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Modelling Blockage Probability and Capacity of Shared Lanes at Signalized Intersections

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Abstract

At signalized intersections, turning vehicles often use the same shared lane together with the through traffic. Since a permitted left-turn movement has to give way to the opposing through movement, it has to wait if necessary and thus impedes the through movement in the same direction. In the real-world, if the left-turn movement is permitted controlled, the through movement at the same approach can be totally blocked by waiting left-turning vehicles during the green time. Thus, the green time for the shared lane cannot be efficiently utilised and the lane capacity under consideration cannot be fully received. In this paper, a mathematical model is presented for an exact calculation of the blockage probability caused by permitted turning vehicles and for the estimation of the capacity of single-shared lanes at signalized intersections. According to the probability and combinatory theory, a mathematically exact model is developed. The proposed model can be applied to shared lanes either with left-turn or with right-turn movement. Respectively, by extending the model, also the capacity for the Right-Turn-On-Right situation can be exactly calculated. Furthermore, in this paper, the model is generalized to turning movements with so-called sneakers within the intersection. The generalized extension provides a more realistic solution for real-world intersections where, in a normal case, there are several places downstream of the stop-line for turning vehicles. The mathematical formulation for the generalization is more complicated. For applications in the practice, monographs are produced for estimating the shared lane capacity under different traffic conditions. In addition, a set of approximation functions are recommended based on the mathematically exact results. Those approximation functions can be more easily used than the exact formulations. The new model is verified by extensive simulation studies. The results of the proposed model fill out a gap in the current procedures of capacity estimations at signalized intersections. Some of the results of the model will be incorporated in the new version of German Highway Capacity Manual.

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1. Introduction

At signalised intersections, there are often traffic lanes which are used by different traffic movements. Those traffic lanes are called share lanes. Normally, the traffic movements in a share lane can obey different departure rules. The turning movements have often other departure rules than the through movements due to different traffic regulations. For example, the permitted left-turn movements have to give way to the opposing through movements while the through movements can depart with a saturation flow rate at stop-lines without hesitation. Here, in general, we deal with a time interval (e.g. the green time) where in a certain probability the departure of through vehicles is blocked by a waiting permitted turning vehicle because of the give way regulations. The capacity of the share lane is reduced by the blockage. In the existing manuals for traffic quality assessment, the reduction of capacity for such a share lane is not sufficiently taken into account. For example, the US Highway Capacity Manual (HCM) (TRB 2000) uses only a simple regression model for the effect of blockage caused by permitted turning movements. There is no accurate theoretical background in that model. That regression model does not satisfy all the necessary boundary conditions. In the Germany Highway Capacity Manual (HBS) (FGSV 2001), the capacity reduction in the share lane is not considered at all. The capacity of share lanes with permitted turning movements cannot be calculated in the HBS.

In this paper, the capacity reduction within a share lane caused by waiting permitted turning vehicles is qualified through a general mathematical model.

2. Literature Review

Previously, Levinson (1989) developed a simple formula for estimating the capacity of a shared left-turn lane and a left-turn blockage factor in the formula. He pointed out, that the capacity of a through lane at a signalized intersection is reduced by factors that reflect permitted left turns or the blockage effect of left turns on through traffic in the same lane. The capacity of a shared lane is reduced according to the number of left turns using the lane, as well as the traffic volumes in the opposing direction. Messer and Fambro (1977) studied the effects of signal phase and length of left-turn bay on left-turn capacity. They modelled the left-turn capacity based on the saturation flow rate that was reduced by left-turn bay blockage under different left-turn signal phases. Kikuchi et al (1993) analyzed the required length of the left-turn lane at signalized intersections for different conditions. Lane lengths are analyzed according to the probability of overflow of vehicles from the turning lane and the probability of blockage of the entrance to the turning lane by the queue of vehicles in the adjacent through lane. Kikuchi et al (2004) evaluated the length of double left-turn lanes using a probabilistic approach considering the blockage and overflow to the dual left-turn lanes. Tian and Wu (2006) proposed a probabilistic model for signalized intersection capacity with a short right-turn lane. Wu (2007) presented an investigation for short lanes using simulation studies for different signal timing configuration. Zhang and Tong (2008) presented a model to estimate the protected left-turn capacity under congested situation by considering the probability of left-turn bay blockage and the average number of vehicles in the left-turn bay. This probabilistic model also estimates the left-turn capacity at signalized intersections with leading protected left-turn signal followed by permitted left-turn phase when protected plus permitted operation is used. Shinya Kikuchi et al (2008) examined the lengths of turn lanes when a single lane approaches a signalized intersection and is divided into three lanes: left-turn, through, and right-turn. A set of formulas to compute the probabilities of lane overflow and lane entrance blockage is developed. In addition, Prassas and Chang (1999) studied the intersection capacity for protected-permitted phasing from shared lanes.

The subject of the proposed new model in this paper distinguishes from most of the models in the earlier works (except Levinson, 1989). In this paper, a shared lane without left-turn pocket is considered mathematically. In such a case, vehicles can only pass the stop-line consecutively. Here, also the number of waiting places (downstream of the stop-line) within the intersection can be taken into account. In contrast, the most of the earlier works investigated the problem of short lanes upstream of stop-lines. In those works, vehicles from different movements can wait at or behind (upstream) the stop-line side-by-side and pass the stop-line simultaneously. The number of waiting places downstream of the stop-line within the intersection was not the subject of those investigations.

