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Estimating Distribution Function of Critical Gaps at Unsignalized Intersections Based on Equilibrium of Probabilities

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ABSTRACT

Critical gap is an important parameter for capacity analysis at unsignalized intersections. This parameter is stochastically distributed and it cannot be obtained directly by field measurements. Thus, many procedures for estimating the critical gap were developed based on different theories. In an early work of the author, a new model the estimation of critical gaps at unsignalized intersection was introduced. Using equilibrium of probabilities for rejected and accepted gaps, a model for estimating the critical gap and its empirical distribution was established. The model did not require any presumptions regarding the distribution function of critical gaps and the driver behaviors. The result of the new model is an un-parameterized empirical distribution of critical gaps. The mathematical function of the critical gaps was not required in advance.

This paper presents a solution accounting for different predefined distribution functions of critical gaps. Using regression analysis two distribution functions a) log-normal distribution and b) Weibull distribution are calibrated to the empirical distribution of critical gaps. The result of this paper shows, that the Weibull distribution is the better one representing the distribution of critical gaps.

Keywords: Unsignalized Intersection, Critical Gap, Distribution Function

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1 INTRODUCTION

Critical gap is an important parameter for capacity analysis at unsignalized intersections. This parameter is stochastically distributed and it cannot be obtained directly by field measurements. The estimation of critical gaps at unsignalized intersections from traffic observation is one of the most challenging tasks in the traffic engineering science. For estimating the critical gaps, statistical models or procedures are required. There exist many different models for estimating critical gaps. Among them the models of Siegloch (*1*), Raff et al. (*2*), Harders (*3*), and Troutbeck (*4*) are the most important. In the practice - for unsaturated conditions - the most common models are that of Raff et al. (*2*) and Troutbeck (*4*).

Brilon et al. (*5*) gave an overview of the most important models. Using microscopic simulations, they also conducted an assessment of those models. They found that the model of Troutbeck (*4*) gives the best results. Thus, this model was recommended for estimating the critical gaps in many standard manuals for traffic engineering (*6,7,8,9,10*).

The model of Troutbeck (*4*) is a microscopic model. That is, single values of the measured gaps are used in the model. The model is based on the theory of Maximum Likelihood Estimation. However, in this model, two presumptions are required: a) a log-normal distribution for critical gaps and b) a homogeneous and consistent behavior of the drivers. That means the rejected gaps need to be smaller than the accepted gaps and only the maximum rejected gap and the accepted gap of single vehicles can be used pairwise. Data pairs with rejected gaps being larger than accepted gaps cannot be used at all. In some cases, more than 50% of the measured gaps cannot be used. This is a huge waste of collected data.

Such presumptions are disadvantages of the model. Furthermore, the model of Troutbeck (*4*) is very complicated and its results are not very robust. This model also requires a large sample size for establishing stable results.

In an early work of the author, a totally new model for estimating the critical gap was presented (*11*). The theoretical background of this new model is the probability equilibrium between the rejected and the accepted gaps. The equilibrium is established *macroscopically* from the cumulative distributions of the rejected and accepted gaps. It turns out that the model from the macroscopic equilibrium is more appropriate for estimating critical gaps. The new model yields similar results as that from Troutbeck's model if the same sample data are used. More importantly, the new model yields directly the empirical distribution of critical gaps. The new model does not require any predefined assumptions and it is easy to use. This new model has already found broad applications in different countries, such as in Germany, Spain, Canada, the Netherlands, and the United States. However, from the new model the mathematical function of critical gaps cannot be estimated explicitly.

This paper introduces a solution accounting for different predefined distribution functions of critical gaps. Using regression analysis two distribution functions a) log-normal distribution and b) Weibull distribution are calibrated to the empirical distribution of critical gaps. The result of this paper shows, that the Weibull distribution is the better one representing the distribution of critical gaps.

2 MODEL DESCRIPTION AND APPLICATIONS

2.1. The method of Raff and Troutbeck

Let $F_r(t)$ and $F_a(t)$ be the probability distribution functions (PDFs) of rejected and accepted gaps, respectively. Then $F_r(t)$ and $F_a(t)$ can be obtained empirically by *in situ* measurements. Thus, the observed probability that a gap of length t is rejected is $F_r(t)$, and that it is not rejected is $1 - F_r(t)$ and the observed probability that a gap of length t is accepted is $1 - F_a(t)$, and that it is not accepted is $F_a(t)$.

More than fifty years ago, Raff (*2*) introduced a macroscopic model for estimating the critical gap. He defined the critical gap t_c as the value of t where the functions $1 - F_r(t)$ and $F_a(t)$ intersect. That is, the value t at which

$$F_a(t) = 1 - F_r(t) \tag{1}$$

is defined as the estimated critical gap t_c. Obviously, the critical gap t_c is only defined if the overall minimum accepted gap $a_{\min}$ is smaller than the overall maximum rejected gap $r_{\max}$. Otherwise the functions $1 - F_r(t)$ and $F_a(t)$ do not intersect at all.

Raff's method was used in many countries in earlier years. Because of its simplicity, it is still being used today in some research projects.

Troutbeck (*4*) gave a procedure for estimating critical gaps based on the Maximum Likelihood technique. This model is a microscopic model. In this model only the maximum rejected gaps which are larger than the corresponding rejected gaps can be taken into account. Thus, for an accepted gap a, there is only a corresponding rejected gap r under consideration. Denote the PDF of critical gaps to be estimated by $F_{tc}(t)$, the likelihood that a driver's actual critical gap is between a and r is given by $F_{tc}(a) - F_{tc}(r)$. The likelihood L^* with a sample of n observed drivers is

$$L^* = \prod_{i=1}^{n} \left[F_{tc}(a) - F_{tc}(r) \right] \tag{2}$$

If the PDF of the critical gaps, $F_{tc}(t)$, is given, the parameters and of the PDF can be obtained by maximizing the likelihood L^*. In the practice, the log-normal distribution is often used as the PDF of the critical gaps. Furthermore, as model assumptions, the driver behavior has to be both homogeneous and consistent. Normally, the maximization of the likelihood L^* can only be done using numerical and iteration techniques (cf. Troutbeck (*4*)).

The maximization of the likelihood L^* delivers only solutions if the overall minimum accepted gap $a_{\min}$ is smaller than the overall maximum rejected gap $r_{\max}$. Otherwise is the mean critical

gap t_c between $a_{\min}$ and $r_{\max}$ and the maximization of the likelihood L^* is not defined. In this case the maximization delivers always the value zero for the standard deviation σ_{tc} of the distribution $F_{tc}(t)$. Then the skewness of the distribution is also zero and the mean critical gap can be approximately calculated as $t_c = (a_{\min} + r_{\max})/2$.

2.2. The model based on the macroscopic probability equilibrium

The author has introduced a new model based on the macroscopic probability equilibrium of the accepted and rejected gaps (*11*). The model was established as follows.

