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Analysis of Capacity Enhancement and Disruption Probability for Freeway Ramp Controls Based on Gap-Acceptance and Queuing Models

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ABSTRACT

Gap-acceptance and queuing theory based models are proposed to model the effect of ramp controls on freeway operations. The models are developed for three types of ramp control and traffic flow patterns, namely the uniform arrival with ramp-metering, the random arrival without ramp-metering, and platoon arrival without ramp-metering but with an upstream signalized intersection. One of the applications of these models is to address, from the theoretical point of view, one of the practical issues regarding freeway ramp-metering thresholds and the corresponding disruption probability. Studies have shown that ramp-metering is effective in reducing vehicle delays only when freeway traffic flow rate reaches a certain flow threshold level. When freeway traffic is low, there will be enough gaps in the freeway flow to accommodate the ramp flow, even when ramp traffic enters the freeway in platoons. The presented models take into account the effect of platoon size resulted from the three ramp controls and arrival flow patterns. The study results clearly indicate that more significant disruption on freeway operations exist due to large platoon arrivals resulting from an upstream traffic signal, compared to when traffic arrives randomly or uniformly. The models are also applied to provide quantitative assessments from the perspectives of freeway capacity, indicating that ramp-metering results in increased freeway capacity and decreased disruption probability.

Keywords: Ramp-metering Threshold, Gap Acceptance, Freeway Operations, Disturbance Probability, Disruption Probability

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INTRODUCTION

Freeway entrance ramps are likely bottleneck locations where most disruptions on freeway operations occur (*1, 2, 3, 4, 5*). Different traffic flow patterns exist at freeway ramps depending on the type of ramp control and location. When ramp-metering is installed at a freeway ramp, it creates nearly uniform vehicle entries to the freeway mainline, thus resulting in fewer disruptions to freeway mainline (*6*). When a ramp is far from an upstream signal and no ramp-metering is installed, vehicles enter the freeway mainline in a nearly random fashion. When the ramp is near an upstream signal and no ramp-metering is installed, traffic tends to enter the freeway mainline in platoons, which results in the highest level of disruption to freeway mainline operations.

Although freeway ramp-metering has been used worldwide as an effective means of improving safety and operations general field studies have indicated that ramp-metering is effective only when freeway traffic flow rate reaches a certain threshold level. When freeway traffic is low, there will be enough gaps in the freeway flow to accommodate the ramp flow, even when ramp traffic enters the freeway in platoons. In the practice, ramp-metering threshold values are typically determined based on empirical studies. This paper develops a new theoretical model based on gap-acceptance and queuing theory that addresses the impacts of the three types of ramp control and traffic arrival patterns, where the model can be used to determine ramp-metering threshold values.

Figure 1 illustrates some of the merging situations of how the ramp traffic can be accommodated without significant disruption on the mainline traffic. These situations involve no more than one vehicle making a lane change or slowing down, which are defined as no disruption events later in this paper. The traffic flow rates on both the freeway mainline and the ramp that would result in no-disruption of freeway operations are defined as the flow thresholds for ramp-metering. Traffic flow rates below the threshold values are not necessary for initiating ramp-metering operations because normal freeway operations would maintain. Gap-acceptance based models to determine the flow threshold values are presented in the following sections of this paper.

GAP DISTRIBUTION AND LANE FLOW DISTRIBUTION

In general, the gaps in a traffic stream follow a distribution function $f(t) = f(t, q)$, where t is the length of the gap and q is the traffic flow rate. For example, the probability density function for partially bunched traffic conditions can be given by the Cowan's M3 model (*7*) shown in Equation (1) in the cumulative form:

$$F(t) = \begin{cases} 1-\alpha\cdot\exp(-\lambda\cdot(t-\Delta)) & for\ \ t \ge \Delta \\ 0 & for\ \ t < \Delta \end{cases} \qquad (-) \qquad (1)$$

where: t is the sample gap (s)
Δ is the minimum gap within bunches (s)
α is the proportion of non-bunched vehicles (-)

$$\lambda = \frac{\alpha \cdot q}{1 - \Delta \cdot q} \quad \text{(veh/s) (2)}$$

q is the stream flow rate (veh/s)
λ is the flow rate within the bunched vehicles (veh/s)

Usually we can use $\alpha = 1 - q \cdot \Delta$ and $\lambda = q$ for normal traffic conditions without impedance of traffic signals (cf. *8*). Thus, we should also use these parameters for freeway traffic flow. In this case Equation (1) yields

$$F(t) = \begin{cases} 1 - (1 - q \cdot \Delta) \cdot \exp(-q \cdot (t - \Delta)) & \text{for } t \geq \Delta \\ 0 & \text{for } t < \Delta \end{cases} \quad \text{(-)} \quad (3)$$

Given the total flow rate of a freeway q_{sum}, the proportion of traffic flow rates, p_1, p_2,..., on different traffic lanes 1, 2... can be calculated based on Equation (4):

$$\begin{cases} p_1 = q_1 / q_{sum} \\ p_2 = q_2 / q_{sum} \\ \ldots \\ p_1 + p_2 + \ldots = 1 \end{cases} \quad .. \quad \text{(-)} \quad (4)$$

In general the lane flow distribution can also be modeled by gap-acceptance theory (*9*). In the practice, regression models are common for describing the lane flow distribution. Of course the lane flow distribution is highly variable from segment to segment and different from country to country because of different traffic behaviors and regulations. For standard motorways, the country-related lane flow distribution should be used for further calculations.

In the North American countries, the regulation of "Keep in Lane" is common. Based on a sample data set collected in Canada and the U.S., Equations (5) and (6) can be established as sample regression functions to represent the lane flow distribution on a 2-lane (each direction) freeway and on a 3-lane (each direction) freeway.

$$\begin{cases} p_1 = 0.332 + (1 - 0.332) \cdot \exp(-1.440 \cdot q_{sum}^{0.5}) \\ p_2 = 1 - p_1 \end{cases} \quad \text{(-)} \quad (5)$$

$$\begin{cases} p_1 = 0.235 + (1 - 0.235) \cdot \exp(-4.758 \cdot q_{sum}^{0.5}) \\ p_3 = 0.420 \cdot (1 - \exp(-2.340 \cdot q_{sum})) \\ p_2 = 1 - p_1 - p_3 \end{cases} \quad \text{(-)} \quad (6)$$

The lane flow distribution from Equations (5) and (6) are illustrated in Figure 2. If the traffic flow rates of different lanes, q_1, q_2, ..., can be obtained directly from field measurements, they should be used for further calculations.

Once the proportion of traffic flow rates, p_1, p_2, ..., are given, the traffic flow rates of different lanes, q_1, q_2, ..., can be calculated by:

$$\begin{cases} q_1 = p_1 \cdot q_{sum} \\ q_2 = p_2 \cdot q_{sum} \\ \ldots \\ \ldots \end{cases} \qquad \text{(veh/s)} \quad (7)$$

THE CAPACITY OF THE ON-RAMP LANE

The capacity for a freeway on-ramp lane is usually calculated using gap-acceptance models (*10, 11, 12, 13, 14*). For example, the capacity, C_{ramp} can be expressed by

$$C_{ramp} = \frac{\alpha_1}{t_{f,ramp}} \cdot \exp\left(-\lambda_1 \cdot (t_{c,ramp} - \frac{t_{f,ramp}}{2} - \Delta_1) \right) \qquad \text{(veh/h) or (veh/s)} \quad (8)$$

where $t_{c,ramp}$ is the critical gap for merging from the on-ramp lane to lane 1, $t_{f,ramp}$ is the follow-up time (cf. *9*).