3. Blockage occurred within a certain interval of time

Theoretical consideration on an example for un-blocked through movements within a share lane

Firstly, m vehicles within a share lane consisting of two movements L (Left-turn) and T (Through) are investigated. These m vehicles can depart in a time interval of the length I if no blockage occurs. The proportions of left-turn and through vehicles are a_L and a_T respectively. We are looking for the average number m_T^* for the through vehicles which arrive consecutively before a waiting left-turn vehicle (Blocker) blocks the share lane (see Figure 1a).

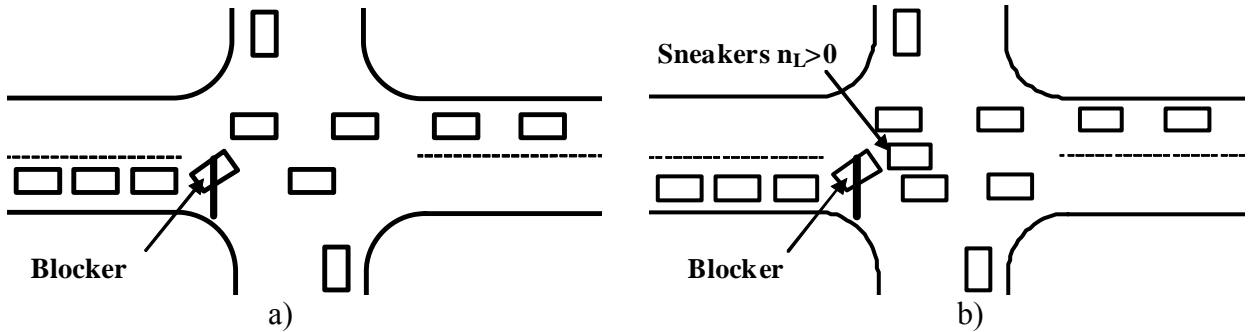


Figure 1 – Effect of blockage caused by a permitted left-turn vehicle at signalised intersections with single lane approaches, a) with sneakers $n_L = 0$, b) with sneakers $n_L > 0$

According to the probability and combinatory theory, the probability that exact n vehicles ($n < m$) in the through movement arrive consecutively before a left-turn vehicle arrives is

$$p_n = a_T^n \cdot a_L = a_T^n \cdot (1 - a_T) \quad \text{für } 0 \leq n < m \quad (1)$$

The probability that all m vehicles are from the through movement is

$$p_m = a_T^m \quad (2)$$

Combining eq. (1) with eq. (2), one obtains the probability that exact $n \leq m$ vehicles in the through movement arrived consecutively before a left-turn vehicle arrives:

$$p_n = \begin{cases} a_T^n \cdot (1 - a_T) & \text{für } 0 \leq n < m \\ a_T^m & \text{für } n = m \end{cases} \quad (3)$$

The following necessary boundary condition for probability holds:

$$\begin{aligned} \sum_{n=0}^m p_n &= a_T^m + \sum_{n=0}^{m-1} a_T^n \cdot (1 - a_T) = a_T^m + (1 - a_T) \sum_{n=0}^{m-1} a_T^n \\ &= a_T^m + (1 - a_T) \frac{(1 - a_T^m)}{(1 - a_T)} = 1 \end{aligned} \quad (4)$$

Therefore, the average number of through vehicles in consecutive order before a left-turn vehicle arrives or the interval I terminates is

$$m_T^* = \sum_{n=0}^m n p_n = (1 - a_T) \left(\sum_{n=0}^{m-1} n a_T^n \right) + m a_T^m = \left(\frac{1 - a_T^{m-1}}{1 - a_T} a_T - (m-1) a_T^m \right) + m a_T^m \quad [\text{veh}] \quad (5)$$

Sorting this equation yields the expectation (mean value) of the number of through vehicles in consecutive order under the given conditions:

$$m_T^* = \frac{a_T(1 - a_T^m)}{1 - a_T} \quad [\text{veh}] \quad (6)$$

For $a_T = 0$ is $m_T^* = 0$ and for $a_T \Rightarrow 1$ we have $m_T^* \Rightarrow m$. For $m \Rightarrow \infty$ one obtains with $a_T < 1$ the upper limit:

$$m_{T,m=\infty}^* = \frac{a_T}{1-a_T} = \frac{a_T}{a_L} \quad [\text{veh}] \quad (7)$$

The here derived function for m_T^* can be very important for different traffic facilities. One of the direct applications of this function is the estimation of capacity for share lanes at signalised intersections with permitted left-turn movements (also for share lanes at signalised intersections with permitted right-turn movements). The value m_T^* means the exact number of through vehicles which can pass the stop-line during the green time (interval under consideration) before being blocked by a permitted left-turn vehicle.