According to the PDFs of the accepted ($F_a(t)$) and rejected ($F_r(t)$) gaps, the observed probability that a gap of length t is accepted is $1 - F_a(t)$ and that it is "not-accepted" is $F_a(t)$. And the observed probability that a gap of length t is rejected is $F_r(t)$ and that it is "not-rejected" is $1 - F_r(t)$. In general, we have $F_r(t) \neq 1 - F_a(t)$ and $1 - F_r(t) \neq F_a(t)$ because an accepted gap in the major stream may not have the exact length of the actual critical gap. In fact, an accepted gap is always larger than the actual critical gap at that specific time instance. The actual critical gap is statistically distributed and can vary from time to time according to a certain distribution. An accepted gap can be larger or smaller than the mean critical gap.

Denote the PDF of critical gaps to be estimated by $F_{tc}(t)$, then the probability $P_{r,tc}(t)$ that a gap of length t in the major stream would be rejected is $F_{tc}(t)$, and the probability $P_{a,tc}(t)$ that it would be accepted is $1 - F_{tc}(t)$.

Considering the observed probability of both acceptance and rejection, we have the probability equilibrium

$$\begin{cases} P_{r,tc}(t) = F_r(t) \cdot P_{r,tc}(t) + F_a(t) \cdot P_{a,tc}(t) \\ P_{a,tc}(t) = \big(1 - F_a(t)\big) \cdot P_{r,tc}(t) + \big(1 - F_r(t)\big) \cdot P_{a,tc}(t) \end{cases} \tag{3}$$

Equation (3) can be rewritten in the following matrix form:

$$\begin{pmatrix} P_{r,tc}(t) \\ P_{a,tc}(t) \end{pmatrix} = \begin{pmatrix} F_r(t) & F_a(t) \\ 1 - F_r(t) & 1 - F_a(t) \end{pmatrix} \begin{pmatrix} P_{r,tc}(t) \\ P_{a,tc}(t) \end{pmatrix} \tag{4}$$

That is exactly the description of the equilibrium state of the probabilities $P_{a,tc}(t)$ and $P_{r,tc}(t)$ as a Markov Chain. In this formulation

$$\begin{pmatrix} P_{r,tc}(t) \\ P_{a,tc}(t) \end{pmatrix}$$

is the state vector and

$$\begin{pmatrix} F_r(t) & F_a(t) \\ 1 - F_r(t) & 1 - F_a(t) \end{pmatrix}$$

the transition matrix. The boundary condition $P_{a,tc}(t) + P_{r,tc}(t) = 1$ holds.

With $P_{r,tc}(t) = F_{tc}(t)$ and $P_{a,tc}(t) = 1 - F_{tc}(t)$, equation (4) yields

$$\begin{pmatrix} F_{tc}(t) \\ 1 - F_{tc}(t) \end{pmatrix} = \begin{pmatrix} F_r(t) & F_a(t) \\ 1 - F_r(t) & 1 - F_a(t) \end{pmatrix} \begin{pmatrix} F_{tc}(t) \\ 1 - F_{tc}(t) \end{pmatrix} \tag{5}$$

Solving equation (5) yields the PDF $F_{tc}(t)$ of the critical gaps:

$$F_{tc}(t) = \frac{F_a(t)}{F_a(t) + 1 - F_r(t)} = 1 - \frac{1 - F_r(t)}{F_a(t) + 1 - F_r(t)} \tag{6}$$

The PDF $F_{tc}(t)$ is always between $F_r(t)$ and $F_a(t)$ (see FIGURE 1).

It should be noted that also here this distribution is only explicitly defined, from the point of view of all vehicles, between the overall minimum accepted gap $a_{\min}$ and the overall maximum rejected gap $r_{\max}$ with $a_{\min} \leq r_{\max}$. For $t_c \leq a_{\min}$ is $F_{tc}(t) = 0$ and for $t_c \geq r_{\max}$ is $F_{tc}(t) = 1$. In case of $a_{\min} > r_{\max}$ the mean critical gap can be approximately calculated as $t_c = (a_{\min} + r_{\max})/2$.

According to Raff's definition for the critical gap (eq. (1)) we have

$$F_{tc}(t) = \frac{F_a(t)}{F_a(t) + 1 - F_r(t)} = \frac{F_a(t)}{F_a(t) + F_a(t)} = 0.5 \tag{7}$$

This means that the critical gap estimated from Raff's method is the median value not the mean value of the critical gap. For lognormal-distributed critical gaps, the median value is always smaller than the mean value. That means Raff's method underestimates the critical gap and thus overestimates the corresponding capacity.

The new model has a solid theoretical background (in terms of the Markov Chain and equilibrium of probabilities) and robust results. It is also independent of any model presumptions. It requires neither predefined distribution function of critical gaps nor the consistency nor the homogeneity of driver behaviors. This model can take into account all relevant gaps (not only the maximum rejected gaps as is the case of the Troutbeck model (*4*) and yields the empirical PDF of the critical gaps directly. Also the limitation that a rejected gap must be smaller than an accepted gap is not more necessary. The calculation procedure of the model is simple and it needs no iteration.

In particular, the property of the new model that all rejected and accepted gaps including accepted gaps which are smaller than the rejected gaps can be taken into account makes the major difference between the new model and the most used model of Troutbeck (*4*). If only the maximum rejected gaps with corresponding accepted gaps larger than the rejected gaps are used for estimating the critical gaps, the new model gives similar results (deviations smaller than 0.2s)

for the mean critical gaps as those from Troutbeck (*4*). If all gaps are used, the estimated mean critical gaps are usually smaller.

For implementing the new model, a useful calculation procedure was recommended. This procedure can be easily implemented into a Spreadsheet (for example, EXCEL or QuatroPro). The procedure is described as follows:

1. insert all measured and relevant (according to whether all or only the maximum rejected gaps with corresponding accepted gaps larger than the rejected gaps are taken into account) gaps t in the major stream into the column 1 of the spreadsheet
2. mark the accepted gaps with "a" and the rejected gaps with "r" in column 2 of the spreadsheet respectively
3. sort all gaps (together with their marks "a" and "r") in an ascending order
4. calculate the accumulate frequencies of the rejected gaps, n_{rj}, in column 3 of the spreadsheet (that is: for a given row j, if mark="r" then $n_{rj}=n_{rj}+1$ else $n_{rj}=n_{rj}$, with $n_{r0}=0$)
5. calculate the accumulate frequencies of the accepted gaps, n_{aj}, in column 4 of the spreadsheet (that is: for a given row j, if mark="a" then $n_{aj}=n_{aj}+1$ else $n_{aj}=n_{aj}$, with $n_{a0}=0$)
6. calculate the PDF of the rejected gaps, $F_j(r)$, in column 5 of the spreadsheet (that is: for a given raw j, $F_j(r)=n_{rj}/n_{max}$ with n_{max}=number of gaps)
7. calculate the PDF of the accepted gaps, $F_a(t_j)$, in column 6 of the spreadsheet (that is: for a given raw j, $F_a(t_j)=n_{aj}/n_{max}$ with n_{max}=number of all gaps)
8. calculate (according to equation (6)) the PDF of the estimated critical gaps, $F_{tc}(t_j)$, in column 7 of the spreadsheet (that is: for a given raw j, $F_{tc}(t_j)=F_a(t_j)/[F_a(t_j)+1-F_r(t_j)]$
9. calculate the frequencies of the estimated critical gaps, $p_{tc}(t_j)$, between the raw j and j-1 in column 8 of the spreadsheet (that is: $p_{tc}(t_j)=F_{tc}(t_j)-F_{tc}(t_{j-1})$)
10. calculate the class mean, $t_{d,j}$, between the raw j and j-1 in column 9 of the spreadsheet (that is: $t_{d,j}=(t_j+t_{j-1})/2$)
11. calculate the mean value and the variance of the estimated critical gaps (that is: ($t_{c,mean}=sum[p_{tc}(t_j)\cdot t_{d,j}]$ and $\sigma^2=sum[p_{tc}(t_j)\cdot t_{d,j}^2]-(sum[p_{tc}(t_j)\cdot t_{d,j}])^2$)

This calculation procedure ensures a monotonic ascending PDF for the critical gaps.