To take into account the effect of the length of the acceleration lane, a correction factor must be applied to the critical gap, $t_{c,\mathrm{ramp}}$. In general, the value of $t_{c,\mathrm{ramp}}$ can be modified according to the length of the acceleration lane and to the possible accelerating rate of the vehicle. The value of $t_{c,\mathrm{ramp}}$ decreases with increasing length of the acceleration lane.

In the existing highway capacity manuals (*15*, *16*), the capacity of the on-ramp is normally expressed by a linear function. The HCM-formula is given by

$$C_{ramp} = 4600 - (q_1 + q_2) \qquad \text{(veh/h)} \quad (9)$$

PROBABILITY OF NO-DISRUPTION IN MAJOR FLOW DUE TO ON-RAMP TRAFFIC

We first define a parameter, B, the required mainline gap to accommodate a particular size of traffic platoon from the ramp. For example, for a platoon of size N, $B = (N+1) \cdot h$, where h is the minimum headway in the on-ramp stream. Considering the merge process as a floating queuing system, the average queue length in the system is $L = (N + x)$ and $B = L \cdot h = (N + x) \cdot h$, where N is the average queue length of the queuing system excluding the vehicle in the counter (= average size of platoon), x is the degree of saturation for the on-ramp. Note, for $q = 0$ is $B = 0$.

The probabilities of no-disruption to freeway mainline traffic are calculated for the three cases illustrated in Figure 1.

Case a - Gap in the merge lane, h_i, is no less than $B = (N + x) \cdot h$ (cf. Figure 1, a)

In this case, the probability of no-disruption $P_{ND,a}$ is equal to the probability that the length of a gap in the major stream is larger than B, i.e., the probability of no-disruption is simply equal to the probability for $t \geq B$. Thus,

$$P_{ND,a} = \Pr(t \geq B) \qquad (\text{-}) \quad (10)$$

From Equation (1) we have

$$P_{ND,a} = 1 - F_t(t = B) = \alpha_1 \cdot \exp\left[-\lambda_1 \cdot (B - \Delta_1)\right] \qquad (\text{-}) \qquad (11)$$

where $F_t(t{=}B)$ is the probability distribution function (cumulative) of gap t with a length B.

Case b - Gap in the first lane is Δ*t* shorter than required. The freeway vehicle can slow down by Δ*t* without affecting the following vehicle (cf. Figure 1, b)

In this case we are looking for the probability of $t^1 + t^2 > B + \Delta_1$ under the condition of $t^1 \le B$, where t^1 and t^2 are two consecutive gaps in the freeway mainline stream in lane 1. Normally, in free flow traffic, we can assume that the probability of $t^1 + t^2 > B + \Delta_1$ and the probability of $t^1 \le B$ are independent of each other for freeway headway distributions. Thus, we have

$$\begin{aligned} P_{ND,b} &= \Pr(t^1 + t^2 > B + \Delta_1 \mid t^1 \le B) \\ &= \Pr(t^1 + t^2 > B + \Delta_1) \cdot \Pr(t^1 \le B) \end{aligned} \qquad (\text{-}) \qquad (12)$$

Since the probability of $t^1 + t^2$ obeys the shifted Erlang distribution, we have (9)

$$F_{t^1+t^2}(t) = 1 - \alpha_1 \cdot \exp(-2 \cdot \lambda_1 \cdot (t - \Delta_1)) \cdot [1 + 2 \cdot \lambda_1 \cdot (t - \Delta_1)] \qquad (\text{-}) \qquad (13)$$

Combining with Equation (1) yields

$$\begin{aligned} P_{ND,b} &= \Pr(t^1 + t^2 > B + \Delta_1) \cdot \Pr(t^1 \le B) \\ &= [1 - F_{t^1+t^2}(B + \Delta_1)] \cdot F_{t^1}(B) \\ &= \alpha_1 \cdot \exp(-2 \cdot \lambda_1 \cdot (B - \Delta_1 + \Delta_1)) \cdot [1 + 2 \cdot \lambda_1 \cdot (B - \Delta_1 + \Delta_1)] \\ &\quad \cdot [1 - \alpha_1 \cdot \exp(-\lambda_1 \cdot (B - \Delta_1))] \\ &= \alpha_1 \cdot \exp(-2 \cdot \lambda_1 \cdot B) \cdot \left(1 + 2 \cdot \lambda_1 \cdot B\right) \cdot [1 - \alpha_1 \cdot \exp(-\lambda_1 \cdot (B - \Delta_1))] \end{aligned} \qquad (\text{-}) \qquad (14)$$

Case c - Gap in the first lane is smaller than required and the freeway vehicle cannot slow down without affecting the following vehicle. But the freeway vehicle can make a lane change to the adjacent lane (cf. Figure 1, c)

In this case, the probability of no-disruption is,

$$\begin{aligned} P_{ND,c} &= \Pr(t_1 < B) \cdot \Pr(t^1 + t^2 < B) \cdot \Pr(t_2 > t_{c,2}) \\ &= F_{t_1}(B) \cdot F_{t^1+t^2}(B + \Delta_1) \cdot [1 - F_{t_2}(t_{c,2})] \\ &= [1 - \alpha_1 \cdot \exp(-\lambda_1 \cdot (B - \Delta_1))] \cdot [1 - \alpha_1 \cdot \exp(-2 \cdot \lambda_1 \cdot B) \cdot \left(1 + 2 \cdot \lambda_1 \cdot B\right)] \\ &\quad \cdot \alpha_2 \cdot \exp(-\lambda_2 \cdot (t_{c,2} - \Delta_2)) \end{aligned}$$

$$(\text{-}) \qquad (15)$$

That total probability of no-disruption is then

$$P_{ND} = P_{ND,\mathrm{a}} + P_{ND,\mathrm{b}} + P_{ND,\mathrm{c}} \qquad (\text{-}) \qquad (16)$$

where Δ_1 is the minimum headway within bunches on lane 1, $t_{c,2}$ is the critical gap for changing from lane 1 to lane 2, Δ_2 is the minimum headway within bunches on lane 2. The lanes are numbered from the shoulder to the median (cf. Figure 1). For further calculation, parameters Δ_1=1.2s, $t_{c,2}$=4s, and Δ_2=1s are used.

In Figure 3, the total probability of no-disruption ($P_{sum} = P_{ND}$) as a function of the average length (B) of time for accommodating the platoon is illustrated both for a two-lane freeway and a three-lane freeway under the sample traffic conditions in North America. It can be recognized, that the total probability of no-disruption decreases with increasing values of B.

CALCULATIONS ON PARAMETER *B* VALUES

Using Equations (11), (14), (15) and (16), the probability of no-disruption can be obtained if the required length of time (B) for accommodating the platoon is known. Because B is a function of platoon size, which is related to the type of ramp controls and ramp traffic arrival patterns, we need first to determine the mean platoon size based on ramp conditions. In general, the size of the platoon can be achieved using a suitable queuing theory.

According to the Pollaczek - Khinchin formula (cf. *17*), the average number of customers L in a M/G/1 queuing system (waiting or in service)

$$L_{M/G/1} = x + N_{M/G/1} = x + \frac{x^2 + q^2\sigma_b^2}{2(1-x)} \qquad \text{(-)} \qquad (17)$$

with x = degree of saturation of the on-ramp, q = flow rate of the on-ramp, and σ_b^2 = variance of the service time for the on-ramp. More general, according to the heavy-traffic approximation (cf. *17*), the average number of customers L in a G/G/1 queuing system can be estimated by

$$L_{G/G/1} \approx x + N_{G/G/1} = x + \frac{q^2(\sigma_a^2 + \sigma_b^2)}{2(1-x)} \qquad \text{(-)} \qquad (18)$$

for x => 1 with σ_a^2 = variance of the headway in the on-ramp flow. Because we are mostly interested in the area of $x \approx 1$ for capacity analysis, this presumption is not critical for our derivation.