Since in the total capacity of the share lane, m_{sh}^* , the proportions of left-turn and through vehicles do not change, the following relationship is always true:

$$m_T^* = m_{sh}^* \cdot a_T \quad [\text{veh}]$$

Thus, the total capacity of the share lane during the green time (corresponding to the capacity of a cycle) with the green time g (i.e., length of the interval $I = g$) before the share lane is blocked by a left-turn vehicle is

$$m_{sh}^* = \frac{m_T^*}{a_T} = \frac{1-a_T^m}{1-a_T} \quad [\text{veh}] \quad (8)$$

Respectively, the capacity of the left-turn movement is

$$m_L^* = m_{sh}^* \cdot a_L = m_T^* \frac{a_L}{a_T} = \frac{1-a_T^m}{1-a_T} a_L = 1-a_T^m \quad [\text{veh}] \quad (9)$$

According to eq. (2), m is the maximum number (capacity) of through vehicles which can pass the stop-line during the green time g ($I = g$). This maximum number m of through vehicles can be achieved if all vehicles are from the through movement. That is,

$$m = g \cdot s_T \quad [\text{veh}] \quad (10)$$

The parameter s_T is the saturation flow rate in veh/s for through vehicles at the stop-line.

In Figure 2, the capacities m_T^* , m_{sh}^* , and m_L^* in veh/cycle as functions of the proportion of through vehicles a_T and the parameter m are depicted. It can be seen that the capacities m_T^* , m_{sh}^* , and m_L^* do not have a linear shape.

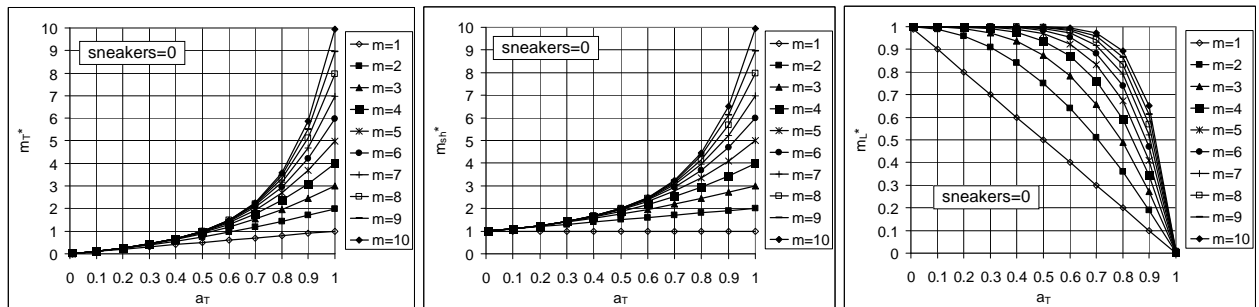


Figure 2 – Capacities m_T^* , m_{sh}^* , and m_L^* as functions of the proportion of the through vehicles a_T and the parameter m

The eqs. (6), (8), and (9) are derived under the assumption that the so-called “blocker” in the left-turn movement can only depart at the end of green time. Therefore, the number of departures for the left-turn vehicles is always 0 or 1 per cycle. The average value is between 0 and 1.

In order to check the equation for the capacity m_T^* , a simplified Monte Carlo simulation study is conducted. In Figure 3, the results of the simulation are depicted together with the values of the theoretical calculations. It can be seen that the derived theoretical model can be confirmed by the simulation study.

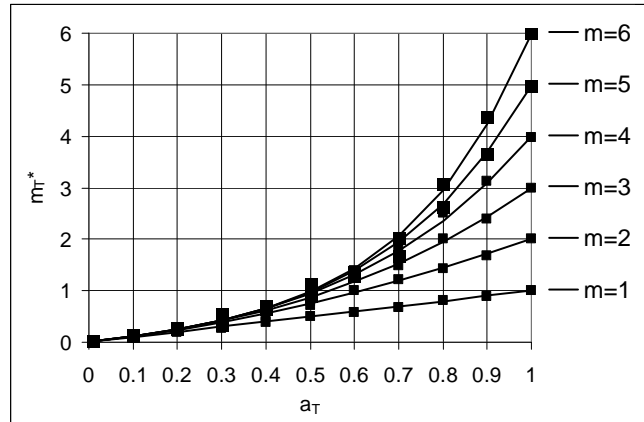


Figure 3 – Number of un-blocked through vehicles m_T^* as a function of a_T and m . Lines = Model (eq. (6)); Points = Monte Carlo Simulation

The proposed model can also be applied for the calculation of Right-Turn-On-Red (RTOR) regulation. The possible capacity for the right-turn vehicles (in case without opposing streams) during the red time r is then:

$$m_{RTOR}^* = \frac{a_R(1 - a_R^{m_r})}{1 - a_R} \quad [\text{veh}] \quad (11)$$

with

$$m_r = r \cdot s_R, \text{ veh}$$

r = red time for the share lane ($I = r$), s

s_R = saturation flow rate for the right-turn vehicles at the stop-line, veh/s

In this case, any arriving through vehicle blocks the departure of right-turn vehicles during the red time by a RTOR regulation.

Consideration of additional waiting places within the intersection

If there are some waiting places (downstream of the stop line) within the signalised intersections, we have to calculate the capacity of share lanes by taking into account the so-called sneakers. Sneakers are vehicles that can wait downstream of the stop-line within the intersection and depart after the green time. In this case the through movement is blocked by the $n_L + 1$ -th ($n_L > 1$) left-turn vehicle (see Figure 1b).