In FIGURE 2, an example of the procedure for estimating the critical gap with a spreadsheet is illustrated.

In FIGURE 3 and FIGURE 4, the results of two examples are presented (data: Weinert (*12*)). In these calculations, only the maximum rejected gaps are used for comparability to the model of Troutbeck (*4*). It can be recognized that the mean values of the critical gaps t_c are similar from both models. Also, the PDF estimated from the new model are comparable to the predefined PDF (log-normal) from Troutbeck's model. This indicates that the predefined log-normal distribution in Troutbeck's model is suitable for describing the distribution of critical gaps. However, the log-normal distribution does not represent the empirical distributions perfectly. It can be seen in the following section, that the Weibull distribution is a better one for representing the distribution of critical gaps.

In Figure 5 and Figure 6, the results for the same examples but using all rejected and accepted gaps are presented. It can be seen that the mean values of the critical gaps t_c are smaller compared to the results in Figure 3 and Figure 4. The average difference is about 15%. To demonstrate this effect clearly, the resulted PDF for both cases are illustrated together in Figure 7 and Figure 8.

3 ESTIMATION OF THE DISTRIBUTION FUNCTION OF CRITICAL GAPS

Using the empirical estimated distributions in Figure 7 and Figure 8, two different distribution functions can be calibrated by conducting non-linear regression analysis. The distribution functions are fitted to the empirical distributions by minimizing the sum of the error squares (The method of least squares). The results of the regression analysis are depicted TABLE 1 and in FIGURE 9 through FIGURE 12. It can be seen, that the Weibull distribution is the better one representing the distribution of critical gaps with better fittings to the empirical distributions although the differences are not very significant. The Weibull distribution has a stronger capability to smooth the empirical distributions (cf. part b in FIGURE 9 through FIGURE 12) and thus to correct the errors in the field data than the log-normal distribution (cf. part a in FIGURE 9 through FIGURE 12). Compared to the empirical distributions, the standard errors from the Weibull distribution is smaller than those from the log-normal distribution (TABLE 1). The standard deviation σ_{tc} of the resulting Weibull distributed critical gaps is always nearly 1.

Thus, for investigations in the future, the Weibull distribution is recommended for representing critical gaps at unsignalized intersections. Compared to the log-normal distribution the Weibull distribution has also a simpler expression.

4 SUMMARY AND CONCLUSIONS

Several important models for estimating the critical gap at unsignalized intersections are presented. The new model developed from the author earlier (*11*) turns out to be the best and simplest one compared to the other models. In addition, this new model does not require any *a priori* assumptions and the results are accurate. Using the new macroscopic model (equation (6)), a generalized procedure for estimating critical gaps is established. With this procedure, the PDF of the critical gaps can be estimated empirically. Using this model, more measurement data can be taken into account.

Using the empirical estimated distributions, two different distribution functions a) log-normal distribution and b) Weibull distribution are calibrated by conducting non-linear regression analysis. The results show, that the Weibull distribution is the better one representing the distribution of critical gaps. The Weibull distribution has a stronger capability to smooth the empirical distributions and thus to correct the errors in the field data than the log-normal distribution.

For investigation in the future, the new model developed from the author earlier (*11*) is recommended for estimating critical gaps and the Weibull distribution is recommended for representing the distribution of critical gaps at unsignalized intersections. The Weibull distribution function can also be used for the Troutbeck's (*4*) procedure for estimating critical gaps based on the Maximum Likelihood method.

The procedure for implementing the new model is simple and robust. It can be carried out using spreadsheet programs (e.g., EXCEL, QuatroPro etc.) without iteration. Thus, with the new model, a useful and promising tool is provided to professionals of traffic engineering. For practical applications, an implemented EXCEL-spreadsheet can be obtained from the author.

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List of Tables and Figures

FIGURE 9 – Calibration of the a) log-normal and b) Weibull distribution. Dada 1: F_{tc}(macro)=PDF of the empirical estimated critical gaps with only the maximum rejected gaps (Data: Weinert (*12*), Bad Nauheim 3, minor right-turn).

FIGURE 10 - Calibration of the a) log-normal and b) Weibull distribution. Dada 2: F_{tc}(macro)=PDF of the empirical estimated critical gaps with only the maximum rejected gaps (Data: Weinert (*12*), Köln 1, major left-turn).

FIGURE 11 – Calibration of the a) log-normal and b) Weibul distribution. Dada 3: F_{tc}(macro)=PDF of the empirical estimated critical gaps with all rejected gaps

FIGURE 12 - Calibration of the a) log-normal and b) Weibull distribution. Data 4 : F_{tc}(macro)=PDF of the empirical estimated critical gaps with all rejected gaps

TABLE 1 – Results of the regression analysis

Distribution function	Data 1 n=289			Data 2 n=663			Data 3 n=198			Data 4 n=300		
	t_c	σ_{tc}	Std. err.	t_c	σ_{tc}	Std. err.	t_c	σ_{tc}	Std. err.	t_c	σ_{tc}	Std. err.
Empirical	6.4	1.11	-	5.5	1.16	-	5.4	1.00	-	5.0	0.99	-
Log-normal	6.4	0.98	0.017	5.6	1.11	0.013	5.4	0.97	0.024	5.0	0.92	0.019
Weibull	6.4	0.97	0.015	5.5	1.07	0.010	5.3	0.88	0.024	4.9	0.84	0.018

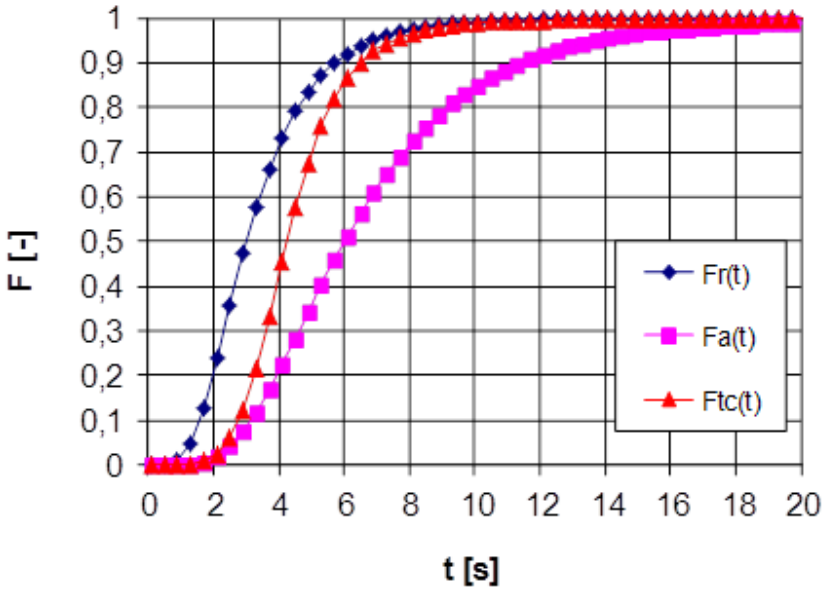


FIGURE 1 – Schematic relationship between the PDF's for the rejected gaps, the accepted gaps, and the estimated critical gaps from the new model

