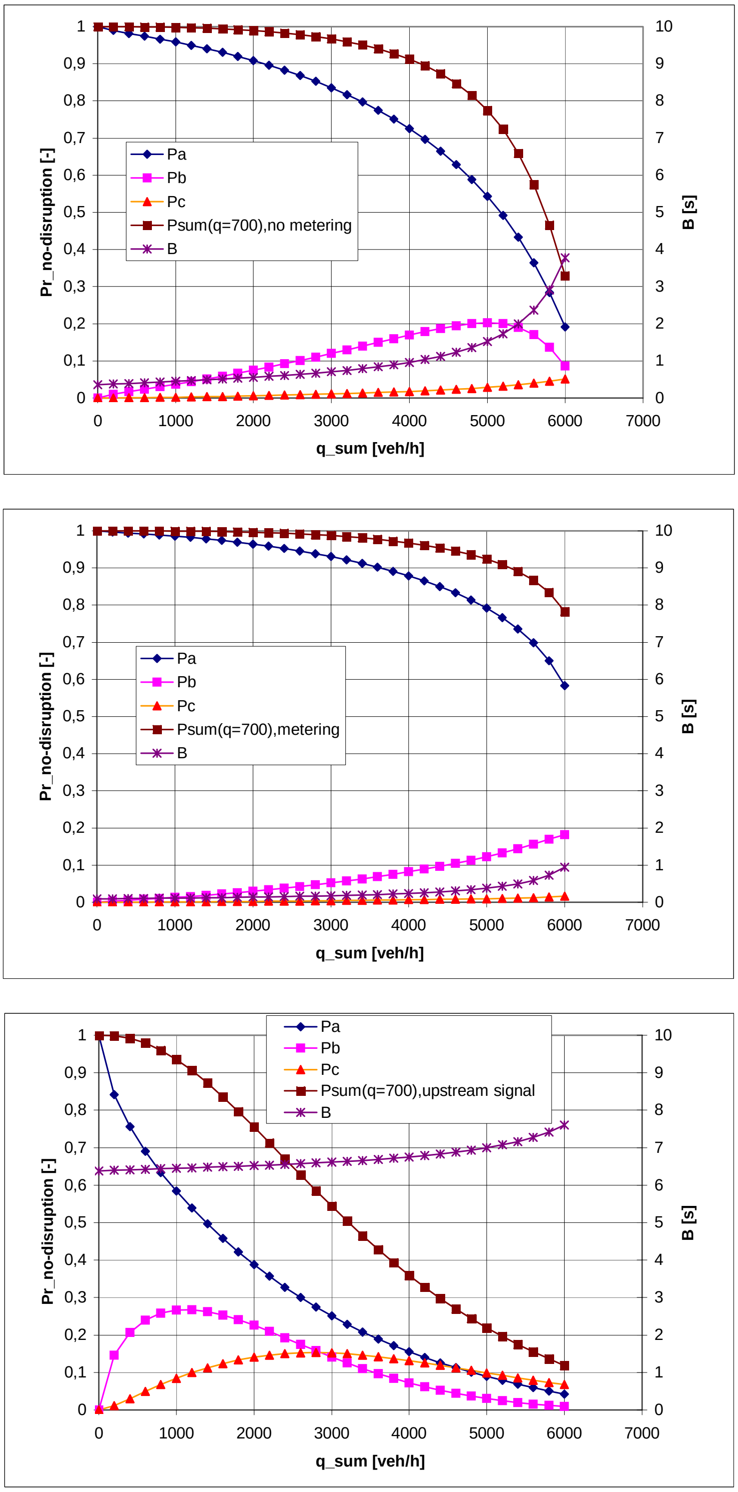


c)

Figure 5 - Average length (B) of time for accommodating the platoon and probability of no-disruption (P_{ND}) for a typical two-lane freeway in North America: a) randomly arriving ramp traffic, b) for uniformly (equivalent to ramp metering) arriving ramp traffic, c) ramp traffic arriving in platoons (e.g., different size of platoons resulted from an upstream signal)



a)

b)