In the real-world, once the opposing queue clears, subject left-turning vehicles can filter through an unsaturated opposing flow at a rate by magnitude of the opposing flow. Obviously, the filtered number $n_{L,perm}$ of permitted left-turn vehicles that can depart during the green time due to gap-acceptance can also be considered as sneakers. The values of $n_{L,perm}$ can be calculated by common procedures (cf. TRB 2000 or FGSV 2001). The maximum number of sneakers (potential capacity for the left-turn movement) per cycle is then $n_L^* = n_L + n_{L,perm}$.

Now, we are looking for the average number m_T^* of through vehicles in consecutive order before the $n_L^* + 1$ -th left-turn vehicle blocks the share lane under consideration before the end of green time. In this case, the through movement is then blocked by a left-turn vehicle if the number of left-turn vehicles is greater than the number of sneakers n_L^* (the $n_L^* + 1$ -th sneaker is the blocker and it stays on the stop-line). The probability that exact n through vehicles ($n \leq m$) arrive consecutively before the number of sneakers $n_L^* + 1 = n_L + 1 + n_{L,perm}$ is reached or the interval I terminates can be obtained by the probability and combinatory theory as well.

The following case is considers first: The number n of through vehicles in consecutive order is smaller than then value of m minus the number of possible sneakers n_L^* downstream of the stop line. In this case a blockage always occurs. That is, we consider at first the case $0 \leq n < m - n_L^*$. In this case, exact n through vehicles and n_L^* left-turn vehicles have arrived and passed the stop-line before the blockage occurs. The number n of through vehicles in consecutive order then follows a binomial distribution, where the arrival of a through vehicle can be considered as an event of success. The probability that the first n through vehicles is un-blocked is according to the probability and combinatory theory

$$p_n = C_{n+n_L}^{(n)} \cdot a_T^n (1-a_T)^{n_L^*} (1-a_T) \quad \text{for } 0 \leq n < m - n_L^*$$

$$= C_{n+n_L}^{n_L^*} \cdot a_T^n (1-a_T)^{n_L^*+1}$$

with $C_l^{(k)} = \binom{l}{k} = \frac{l!}{k!(l-k)!} = \frac{l(l-1)(l-2)\dots(l-k+1)}{k!}$ = combination of k elements from l elements

In the case of $m - n_L^* \leq n < m$, the through movement is not blocked by the left-turn movement at all because the number of left-turn vehicles is smaller than the possible number of sneakers. The through vehicles can pass the stop-line until the green time is terminated. Since the maximum capacity during the green time is limited to the maximum number m of through vehicles, the number of departing through vehicles cannot be larger than m . Within the m vehicles there are n through vehicles and $m - n$ left-turn vehicles. Here, the order of the through and left-turn vehicles is not important, it can be considered as random. The probability that n through vehicles can depart during the green time is then

$$p_n = C_m^{(n)} a_T^n (1-a_T)^{m-n} = C_m^{(m-n)} a_T^n (1-a_T)^{m-n} \quad \text{for } m - n_L^* \leq n \leq m$$

Summarising the results yields

$$p_n = \begin{cases} C_{n+n_L}^{n_L^*} \cdot a_T^n (1-a_T)^{n_L^*+1} & \text{for } 0 \leq n < m - n_L^* \\ C_m^{(m-n)} a_T^n (1-a_T)^{m-n} & \text{for } m - n_L^* \leq n \leq m \end{cases} \quad (12)$$

Also here the following necessary boundary condition holds:

$$\sum_{n=0}^m p_n = 1$$

The correctness of eq. (12) can be confirmed by the Monte Carlo simulation as well. For $n_L^* = 0$, eq. (12) becomes eq. (3).

The average number of through vehicles which can pass the stop-line during the interval I under consideration is then

$$m_T^* = \sum_{n=0}^m n p_n = \sum_{n=0}^{m-n_L^*-1} n C_{n+n_L}^{n_L^*} a_T^n (1-a_T)^{n_L^*} + \sum_{n=m-n_L^*}^m n C_m^{(m-n)} a_T^n (1-a_T)^{m-n} \quad (13)$$

Correspondingly,

$$m_{sh}^* = \frac{m_T^*}{a_T} \quad \text{and} \quad m_L^* = m_T^* \frac{a_L}{a_T} \quad (14)$$

Unfortunately, eq. (13) cannot be summarised analytically. But one can solve the summation numerically using a spreadsheet. In Figure 4 and Figure 5, the capacities m_T^* , m_{sh}^* , and m_L^* as functions of the proportion of through vehicles a_T and the parameter m are depicted for Sneakers $n_L^*=1$ and $n_L^*=2$. The curves in Figure 2 are valid for $n_L^*=0$ correspondingly.