(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	1/10*s	accepted or rejected	if (2)="r", nr=nr+1	if (2)="a", na=na+1	(3)/nr,max	(4)/nr,max	(6)/[(6)+1-(5)]	(7)_j-(7)_j-1	[(1)_j-(1)_j-1]/2
index j	gap t		nr	na	Fr	Fa	Ftc	ftc	cl.m
1	5	r	1	0	0,00694444	0	0	0	2,5
2	7	r	2	0	0,01388889	0	0	0	6
3	7	r	3	0	0,02083333	0	0	0	7
4	7	r	4	0	0,02777778	0	0	0	7
5	7	r	5	0	0,03472222	0	0	0	7
6	8	r	6	0	0,04166667	0	0	0	7,5
7	9	r	7	0	0,04861111	0	0	0	8,5
8	10	r	8	0	0,05555556	0	0	0	9,5
9	10	r	9	0	0,0625	0	0	0	10
10	11	r	10	0	0,06944444	0	0	0	10,5
11	11	r	11	0	0,07638889	0	0	0	11
12	11	r	12	0	0,08333333	0	0	0	11
13	12	r	13	0	0,09027778	0	0	0	11,5
	…								
	…								
	…								
138	60	r	133	5	0,92361111	0,034722222	0,3125	0,018382353	60
139	61	r	134	5	0,93055556	0,034722222	0,333333333	0,020833333	60,5
140	62	r	135	5	0,9375	0,034722222	0,357142857	0,023809524	61,5
141	63	a	135	6	0,9375	0,041666667	0,4	0,042857143	62,5
142	63	a	135	7	0,9375	0,048611111	0,4375	0,0375	63
143	64	a	135	8	0,9375	0,055555556	0,470588235	0,033088235	63,5
144	64	a	135	9	0,9375	0,0625	0,5	0,029411765	64
145	64	a	135	10	0,9375	0,069444444	0,526315789	0,026315789	64
146	64	a	135	11	0,9375	0,076388889	0,55	0,023684211	64
147	65	r	136	11	0,94444444	0,076388889	0,578947368	0,028947368	64,5
148	66	r	137	11	0,95138889	0,076388889	0,611111111	0,032163743	65,5
149	67	a	137	12	0,95138889	0,083333333	0,631578947	0,020467836	66,5
150	67	a	137	13	0,95138889	0,090277778	0,65	0,018421053	67
151	68	a	137	14	0,95138889	0,097222222	0,666666667	0,016666667	67,5
152	69	r	138	14	0,95833333	0,097222222	0,7	0,033333333	68,5
	…								
	…								
	..								
279	328	a	144	135	1	0,9375	1	0	327
280	363	a	144	136	1	0,944444444	1	0	345,5
281	368	a	144	137	1	0,951388889	1	0	365,5
282	387	a	144	138	1	0,958333333	1	0	377,5
283	439	a	144	139	1	0,965277778	1	0	413
284	461	a	144	140	1	0,972222222	1	0	450
285	467	a	144	141	1	0,979166667	1	0	464
286	633	a	144	142	1	0,986111111	1	0	550
287	642	a	144	143	1	0,993055556	1	0	637,5
288	656	a	144	144	1	1	1	0	649
		sum	144	144				tc,mean	6,38
								sigma	1,11

FIGURE 2 - Example of a spreadsheet for estimating the critical gap

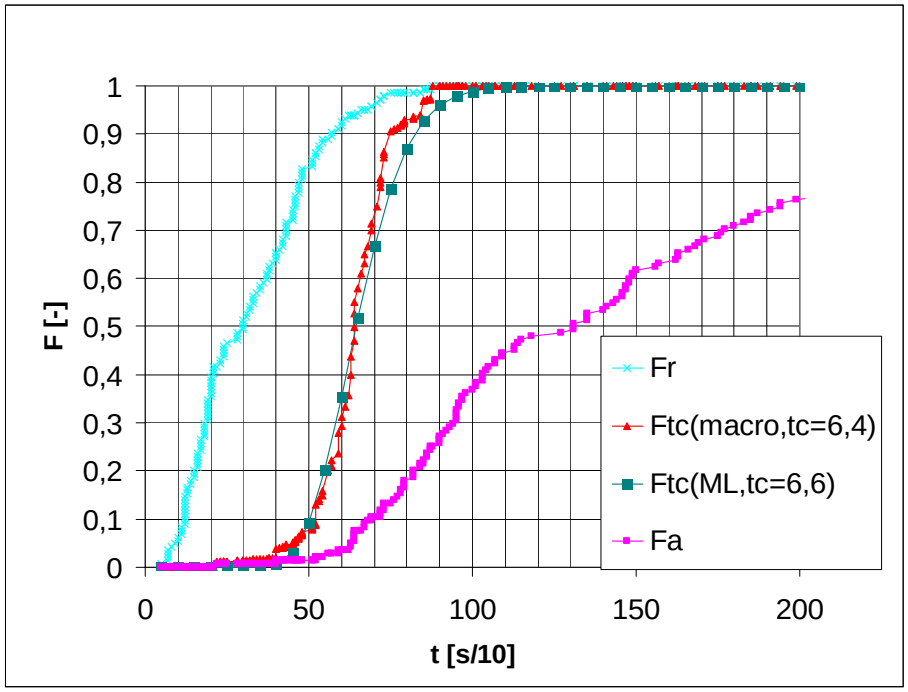


FIGURE 3 - Example for critical gap estimation.
F_r=PDF of the maximum rejected gaps, F_a=PDF of the accepted gaps, F_{tc}(ML)=PDF of the estimated critical gaps from the Maximum Likelihood model of Troutbeck, F_{tc}(macro)=PDF of the estimated critical gaps from the new model for macroscopic equilibrium (Data: Weinert (*12*), Bad Nauheim 3, minor right-turn).