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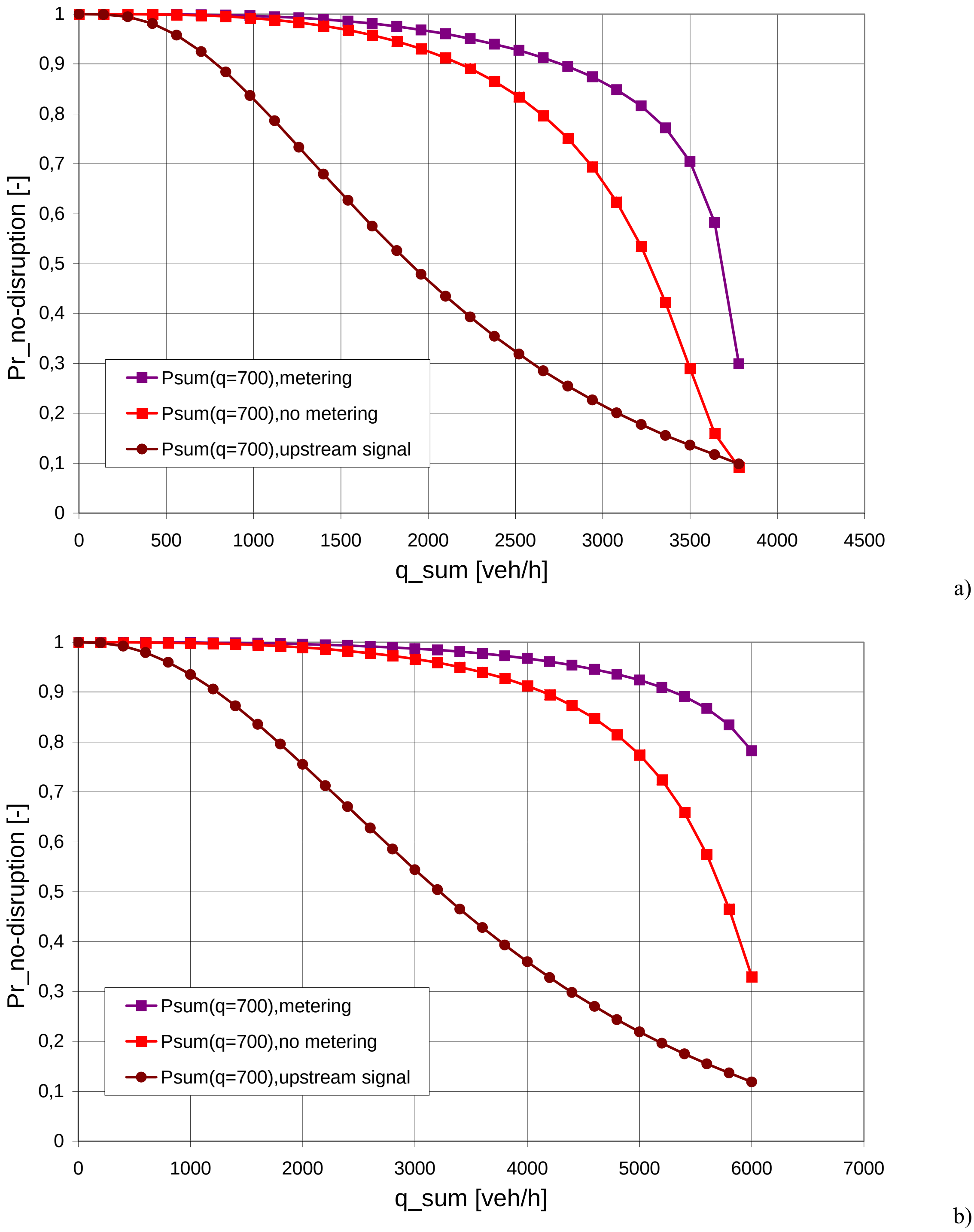
Figure 6 - Average length (B) of time for accommodating the platoon and probability of no-disruption (P_{ND}) for a typical three-lane freeway in North America: a) randomly arriving ramp traffic, b) for uniformly (equivalent to ramp metering) arriving ramp traffic, c) ramp traffic arriving in platoons (e.g., different size of platoons resulted from an upstream signal)