If the variance of the service time σ_b^2 and the variance of the headway in the on-ramp flow σ_a^2 are known, the number of customers L in the system and thus the average length of the platoon B can achieved.

Uncontrolled Ramp – Random Arrival

When a ramp is uncontrolled and is far from upstream signals, traffic enters the freeway randomly. The system can be represented by an M/M/1 - queuing system for simplification. Here, the average queuing length in the on-ramp stream is given by

$$L_{M/M/1} = x + N_{M/M/1} = x + \frac{2x^2}{2(1-x)} = \frac{x}{1-x} \qquad \text{(-)} \qquad (19)$$

Thus, we have the average length of time for accommodating the platoon

$$B_2 = \frac{x \cdot h}{1-x} \qquad \text{(s)} \qquad (20)$$

Ramp with Metering – Uniform Arrival

When a ramp has ramp-metering in operation, the output of the metering is the input of the queuing system (ramp). This input is in this case a uniform input. The system can be then classified as a D/M/1-queuing system for simplification. Despite of the metering, there still will be platoon before the merge point if the capacity of the merge point is relatively low. For a D/M/1-queuing system, the average queuing length in the on-ramp stream (before the merge point) is given by

$$L = x + N_{D/M/1} \approx x + \frac{x^2}{2(1-x)} = \frac{x(2-x)}{2(1-x)} = \frac{x}{1-x}(1-\frac{x}{2}) \qquad (-) \qquad (21)$$

where x is the degree of saturation of the on-ramp. Thus, we have the average length of time for accommodating the platoon (the headway in the queuing platoon remains h)

$$B_1 = \frac{x}{1-x}(1-\frac{x}{2}) \cdot h \qquad (s) \qquad (22)$$

We can recognize that B_2 is always smaller than B_1. That is, the average length of platoon can be reduced by ramp-metering.

Ramp with Upstream Signal – Large Platoon Arrival

When a ramp is located close to an upstream traffic signal, traffic enters the freeway in bunches (i.e., large platoons). In this case, the average length of time for accommodating the platoon can be calculated by (cf. Figure 4)

$$B_3 \approx [m \cdot P_{bunch} + N \cdot P_{free} + x] \cdot h \qquad (s) \qquad (23)$$

where P_{bunch} is the proportion of bunched time, m is the number of bunched traffic flow, and P_{free} is the proportion of time for free traffic. N can be calculated from an M/M/1 queuing system by

$$N_{M/M/1} = \frac{x^2}{1-x} \qquad (-) \qquad (24)$$

For normal signalized, not coordinated intersections we have

$$P_{free} = 1 - \frac{r/c}{1-q/q_s} \qquad (-) \qquad (25)$$

$$P_{bunch} = \frac{r/c}{1-q/q_s} - \frac{r}{c} = \frac{r}{c}\left(\frac{q/q_s}{1-q/q_s}\right) \qquad (-) \qquad (26)$$

and

$$m = \frac{r/c}{1/q - 1/q_s} \qquad (-) \qquad (27)$$

where q_s is the saturation flow rate, c is the cycle length, and r is the red time.

Here, only the case for one single movement is considered. However, it can be extended to model a more general case with more than one feeding traffic movement based on the same principle.

MODEL APPLICATIONS

For all the three cases mentioned in the previous section, the freeway flow threshold can be obtained implicitly from Equation (16) based on certain P_{ND} values. It is noted that these threshold values are determined based on the conditions of no-disruption to freeway traffic as defined early in this paper. They are not exactly the flow threshold for ramp-metering applications. In order to derive the threshold for ramp-metering applications, the relationship between freeway breakdown and the probability of no-disruption must be established. Such a relationship needs to be verified based on field studies.

The freeway flow threshold is a function of the on-ramp flow rate. The threshold can be easily obtained from the graphs developed below. The graphs are developed based on the sample traffic characteristics in North America. In General, the value of the flow threshold depends on the major flow rate on the freeway, the flow rate of the on-ramp and the predefined probability P_{ND} for no-disruption. To obtain the practical flow threshold values for ramp-metering, these parameters must be calibrated based on field conditions.

Two-lane freeway

Using the following parameter values and based on the sample traffic characteristics in North America: $q_{ramp} = 700$ veh/h, $c = 60$s and $r = 30$s, p_1 and p_2 from Equation (5) and C_{ramp} from Equation (9), the average length (B) of time for accommodating the platoon and the probability of no-disruption (P_{ND}) for different on-ramp traffic conditions are depicted in Figure 5.

If a required probability of no-disruption $P_{ND} = 0.8$ is predefined, we have a freeway flow threshold of 2648 ($q_{ramp} + q_{sum} = 3348$) veh/h for randomly arriving ramp traffic (Figure 5, a), 3274 ($q_{ramp} + q_{sum} = 3974$) veh/h for uniformly (equivalent to ramp-metering) arriving ramp traffic (Figure 5, b), and only 1082 ($q_{ramp} + q_{sum} = 1782$) veh/h for ramp traffic arriving in platoons due to upstream traffic signals (Figure 5, c).

Three-lane freeway

Using the same parameter values as in the case of two-lane freeway (except for p_1, p_2, and p_3 from Equation (6)), the average length (B) of time for accommodating the platoon and the probability of no-disruption (P_{ND}) for different on-ramp traffic conditions are depicted in Figure 6.

Again, if a required probability of no-disruption $P_{ND} = 0.8$ is predefined, we have a freeway flow threshold of 4875 ($q_{ramp} + q_{sum} = 5575$) veh/h for randomly arriving ramp traffic (Figure 6, a) 5932 ($q_{ramp} + q_{sum} = 6632$) veh/h for uniformly (equivalent to ramp-metering) arriving ramp traffic (Figure 6, b), and only 1780 ($q_{ramp} + q_{sum} = 2480$) veh/h for ramp traffic arriving in platoons due to upstream traffic signals (Figure 6, c).

METERING THRESHOLD

Metering threshold for random on-ramp flow

In the United States, different flow thresholds have been used for ramp-meriting on freeways. For example, Wisconsin uses the flow-to-capacity (*q/C)* ratio 0.7 for urban and 0.6 - 0.65 for rural areas. The capacity *C* is based on HCM, the flow rate *q* includes both the on-ramp flow rate and the mainline flow rate. Illinois uses occupancy of 11.7% (upstream of meter). Denver uses the flow rate, occupancy, and speed, whichever is controlling, and the values are measured downstream of meter and must last for 3 consecutive minutes. The threshold for flow rate is 1900 veh/h/ln (average of all lanes), for occupancy is 20%, and for speed is 35 mph. In general, the downstream flow rate or the *q/C* ratio is considered to be suitable parameters for metering control. The occupancy can be considered as a function of the flow rate *q*.

From the proposed method in this paper, the probability of no-disruption P_{ND} as a function of the total downstream flow rate $q_{sum}+q_{ramp}$ can be estimated. In Figure 7, this functional relationship is illustrated. In case of a two-lane freeway (cf. Figure 7, a), the relationship is nearly independent of the individual on-ramp flow rate q_{ramp}. Thus, for any on-ramp flow rate, the downstream flow threshold can be simply defined as a function of the probability of no-disruption. For example, using a predefined probability of no-disruption $P_{ND} = 0.8$, the downstream flow threshold is ca. 3400 veh/h. This correspondents to a lane flow rate of 1700 veh/h/ln and a *q/C* ratio of 0.74 (for $C = 2300$ veh/h/ln). Unfortunately, for a three-lane freeway, the independence between P_{ND} and q_{ramp} does not exist (cf. Figure 7, b). Thus, for different on-ramp flow rates, different downstream flow thresholds must be used if the constant value of P_{ND} is predefined. For example, with $P_{ND} = 0.8$, the downstream flow threshold varies from 5100 through 5900 veh/h or 1700 through 1967 veh/h/ln (q_{ramp} from 1100 to 500 veh/h). This correspondents to *q/C* ratios ranging between 0.74 and 0.85 (for $C = 2300$ veh/h/ln). On the other hand, if a downstream flow threshold of 1700 veh/h/ln is used, the value of the probability of no-disruption P_{ND} would vary from 0.8 through 0.91 (cf. Figure 7, b). For simplification, a downstream flow threshold of 1700 veh/h/ln can be used both for two-lane and three-lane freeway. In this case, the probability of no-disruption is always equal to or greater than 0.8.