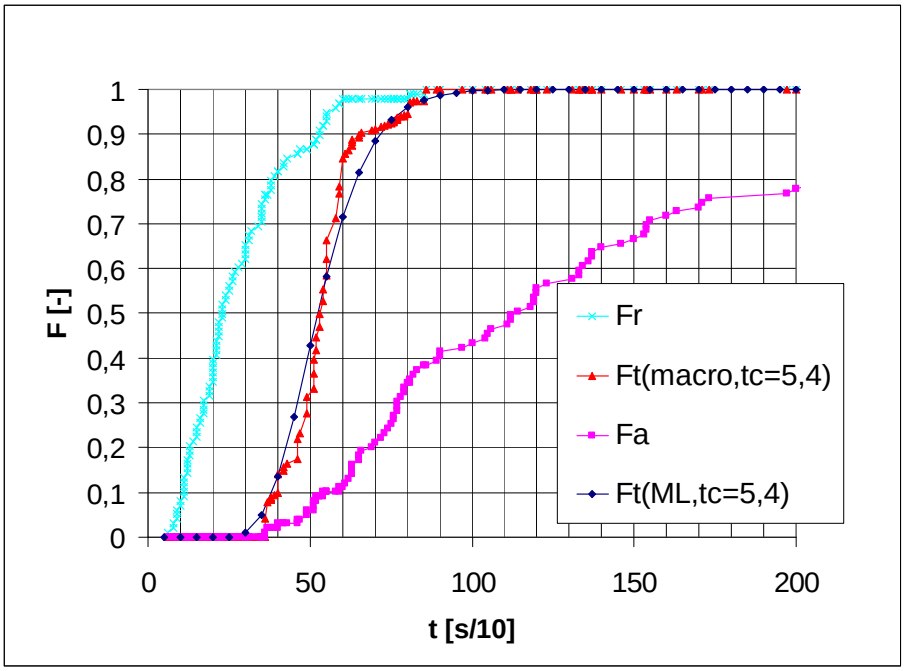


FIGURE 4 - Example for critical gap estimation.
F_r=PDF of the maximum rejected gaps, F_a=PDF of the accepted gaps, F_{tc}(ML)=PDF of the estimated critical gaps from the Maximum Likelihood model of Troutbeck, F_{tc}(macro)=PDF of the estimated critical gaps from the new model for macroscopic equilibrium (Data: Weinert (*12*), Köln 1, major left-turn).

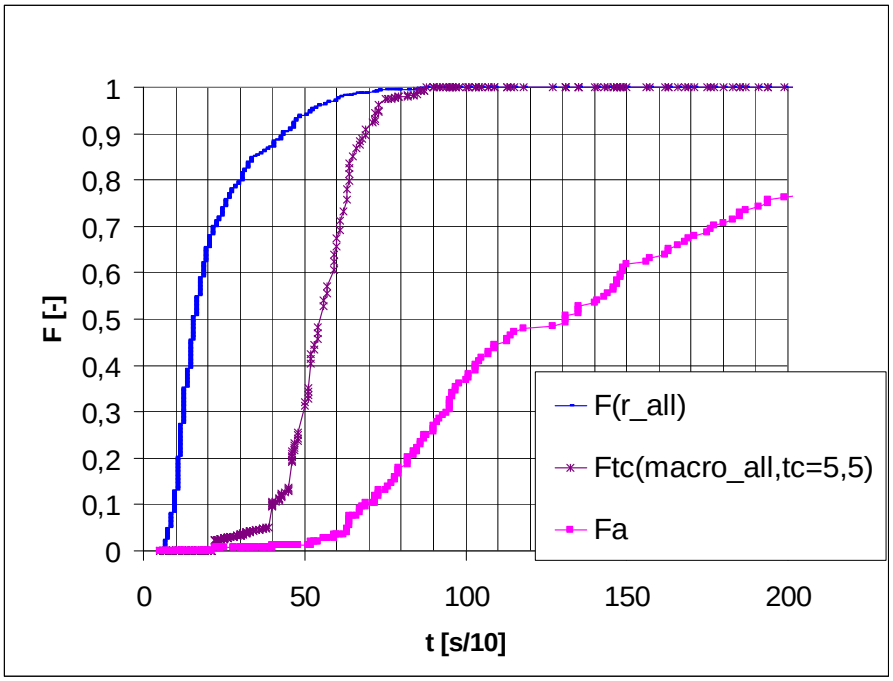


FIGURE 5 - Example for critical gap estimation.
$F_{r,all}$=PDF of all gaps, F_a=PDF of the accepted gaps, F_{tc}(macro)=PDF of the estimated critical gaps from the new model for macroscopic equilibrium
(Data: Weinert (*12*), Bad Nauheim 3, minor right-turn).

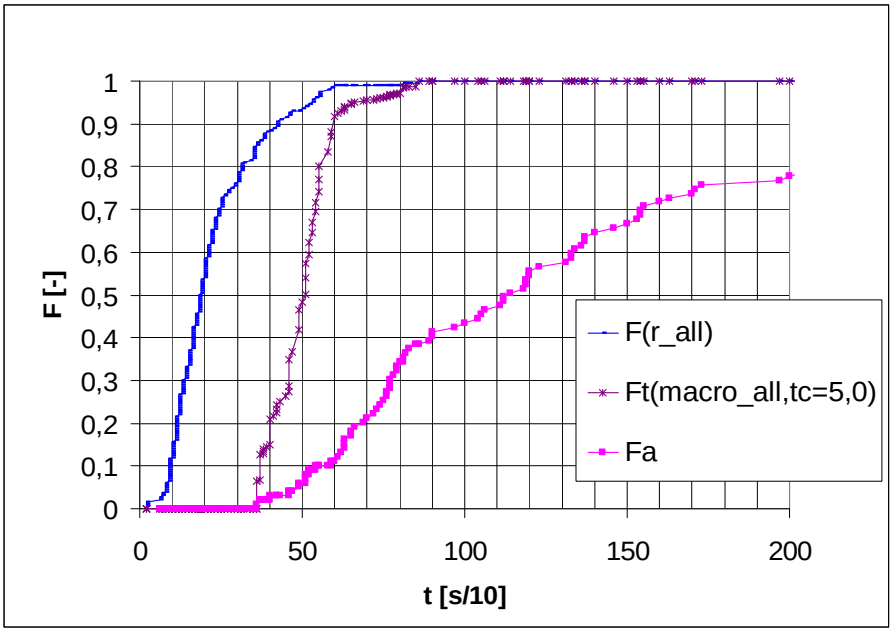


FIGURE 6 - Example for critical gap estimation.
F_r=PDF of all rejected gaps, F_a=PDF of the accepted gaps, F_{tc}(macro)=PDF of the estimated critical gaps from the new model for macroscopic equilibrium (Data: Weinert (*12*), Köln 1, major left-turn).