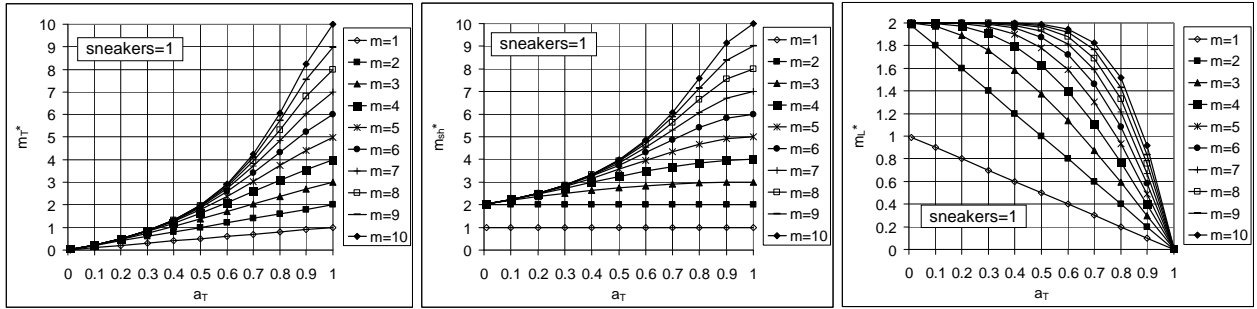


Figure 4 – Capacities m_T^* , m_{sh}^* , and m_L^* as functions of the proportion of through vehicles a_T and the parameter m for sneakers $n_L^* = 1$

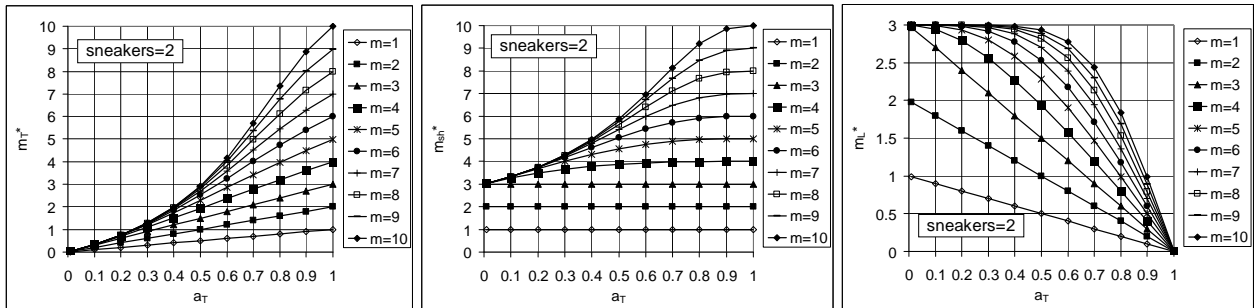


Figure 5 – Capacities m_T^* , m_{sh}^* , and m_L^* as functions of the proportion of through vehicles a_T and the parameter m for sneakers $n_L^* = 2$

For calculating the capacity of share lanes with permitted right-turn movements, one can use $n_L^* \Rightarrow n_R + n_{R,perm}$ respectively. That is, the curves in Figure 4, Figure 5, and Figure 2 are also valid for share lanes with permitted right-turn movements. For a share lane with additional RTOR regulation, the parameter $n_L^* \Rightarrow n_R + n_{R,perm} + m_{RTOR}^*$ and $m \Rightarrow m + m_{RTOR}^*$ can be used with m_{RTOR}^* from eq. (11).

4. Approximation functions

The additional capacity $m_{sneaker}$ per cycle resulting from the sneakers of the number n_L^* can be also approximately calculated according to the so-called share-lane formula from Harders (1968):

$$m_{sneaker} = \frac{1}{\frac{a_T}{m_T} + \frac{a_L}{n_L^*}} = \frac{1}{\frac{a_T}{g \cdot s_T} + \frac{a_L}{n_L^*}} \quad [\text{veh}] \quad (15)$$

The total capacity of the share lane per cycle is therefore (cf. eq.(8)):

$$m_{sh}^* = m_{sh, n_L^*=0}^* + m_{sneaker} = \frac{1 - a_T^m}{1 - a_T} + \frac{1}{\frac{a_T}{g \cdot s_T} + \frac{a_L}{n_L^*}} \quad [\text{veh}] \quad (16)$$

In addition, the capacity at the stop-line is limited to the boundary condition:

$$m_{sh}^* \leq \frac{g}{\frac{a_T}{s_T} + \frac{a_L}{s_L}} \quad [\text{veh}] \quad (17)$$

That is,

$$m_{sh}^* = \min \left(\frac{g}{\frac{a_T}{s_T} + \frac{a_L}{s_L}}, \frac{1 - a_T^m}{1 - a_T} + \frac{1}{\frac{a_T}{g \cdot s_T} + \frac{a_L}{n_L^*}} \right) \quad [\text{veh}] \quad (18)$$

$$m_T^* = m_{sh}^* \cdot a_T \quad [\text{veh}] \quad (19)$$

$$m_L^* = m_{sh}^* \cdot a_L \quad [\text{veh}] \quad (20)$$

with

$$m = \max \left(0, g \cdot s_T - m_{sneaker} \right) = \max \left(0, g \cdot s_T - \frac{1}{\frac{a_T}{g \cdot s_T} + \frac{a_L}{n_L^*}} \right) \quad [\text{veh}] \quad (21)$$

$$n_L^* = n_L + n_{L,perm}, \text{ veh}$$

n_L = number of waiting places within the intersection for the permitted left-turn movement, veh

$n_{L,perm}$ = capacity of the permitted left-turn movement during the green time, veh

This is an approximation because the dependence between $m_{sh,nl}=0$ and $m_{sneaker}$ was not taken into account.

For $g \cdot s_L \leq n_L^*$ the through vehicles can not be blocked by left-turn vehicles. In this case is $m = 0$, $m_{sh} = 0$ and $m_{sh}^* = m_{sneaker}$.