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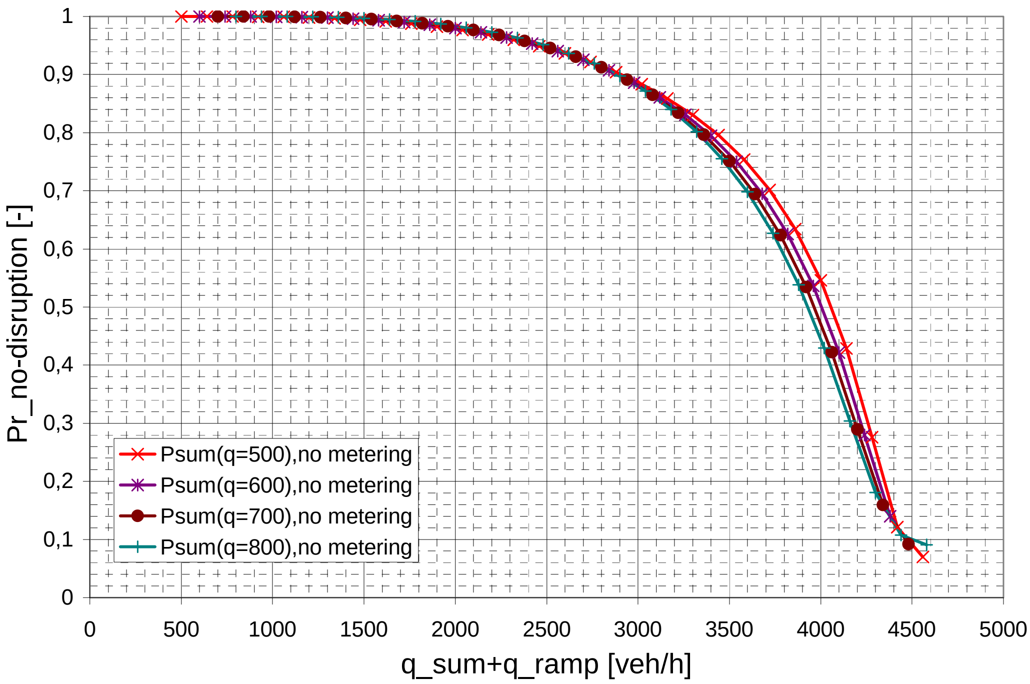

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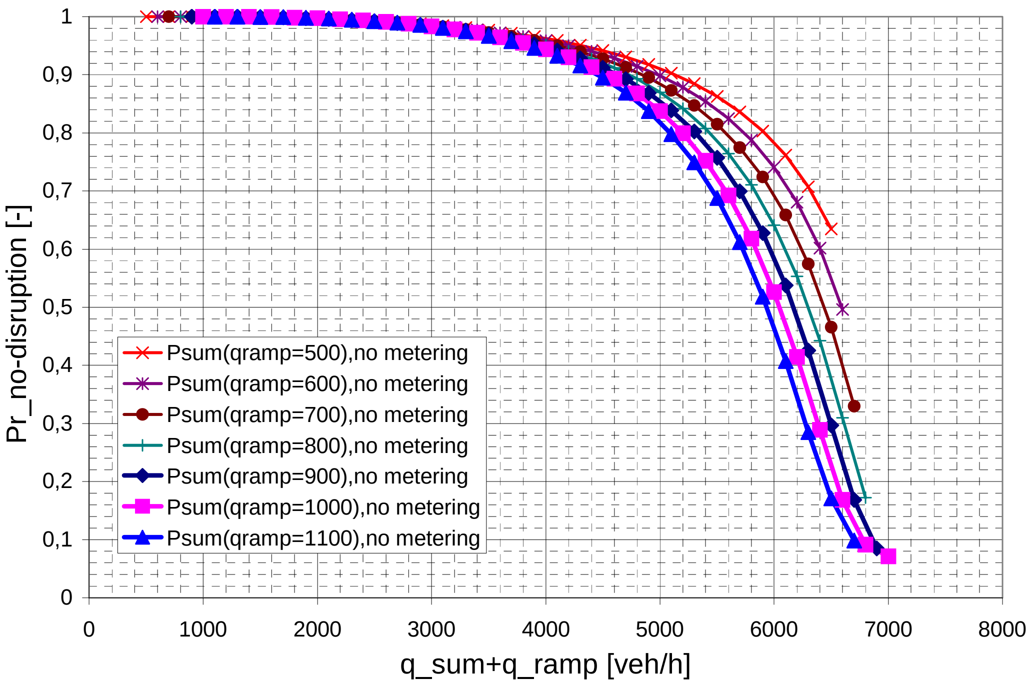