Metering threshold for ramp traffic arriving in platoons (upstream signal)

The probability of no-disruption P_{ND} is dependent on the cycle time *c* and the red time *r* of the upstream signal. As an example for the sample North America conditions, the probability of no-disruption P_{ND} as a function of the total downstream flow rate $q_{sum}+q_{ramp}$ is illustrated in Figure 8 for $c = 60$s and $r = 30$s. It can be seen, that the independence between P_{ND} and q_{ramp} does not exist both for the two-lane and the three-lane freeway. With $P_{ND} = 0.8$, the downstream flow threshold varies from 1700 through 2300 veh/h/ln or 750 through 1150 veh/h/ln (q_{ramp} from 800 to 500 veh/h) for a two-lane freeway. For a three-lane freeway, the downstream flow threshold varies from 1600 through 3900 veh/h or 533 through 1300 veh/h/ln (q_{ramp} from 1100 to 500 veh/h).

Because the thresholds depend strongly on the upstream signal parameters, no general recommendations can be made for the metering control. For practical applications, Figure 3 (for sample North America conditions) can be used for estimating the downstream flow threshold if the average size of platoon *B* can be measured or estimated prior to the calculation.

CONCLUSIONS AND OUTLOOK

Using the gap-acceptance and queuing theory, the probability of no-disruption caused by on-ramp traffic can be evaluated in details. Equations (11), (14), (15), (22), (20), and (23) are the most critical elements for the developed models. From these equations, the probability of no-disruption for three different on-ramp traffic conditions of a) randomly arriving, b) uniformly arriving (equivalent to ramp metering), and c) arriving in platoons (e.g., different size of platoons resulted from an upstream signal) can be obtained. From these probabilities of no-disruption, the control thresholds of the freeway flow rate can be determined. Furthermore, these probabilities of no-disruption can also be associated to the bottleneck capacity and safety measures in the merge area. The results indicate that ramp-metering significantly enhances the probability of no-disruption and thus improve the operations and safety for freeways. Another important finding is that the effect of upstream signals on the merge operation can be positive or negative, depending on the traffic flow level on the freeway. For normal traffic conditions, thresholds of freeway flow rate for the case with ramp-metering control are lower than those with upstream signals and uncontrolled random arrivals.

Most importantly, the study delivers a theoretical framework for estimating the probability of no-disruption in the merge area on freeways, thus providing a theoretical basis for determining ramp-metering threshold values in practice. It is recommended that the models be calibrated based on country-specific conditions before being applied in the practice. Furthermore, for every particular traffic regulation, the capacity formula (cf. eq.(8)) and the formulae for calculation of the lane flow distribution must be carefully calibrated against more field measurements.

As a reference, Figure 7 is useful for estimating downstream flow thresholds for freeways in North America with random on-ramp flows. For on-ramp flow under other conditions (e.g. upstream signals), Figure 3 can be used in case that the average size of the platoon can be measured or estimated prior to the calculation.

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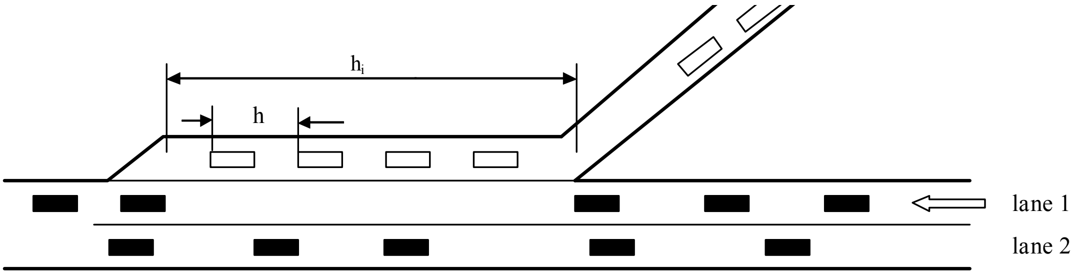
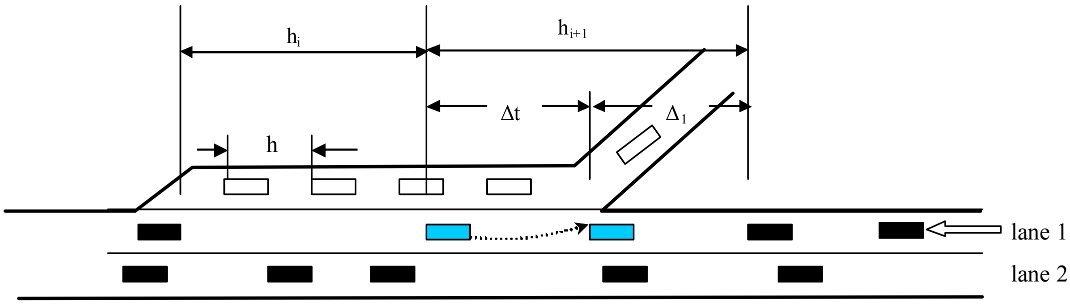
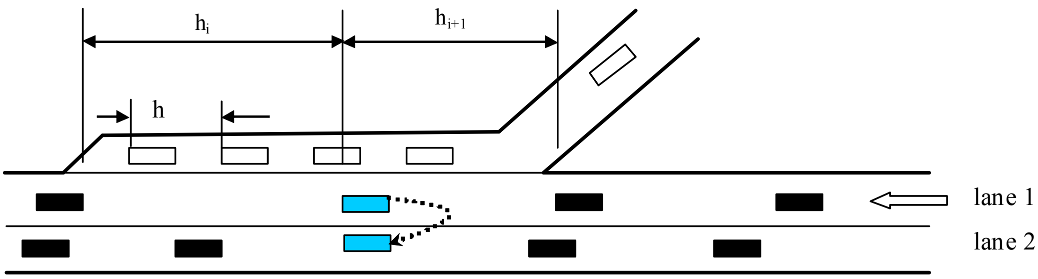