In Figure 6, the curves for m_T^* and m_{sh}^* according to the approximation formulas as functions of the proportion of through vehicles a_T , the parameter m and the number of sneakers m_L^* are depicted. It can be seen that the difference to the exact model (Figure 5) can be neglected. The difference is always smaller than 0.5 vehicles per cycle.

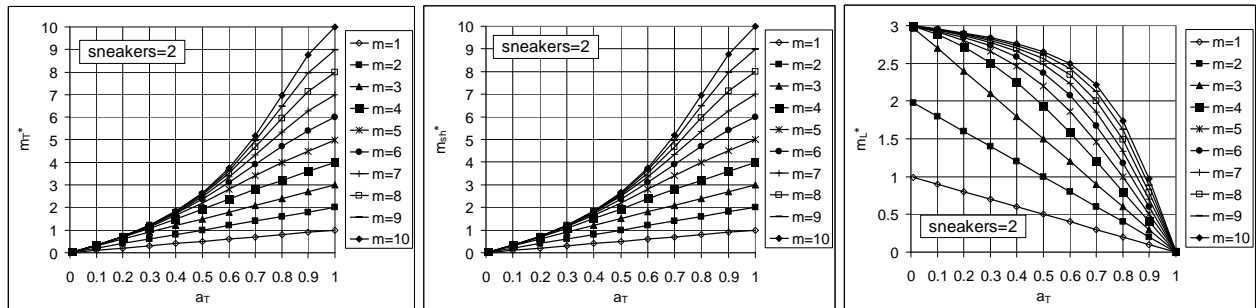


Figure 6 –Curves for the capacity m_T^* (eq. (19)) of un-blocked through vehicles, the capacity of the share lane m_{sh}^* (eq. (18)), and the capacity of the left-turn vehicles m_L^* (eq. (20)) as functions of the proportion of through vehicles a_T and the parameter m for sneakers $n_L^* = 2$ according to approximation formulas

In Figure 7, the differences between the approximation formula and the mathematically exact model are depicted in details. It can be seen that the deviations increase with increasing number of sneakers n_L^* . The deviation is the largest at ca. $n_L^* = 0,5 \cdot m$. Beyond that value the deviation goes back again.

Respectively, the approximation formula for share lanes with permitted right-turn movements is

$$m_{sh}^* = \min \left(\frac{g}{\frac{a_T}{s_T} + \frac{a_R}{s_R}}, \frac{1 - a_T^m}{1 - a_T} + \frac{1}{\frac{a_T}{g \cdot s_T} + \frac{a_R}{n_L^*}} \right) \quad [\text{veh}] \quad (22)$$

$$m_T^* = m_{sh}^* \cdot a_T \quad [\text{veh}] \quad (23)$$

$$m_R^* = m_{sh}^* \cdot a_R \quad [\text{veh}] \quad (24)$$

with

$$m = \max \left(0, g \cdot s_L + m_{RTOR}^* - \frac{1}{\frac{a_T}{g \cdot s_T} + \frac{a_R}{n_R^*}} \right), \text{veh} \quad (25)$$

$$n_R^* = n_R + n_{R,perm} + m_{RTOR}^*, \text{veh}$$

n_R = number of waiting places within the intersection for the permitted right-turn movement, veh

$n_{R,perm}$ = capacity of the permitted right-turn movement during the green time, veh

m_{RTOR}^* = capacity of the permitted right-turn movement with RTOR regulation during the red time (cf. eg. (11)), veh

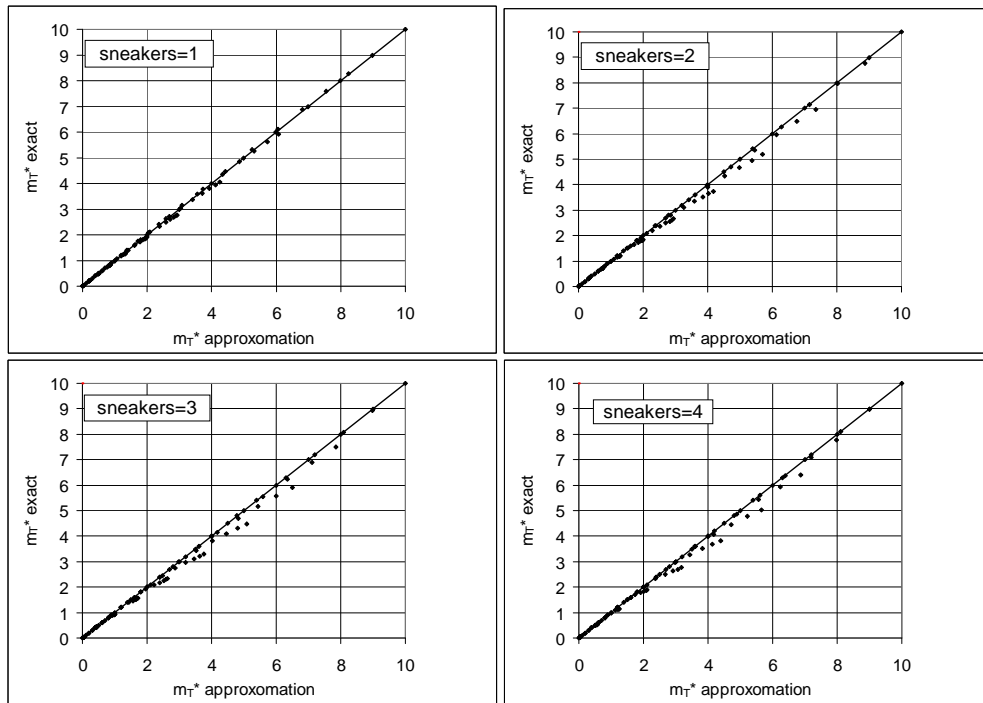


Figure 7 – Differences between the exact (eq. (13)) and the approximation model (eq. (19)) for the number m_T^* of un-blocked through vehicles