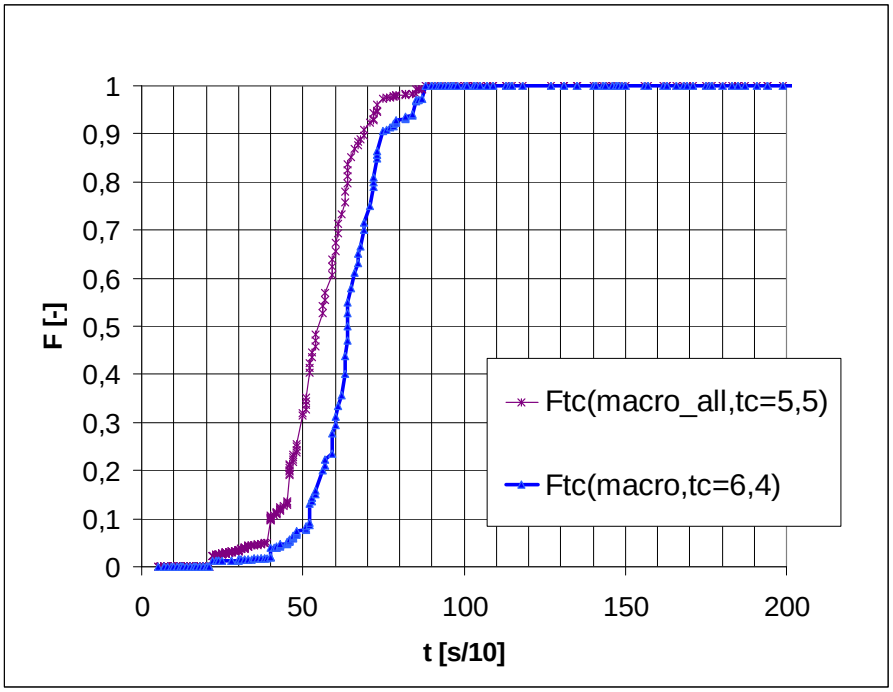


FIGURE 7 – Comparison of the estimated distributions of critical gaps. F_{tc}(macro)=PDF of the estimated critical gaps from the new model with only the maximum rejected gaps, F_{tc}(macro_all)=PDF of the estimated critical gaps from the new model with all rejected gaps (Data: Weinert (*12*), Bad Nauheim 3, minor right-turn).

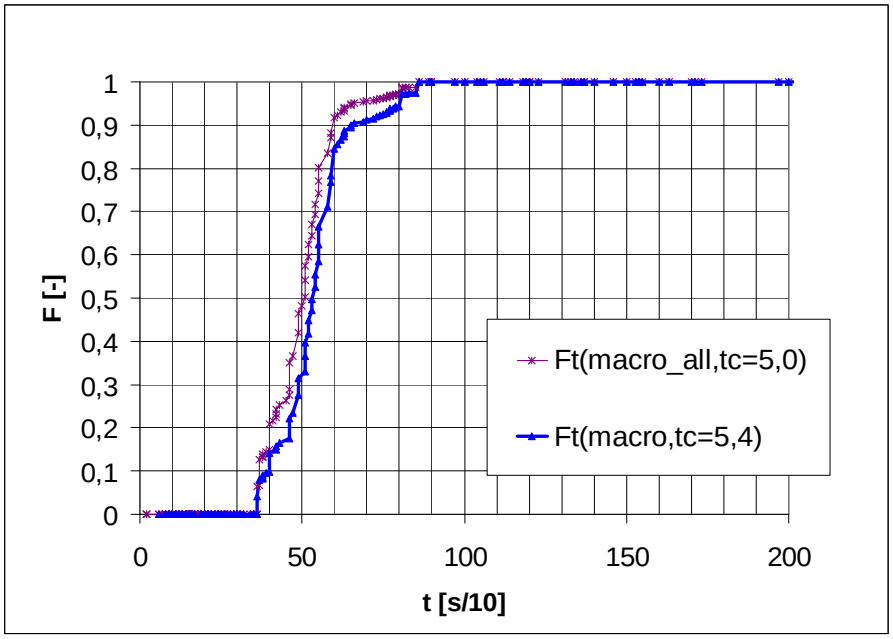
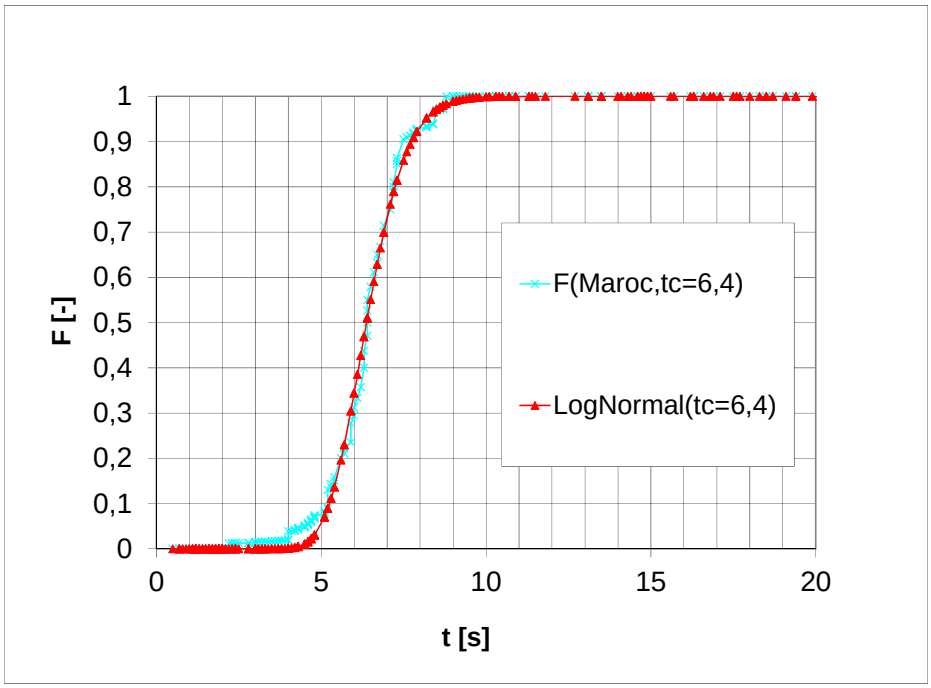
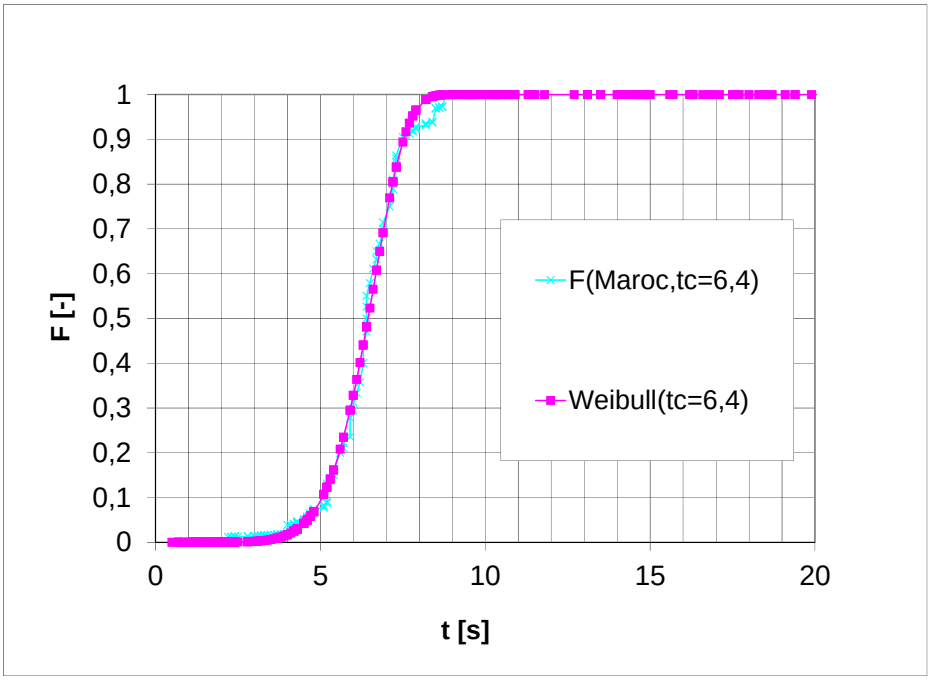


FIGURE 8 - Comparison of the estimated distributions of critical gaps. F_{tc}(macro)=PDF of the estimated critical gaps from the new model with only the maximum rejected gaps, F_{tc}(macro_all)=PDF of the estimated critical gaps from the new model with all rejected gaps (Data: Weinert (*12*), Köln 1, major left-turn).