a) Platoon size n = 4; Gap in the merge lane, h_i is no less than $(n + 1)$ h

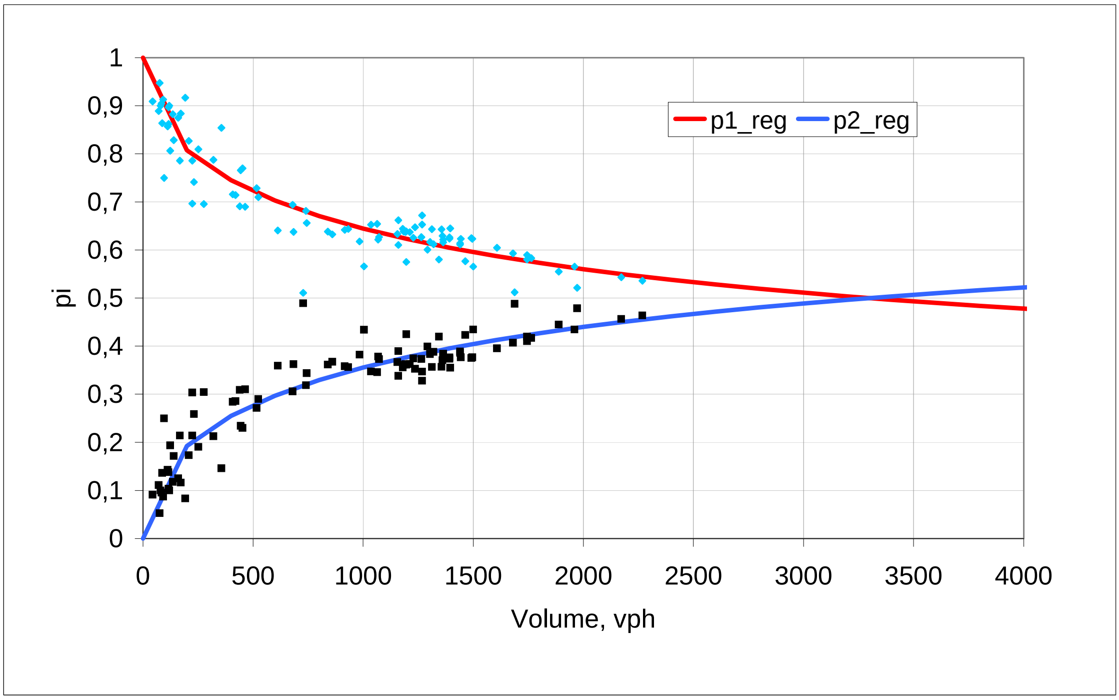


b) Gap in the first lane is Δt shorter than required. The freeway vehicle can slow down by Δt without affecting the following vehicle



c) Gap in the first lane is smaller than required. The freeway vehicle can make lane change to the adjacent lane.

FIGURE 1 - Three different cases of disruption in major stream (here we use *h*=gap=headway)



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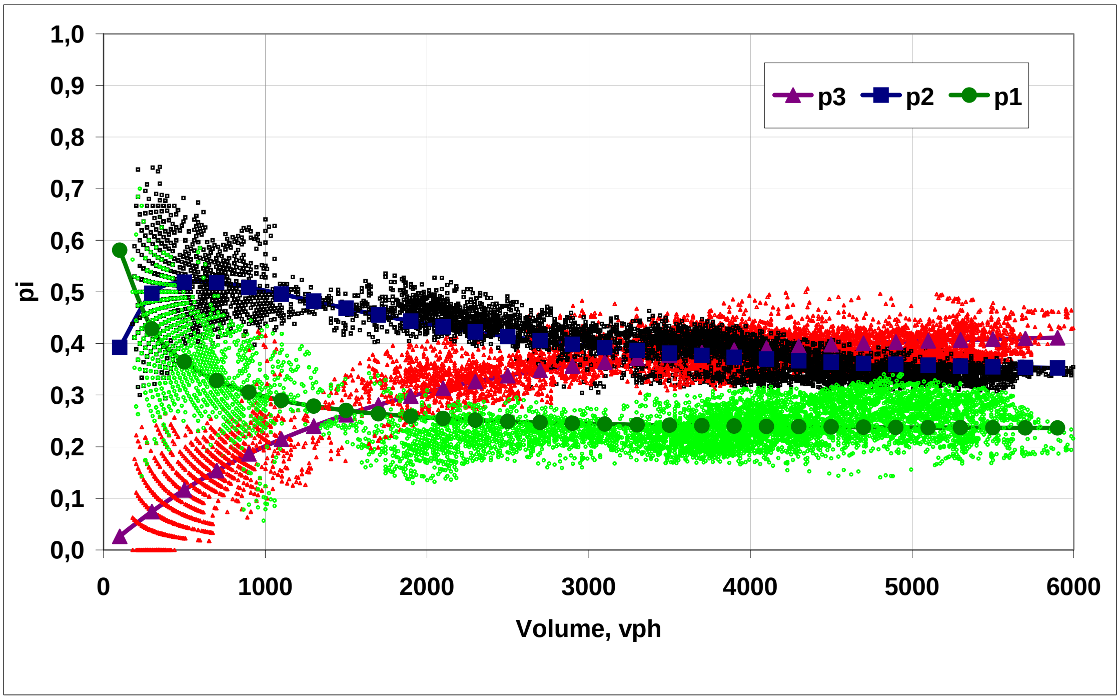
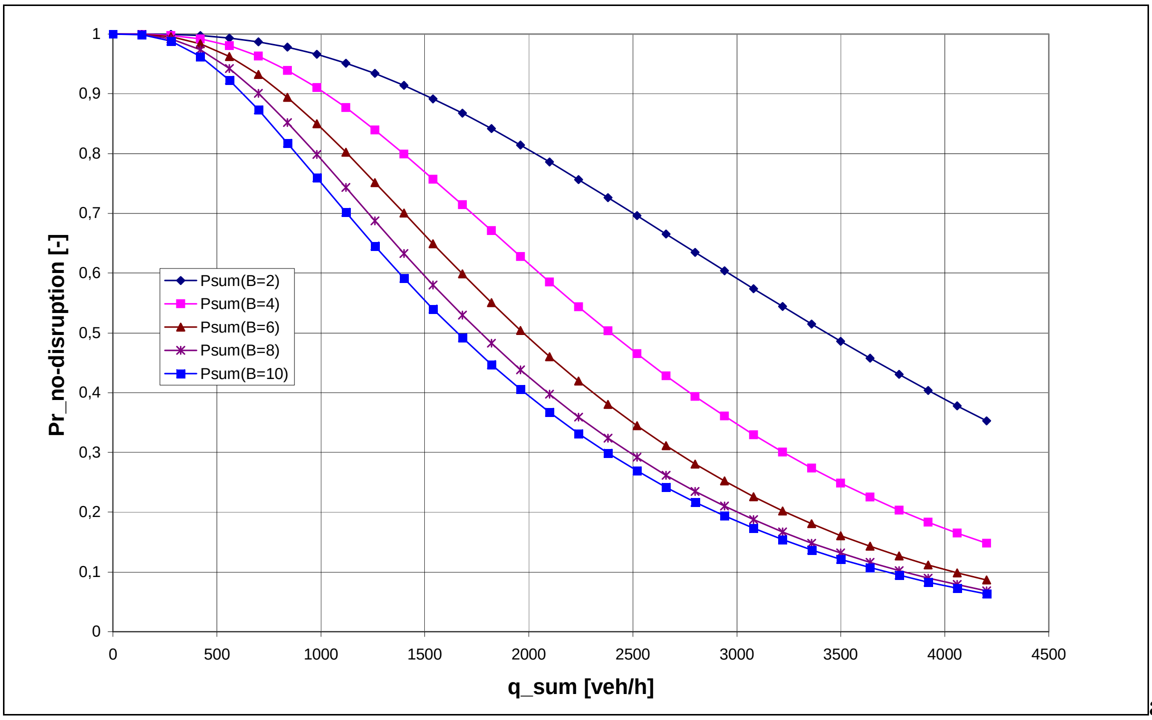


b)

FIGURE 2 - Lane flow (volume) distribution for typical freeways in North America: a) two-lane freeway (Regression: eq.(5), 15-min-Intervals), b) three-lane freeway (Regression: eq.(6), 20-sec-Intervals)