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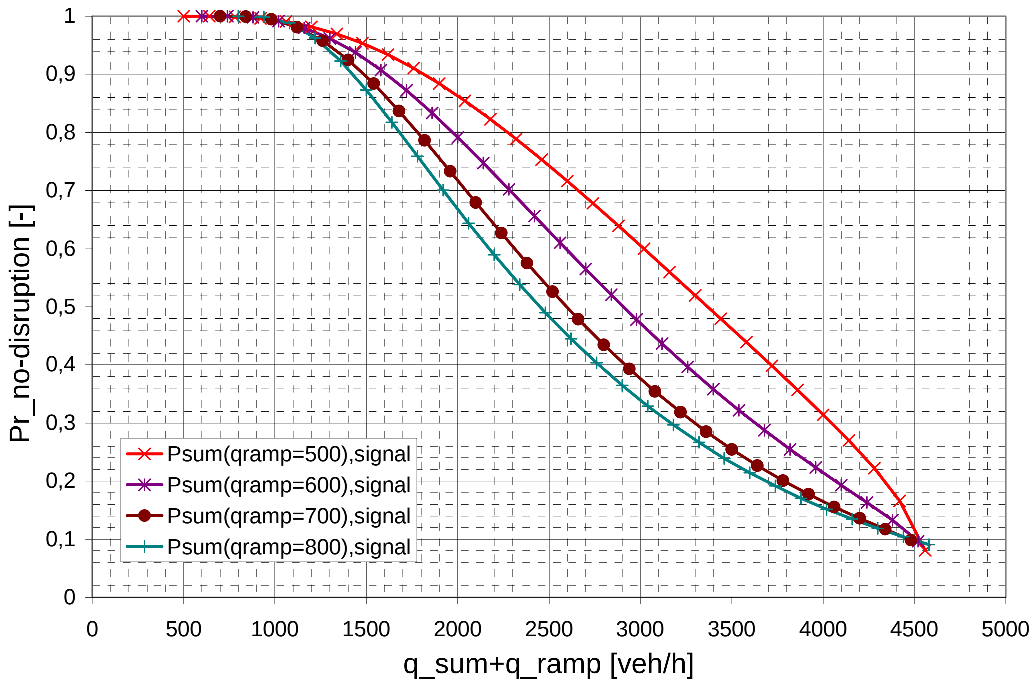

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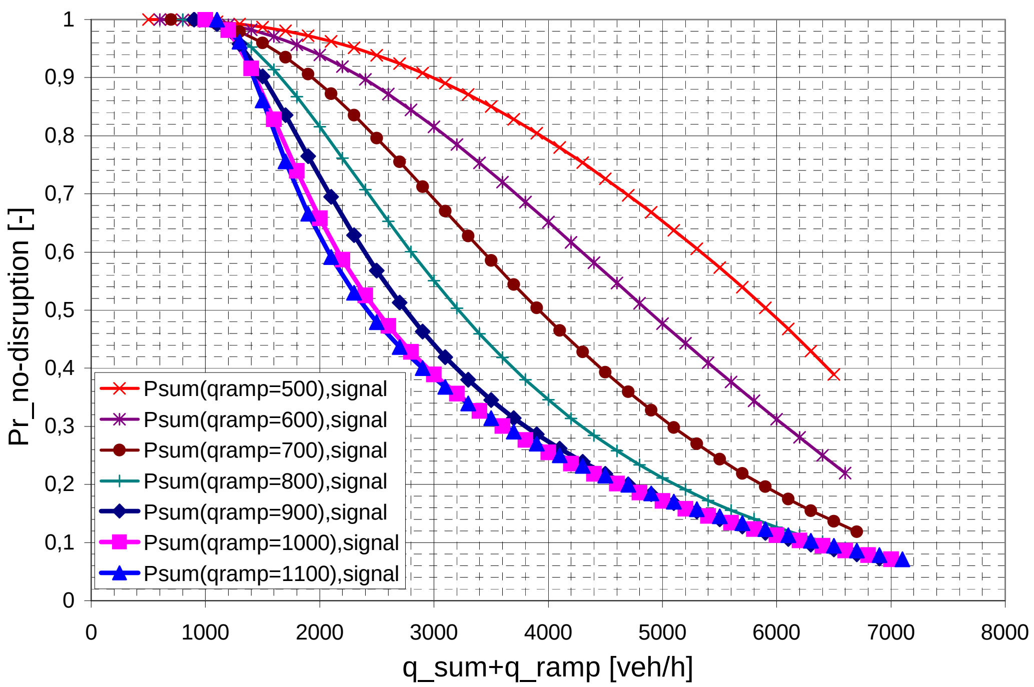

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