5. Comparison with the Regression Formula in the HCM

In appendix C of the HCM, the Left-Turn Adjustment Factor for permitted phasing is considered explicitly. There are two formulas for accounting the un-blocked green time for approaches with shared permitted left-turn lanes, one for multilane approaches with opposing multilane approaches (eq. C16-5 in the HCM) and one for single lane approaches opposed by single lane approaches (eq. C16-10 in the HCM). Both equations are derived by regression analysis. In the HCM, the portion of effective green time until the arrival of the first left-turn vehicle is designated g_f by the following formula:

$$g_f = g \cdot e^{-a \cdot LTC^b} - t_L \quad [\text{s}] \quad (26)$$

where

- g = actual green time for the permitted phase, s
 LTC = left turns per cycle
 t_L = lost time for subject left-turn lane group, s
 a, b = model parameters, for multilane approaches $a = 0.822$ and $b = 0.717$ (eq. C16-5 in HCM), for single lane approaches $a = 0.860$ and $b = 0.629$ (eq. C16-10 in HCM).

Compared to the new model, the parameter LTC can be calculated from the parameter m in combination with the proportion of left-turn vehicles a_L . Because the eq. C16-10 in the HCM (for single lane approaches opposed by single lane approaches) considers similar preconditions as the proposed model in this paper, the formula with $a = 0.860$ and $b = 0.629$ is used for further comparison.

Setting $LTC = a_L \cdot m = (1 - a_T) \cdot m$ in eq. (26) yields

$$g_f = g \cdot e^{-a \cdot [(1 - a_T) \cdot m]^b} - t_L = g_f = g \cdot a_T^* - t_L \quad [s] \quad (27)$$

with

$$a_T^* = e^{-a \cdot [(1 - a_T) \cdot m]^b} \quad (28)$$

This term represents the proportion of green time in which the through vehicles are not blocked by the first left-turn vehicle. According to the definition in the proposed new model this proportion is expressed by

$$a_T^* = \frac{m_T^*}{m} = \frac{a_T(1 - a_T^m)}{m(1 - a_T)} \quad (29)$$

The proportion of green time, a_T^* , in which the through vehicles are not blocked by the first left-turn vehicle, must satisfies the boundary conditions:

$$\begin{aligned} a_T^* &= 1 & \text{for } a_T &= 1 \\ a_T^* &= 0 & \text{for } a_T &= 0 \end{aligned} \quad (30)$$

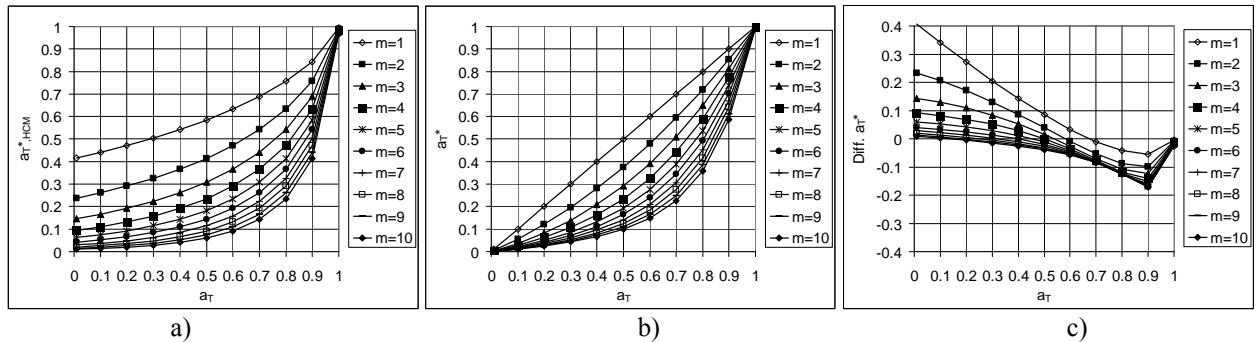


Figure 8 – The proportions a_T^* of green time where the through vehicles are not blocked by the first left-turn vehicle: a) from the HCM formula (eq.(28)), b) from the new model (eq.(29)), and c) differences between a) and b)

In Figure 8 the proportions a_T^* of green time where the through vehicles are not blocked by the first left-turn vehicle, both from the HCM formula (eq. (28)) and from the new model (eq. (29)) as well the differences, are presented. The proportions a_T^* are depicted as functions of the proportion of through vehicles a_T , and the parameter m . It can be seen that the difference are not negligible. For $a_T < 0.5$ the HCM formula gives totally different (too high) values. For $a_T > 0.5$, which corresponds to realistic traffic conditions, the HCM formula has similar but not the same values as the new model. It can also be seen that the deviations increase with increasing value of the parameter m . The differences are positive (overestimation of capacities) for $a_T < 0.5$ and negative (underestimation of capacities) for $a_T > 0.5$.