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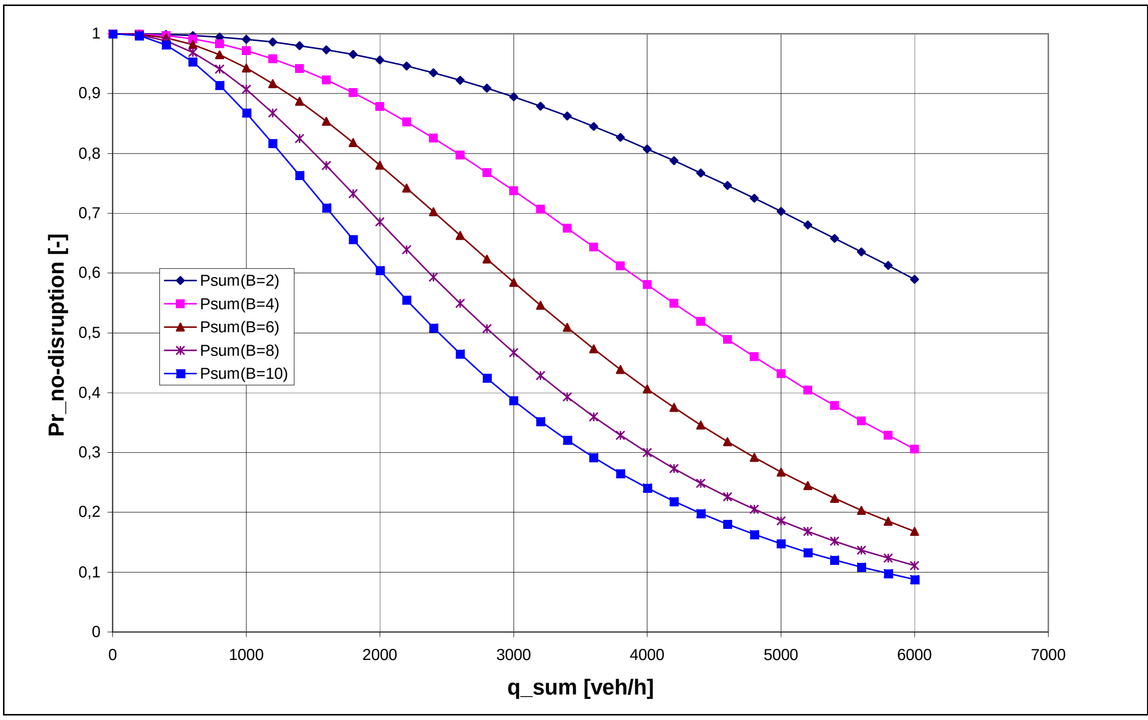

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FIGURE 3 - Total probability of no-disruption ($P_{sum} = P_{ND}$) as a function of parameter *B* for typical freeways in North America: a) two-lane freeway, b) three-lane freeway

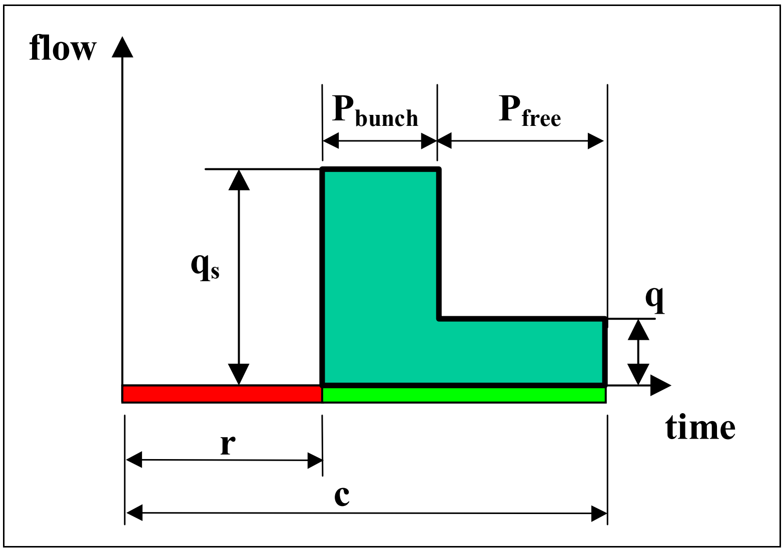


FIGURE 4 - Output behind a traffic signal

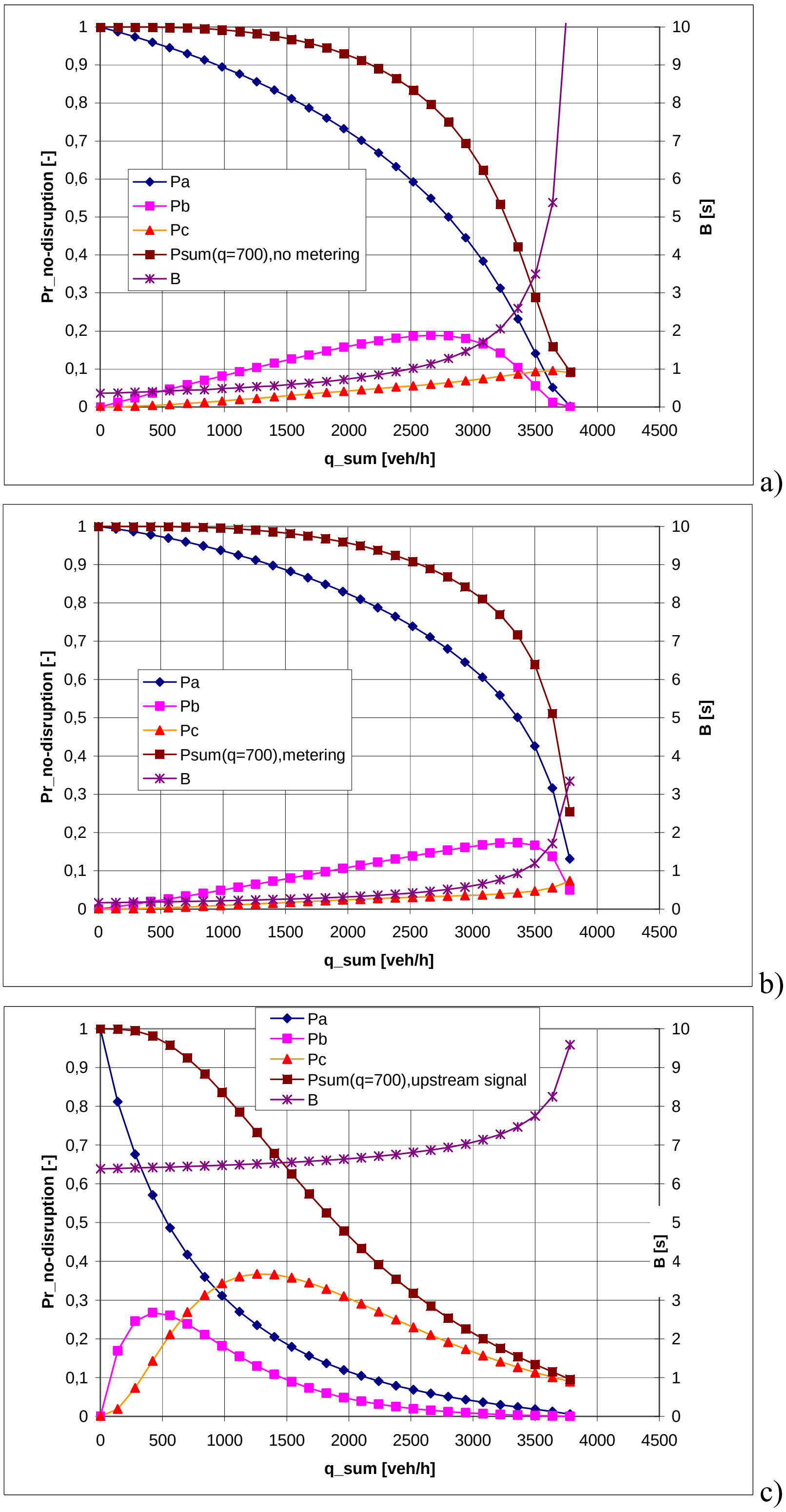


FIGURE 5 - Average length (*B*) of time for accommodating the platoon and probability of no-disruption (P_{ND}) for a typical two-lane freeway in North America: a) randomly arriving ramp traffic, b) for uniformly (equivalent to ramp metering) arriving ramp traffic, c) ramp traffic arriving in platoons (e.g., different size of platoons resulted from an upstream signal)