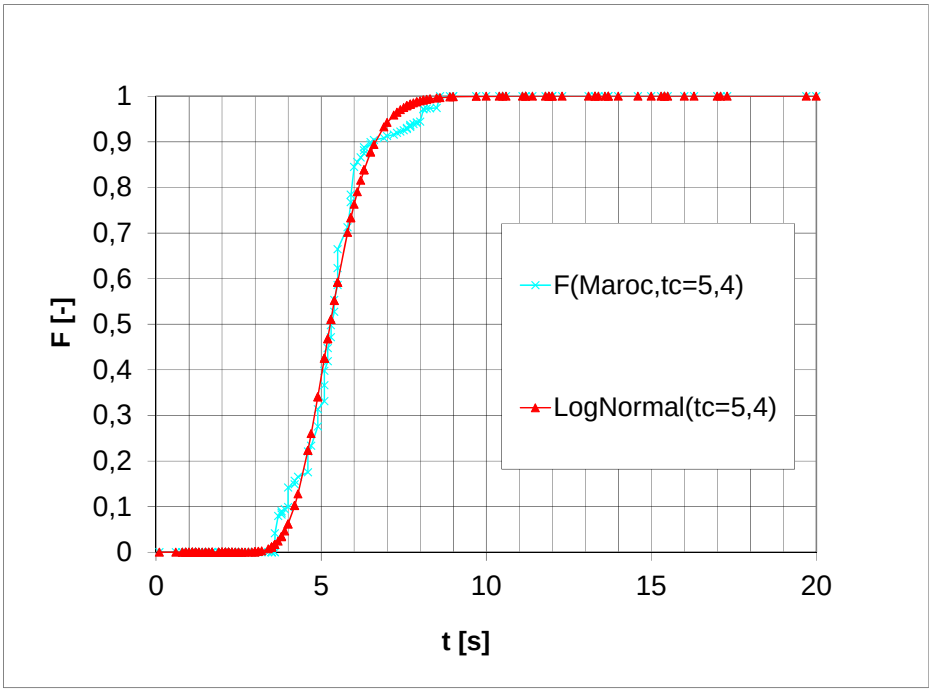


a)

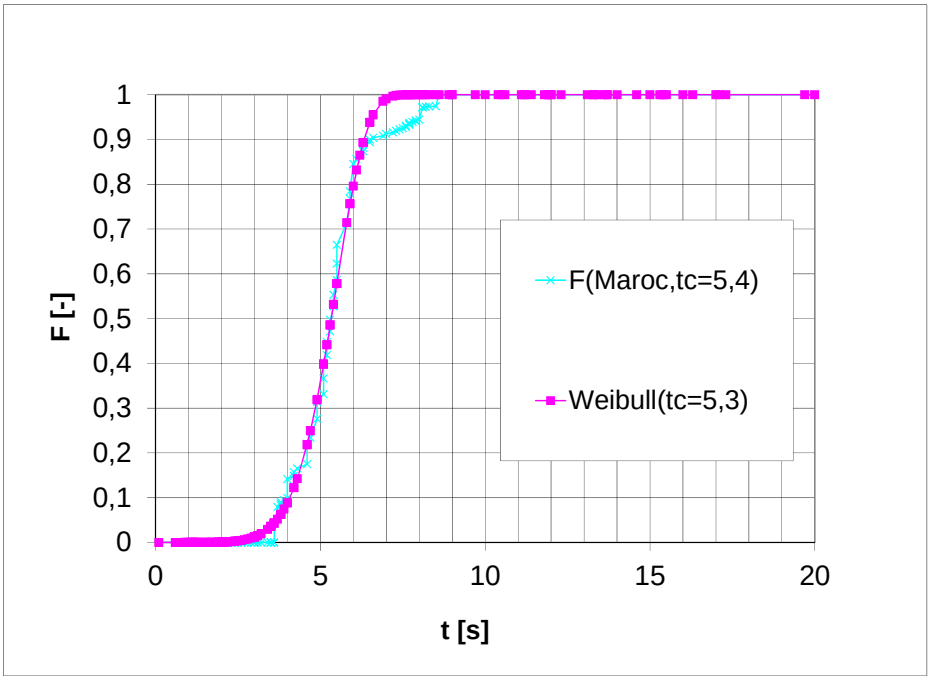


b)

FIGURE 9 – Calibration of the a) log-normal and b) Weibull distribution.
Dada 1: F_{tc}(macro)=PDF of the empirical estimated critical gaps with only the maximum rejected gaps (Data: Weinert (*12*), Bad Nauheim 3, minor right-turn).

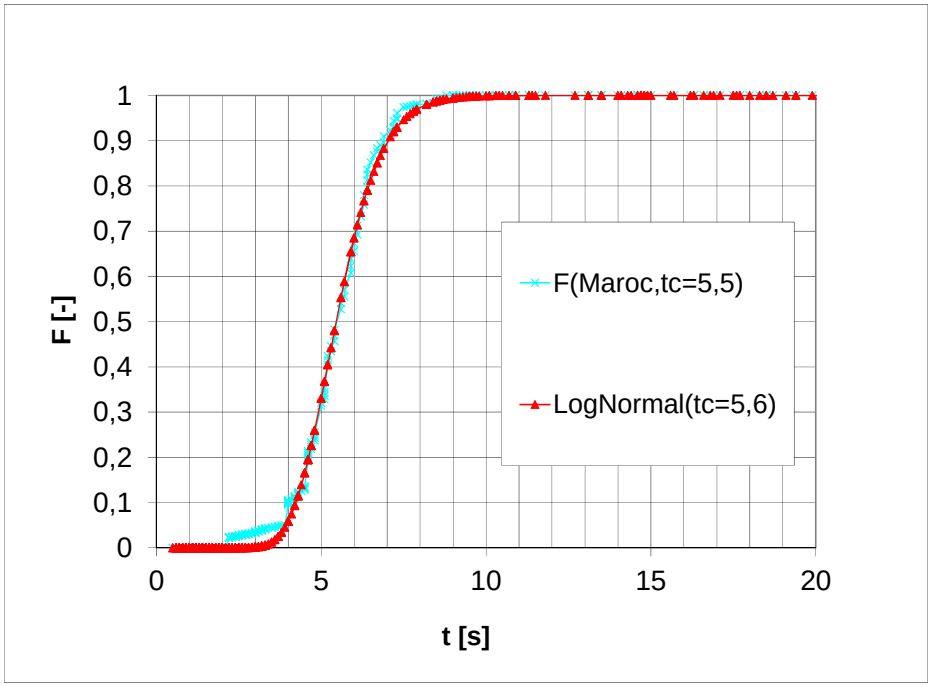


a)