Eq. (28) does not satisfy all the necessary boundary conditions. For $a_T = 1$ we have $a_T^* = \exp(-a \cdot 0^b) = 1$ and that is correct. For $a_T = 0$ is $a_T^* = \exp(-a \cdot m^b) \neq 0$. The boundary condition by $a_T = 0$ is not fulfilled.

In Figure 9, the proportions a_T^* of green time where the through vehicles are not blocked by the first left-turn vehicle are depicted as a function of LTC . Here, the HCM formula (eq.(28)) with parameters both for single-lane

and multilane approaches is depicted with the results from the new model (eq.(29)) for $m = 4$ through 10. It can be clearly seen, that the value of a_T^* decreases with increasing LTC for all functions under consideration. According to the new model, the value of a_T^* increases with increasing m . On the other side, according to the HCM-formula, the value of a_T^* is lower for multilane approaches than for single-lane approaches. Because a multilane approach has in general a lower total saturation flow rate (per lane) than a single-lane approach due to un-equal lane utilization, a multilane approach has also smaller value of m per lane than a single-lane approach for the same green time. Thus, multilane approaches can be taken in to account using adapted, smaller values for the parameter m .

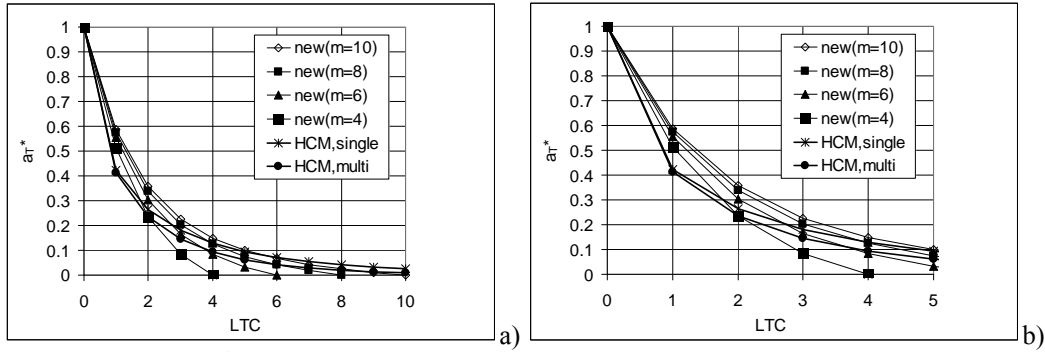


Figure 9 – The proportions a_T^* of green time where the through vehicles are not blocked by the first left-turn vehicle as a function of LTC , a): overview, b): detail

Using the new theoretical model, the portion of effective green time until the arrival of the first left-turn vehicle can be rewritten as:

$$\begin{aligned}
 g_f &= g \cdot \frac{m_T^*}{m} - t_L \\
 &= g \cdot \frac{a_T(1 - a_T^m)}{m(1 - a_T)} - t_L
 \end{aligned}
 \quad [s] \quad (31)$$

where

- g = actual green time for the permitted phase, s
- m = maximum number of through vehicles per cycle, veh
= $g \cdot s$
- s = saturation flow rate, veh/s
- a_T = proportion of through vehicles
- t_L = lost time for subject left-turn lane group, s

This equation could be incorporated into HCM in place of eq. (26) both for multilane and single-lane approaches.

6. Summary and Conclusions

In this paper, the influence of permitted turning vehicles on the total capacity of share lanes at signalised intersections is quantified through a mathematical model. With this model, the probability that the share lane is blocked by a permitted turning vehicle can be exactly calculated. Based on this probability, the average capacity of the share lane can be estimated. The proposed model can be used for share lanes with either permitted left-turn or permitted right-turn movements. Also the Right-Turn-On-Red regulation can be calculated by the proposed model.

The derivation of the model is based on the assumption that the permitted turning vehicles on and downstream of the stop line can clear the intersection after the green time. This assumption is necessary because the model is only valid for the case that at the end of green time the blockage is cleared and the arrivals of the through and turning vehicles in the new interval under consideration are random. This assumption is not critical since in the reality traffic regulations allow the permitted waiting vehicles to clear the intersection immediately after the end of green time.

The major findings of the paper are the derivations of eqs.(3), (6), (12), and (13). According to those equations, the number of un-blocked through vehicles in the share lane and therefore the total capacity of the shared lane before a blockage can be calculated exactly.

Unfortunately, eq. (13) can not be solved analytically and thus, it is not easily applicable. However, for practical applications, Figure 4, Figure 5, and Figure 8 can be used directly. In addition, approximation formulas which can be easily calculated are recommended. The recommended approximation formulas are very close to the exact solutions and they can be easily incorporated into the existing highway capacity manuals. In contrast to the regression functions in the Highway Capacity Manual, the proposed approximation functions satisfied all necessary boundary conditions. Using the new theoretical model, the portion of effective green time until the arrival of the first left-turn vehicle can be simplified.

The model in this paper is developed for fixed time controlled and isolated intersections. In the future, an extension to actuated, adapted or coordinated intersections is possible.

7. References

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