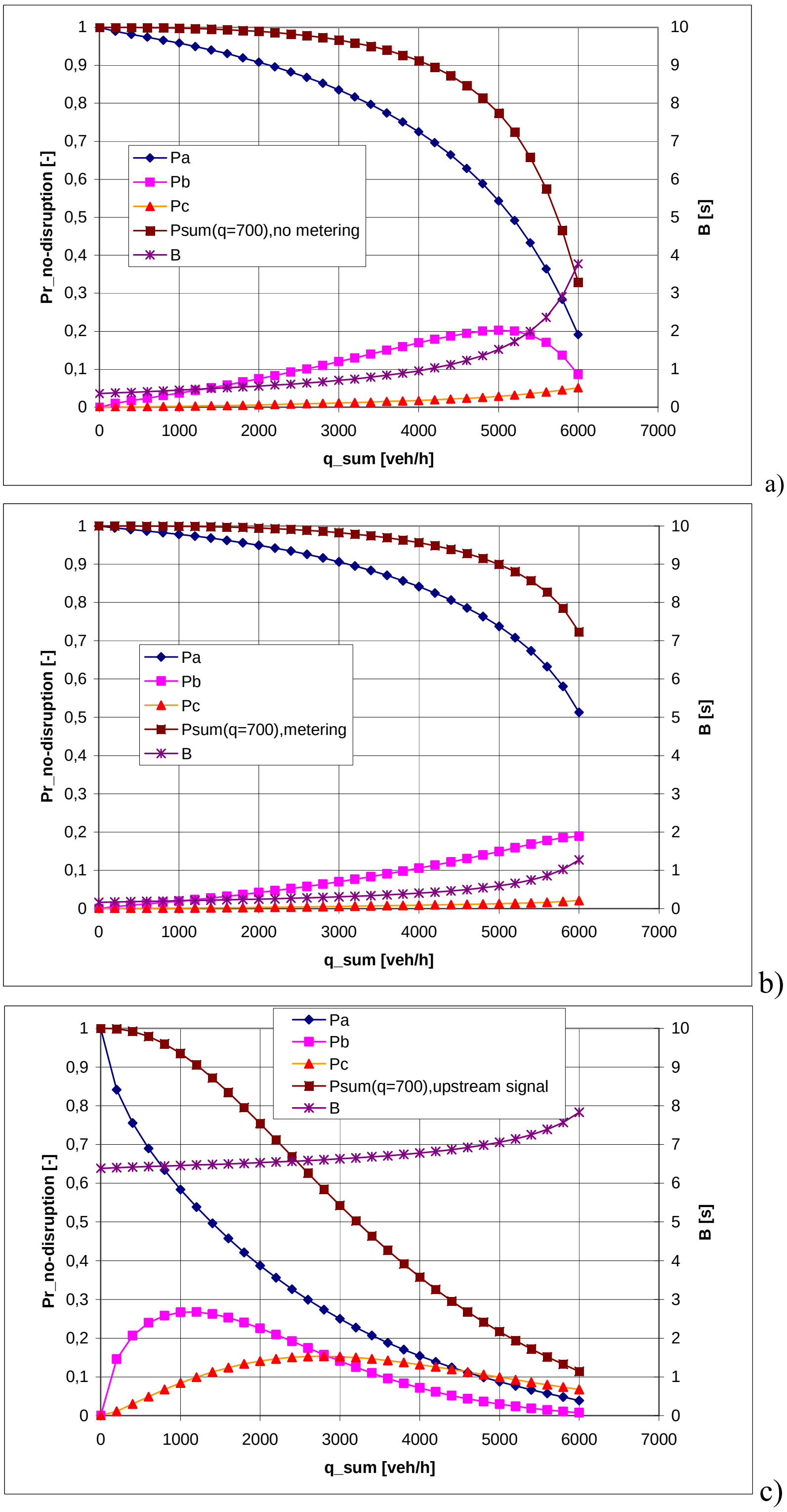
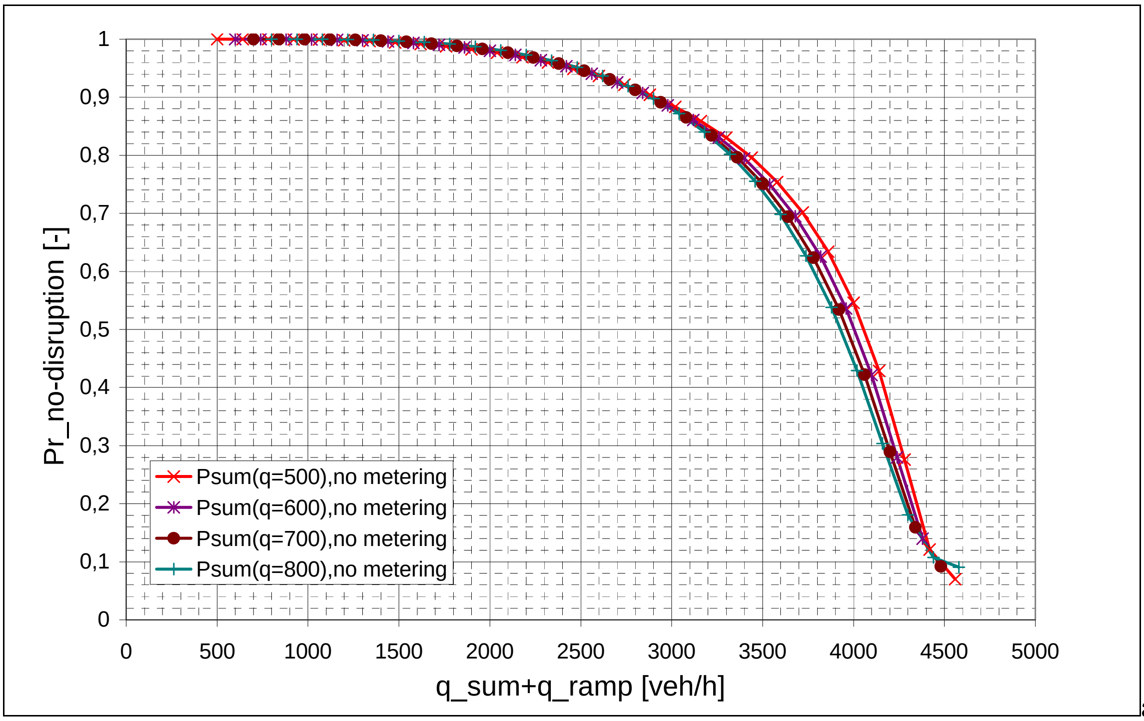


FIGURE 6 - Average length (*B*) of time for accommodating the platoon and probability of no-disruption (P_{ND}) for a typical three-lane freeway in North America: a) randomly arriving ramp traffic, b) for uniformly (equivalent to ramp metering) arriving ramp traffic, c) ramp traffic arriving in platoons (e.g., different size of platoons resulted from an upstream signal)