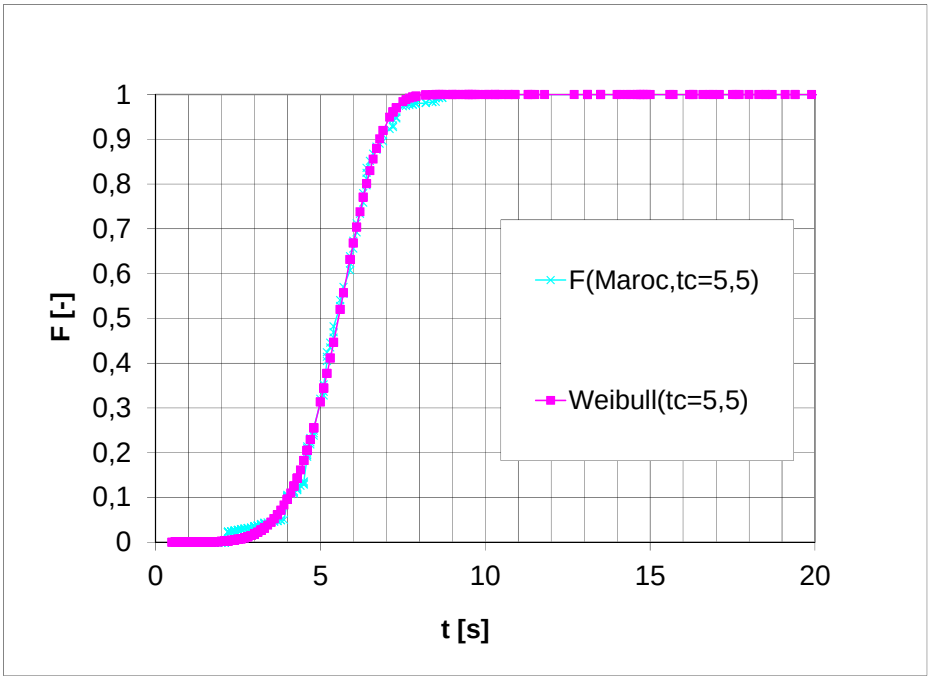


b)

FIGURE 10 - Calibration of the a) log-normal and b) Weibull distribution.
Dada 2: F_{tc}(macro)=PDF of the empirical estimated critical gaps with only the maximum rejected gaps (Data: Weinert (*12*), Köln 1, major left-turn).

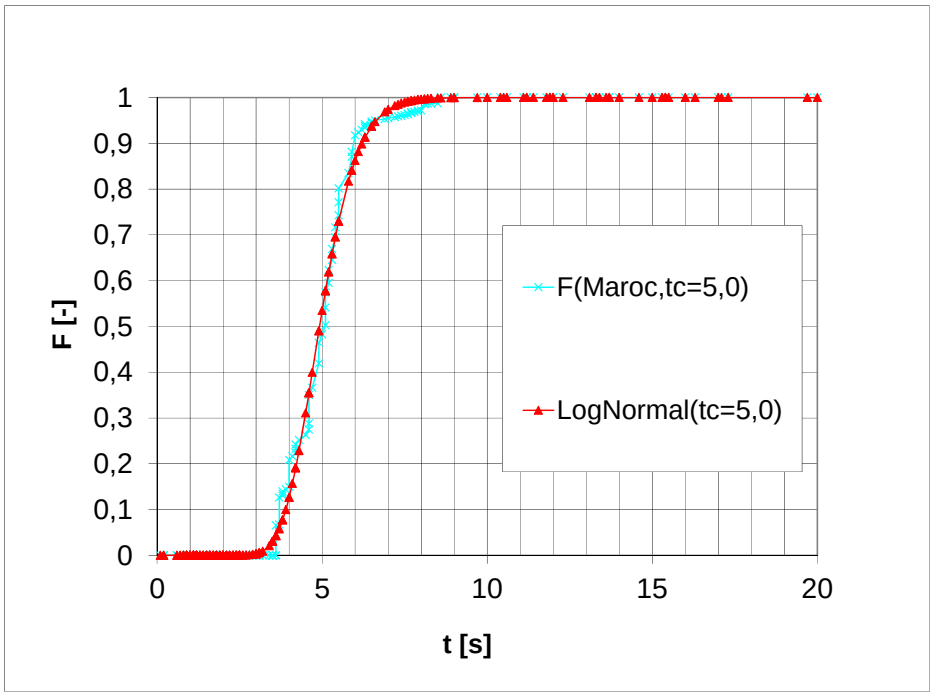


a)

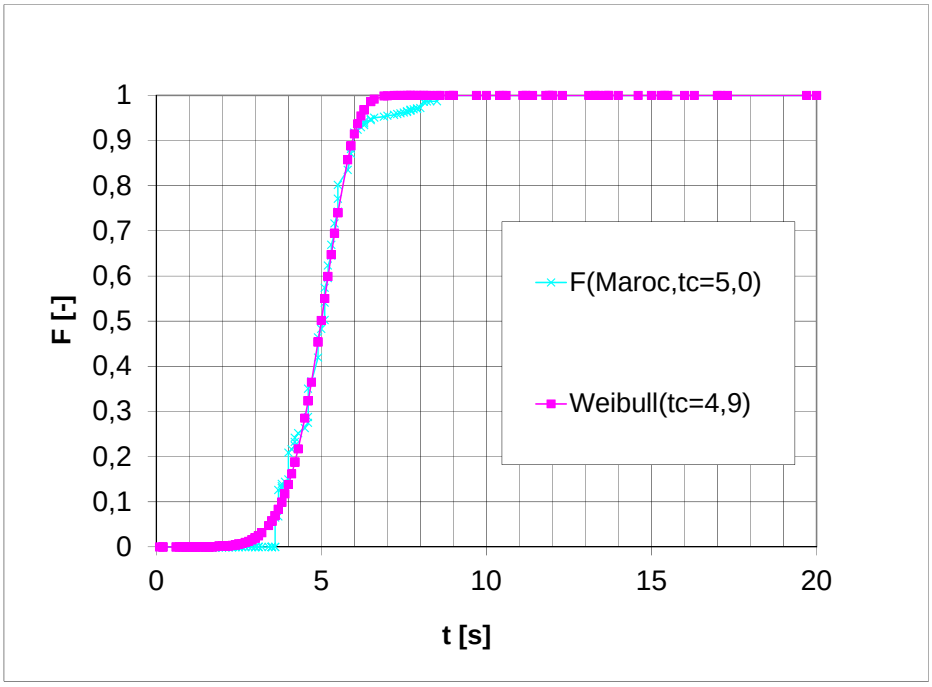


b)

FIGURE 11 – Calibration of the a) log-normal and b) Weibul distribution.
Dada 3: F_{tc}(macro)=PDF of the empirical estimated critical gaps with all rejected gaps (Data: Weinert (*12*), Bad Nauheim 3, minor right-turn).



a)



b)

FIGURE 12 - Calibration of the a) log-normal and b) Weibull distribution. Data 4 : F_{tc}(macro)=PDF of the empirical estimated critical gaps with all rejected gaps (Data: Weinert (*12*), Köln 1, major left-turn).