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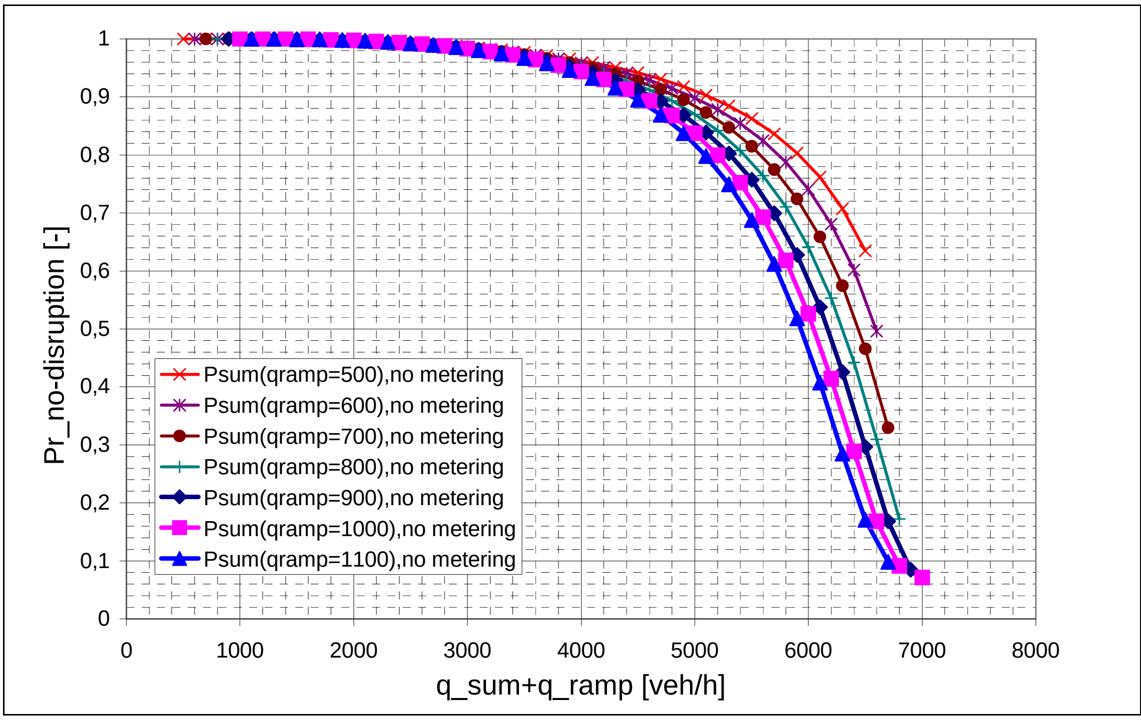
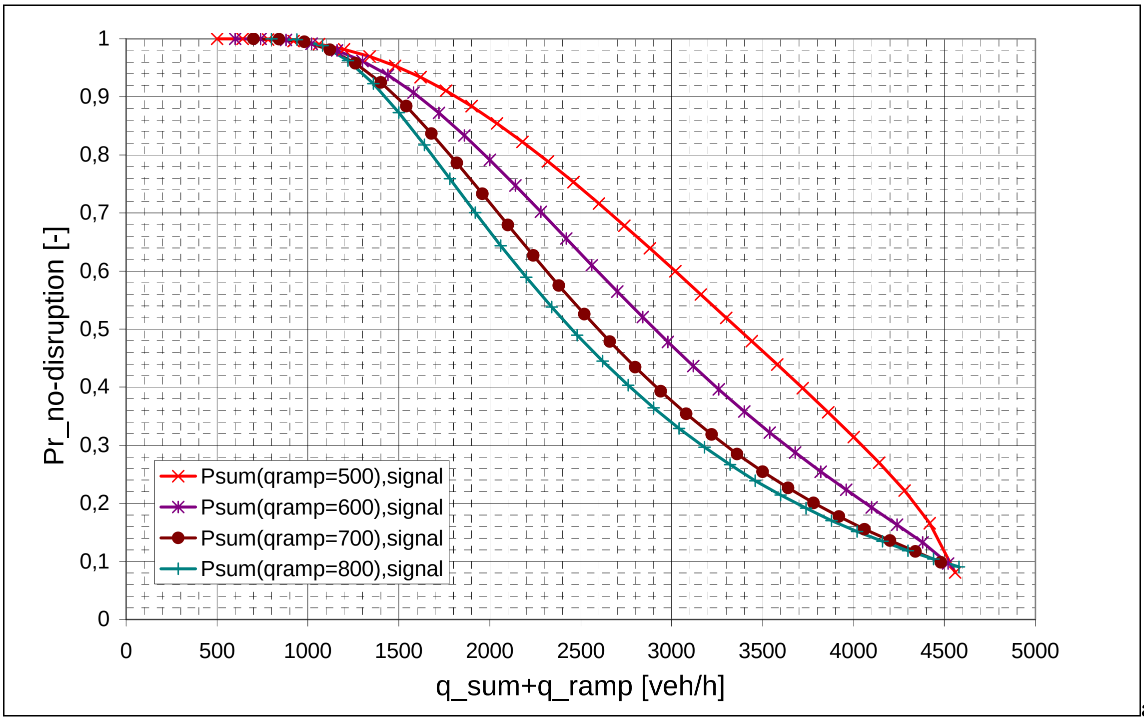
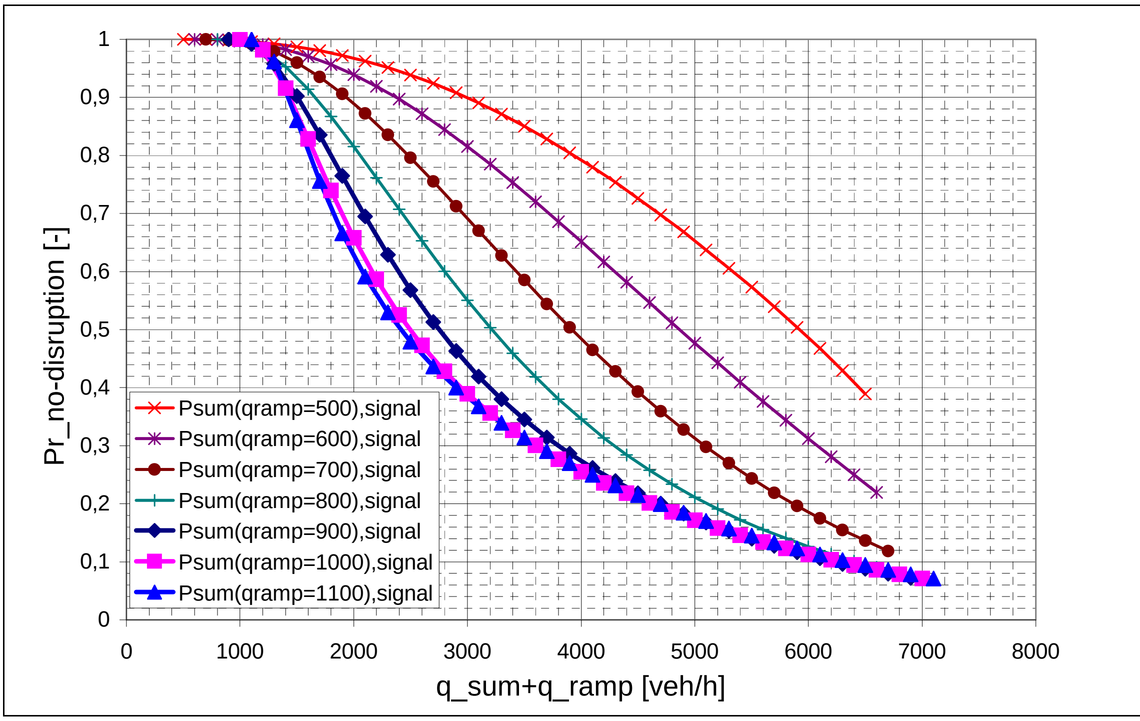


b)

FIGURE 7 - Total probability of no-disruption (P_{ND}= P_{sum}) in case without metering as a function of the total downstream flow rate q_{sum}+q_{ramp} for typical freeways in North America: a) two-lane freeway, b) three-lane freeway



a)



b)

FIGURE 8 - Total probability of no-disruption (P_{ND}= P_{sum}) for ramp traffic with upstream signal as a function of the total downstream flow rate q_{sum}+q_{ramp} for ramp traffic arriving in platoons for typical freeways in North America (*c*=60s, *r*=30s): a) two-lane freeway, b) three-lane freeway