Two-Stage Gap Acceptance – some Clarifications

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1. ABSTRACT

Chapter 17 of the HCM 2000 provides a method for analyzing a two-stage priority process at unsignalized intersections, which provide a wide median in the major street. In the HCM itself, the method is, however, described rather briefly. This could give rise to misunderstandings, and practitioners might avoid using the procedure altogether. This being so, the analytical background of the procedure and the correct application of the parameters will be explained in greater detail. Moreover, graphical representations have been developed to replace the difficult formulas, simplifying application in the field. With these graphs, application becomes significantly easier. The method also has some limitations, which should be taken into account in practical application. Thus, this paper is something like a comment on the method described in the HCM 2000, besides providing some enhancements which allow easier application in practice.

2. INTRODUCTION

Unsignalized intersections located in streets or highways containing a wide median offer enough space to allow the minor street drivers to cross the two major streams coming from both directions independently (Fig. 1). The central area can be used by the minor drivers for queuing, with *m* representing the number of spaces, which this area provides for passenger cars in each of the two minor directions.

Due to this central storage space, minor-road drivers do not need coinciding gaps in both major directions, as would be the case at an undivided intersection. Due to this effect, two-stage priority usually has the tendency to increase an intersection's capacity for minor movements compared to the undivided case.

HCM 2000 *(1)* contains procedures for calculating two-stage priority capacity from *(2)*. However, some more detailed explanations and critical remarks might be helpful for the use of these procedures and for the interpretation of the results. Therefore, this paper attempts to provide such comments to users of Chapter 17 at unsignalized intersections within HCM 2000.

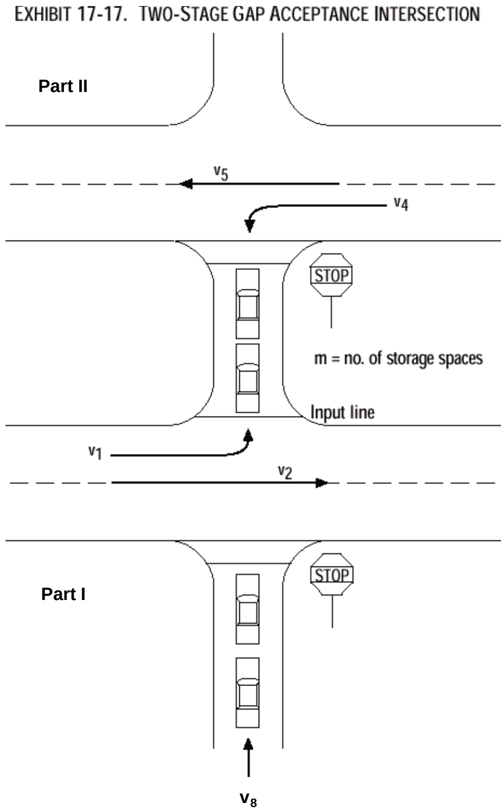

Figure 1: Sketch of a 2-stage priority intersection (source: HCM 2000) with its movements.

3. THEORETICAL BACKGROUND OF THE TOTAL CAPACITY C_T

Originally, the idea to analyze this problem dates back to Harders *(3)*. The theory was significantly expanded in *(2)*. At this stage, the concept was accepted for HCM 2000. As the method is explained in full detail in *(2)*, only a rather brief outline will be given in this paper.

The capacity of the two parts (I and II) of the intersection is explained in 4 different stages: Let us look at movement 8 (minor through) and try to estimate the capacity for this minor movement. A vehicle belonging to this flow is designated as an "8-vehicle". Probabilities for different events are abbreviated as "*p*".

Stage 1 (part I): An 8-vehicle can only cross part I of the intersection if

a) at least 1 space in the center is empty. The probability for this stage is $p_a = 1 - w_m$.

b) The capacity limited by movements 1 and 2 allows the departure of the 8-vehicle.

Thus, the total contribution of stage 1 to the movement-8 capacity is

$$C_1 = (1 - w_m) \cdot C_I \qquad \text{[veh/h]} \qquad (1)$$

where:

C_1	=	capacity for movement 8 provided during stage 1 at part I	[veh/h]
C_I	=	capacity for movement 8 when crossing part I of the intersection assuming that part I is an isolated intersection of its own. C_I is a function of volumes v_1 and v_2 .	[veh/h]
w_m	=	probability that all *m* spaces of the central storage area are occupied, where $\sum_{i=0}^{m} w_i = 1$	[-]

w_i	=	probability that i spaces of the central storage area are occupied	[-]

Stage 2 (part I and II): When the storage area is filled with m vehicles, an 8-vehicle may, alternatively, enter the central area by crossing part I , if a vehicle is departing at the same time from the through part II of the intersection.

c) The probability for this stage is $p_c = w_m$.

d) During this stage, the capacity is identical with the capacity $C_{m,x}$ of an undivided intersection carrying the same amount of traffic as the 2-stage intersection.

Thus, the contribution of stage 2 to the capacity of part I for movement 8 is

$$C_2 = w_m \cdot C_{m,x} \qquad \text{[veh/h]} \qquad (2)$$

where:

C_2	=	capacity for movement 8 provided during stage 2 at part I	[veh/h]
$C_{m,x,}$	=	capacity for movement 8 when crossing part I and part II of the intersection in one step without stopping in the central storage area. $C_{m,x,}$ is a function of volumes v_1 , v_2 , v_4 , v_5 , and v_6 .	[veh/h]
w_m	=	probability that m spaces of the central storage area are occupied	[-]

Stage 3 (part II): In part II, an 8-vehicle can only depart if

e) at least 1 vehicle is stopping in the central storage area. The probability for this is $p_e = 1 - w_0$.

f) the capacity limited by movements 4, 5 and 6 allows the departure of the 8-vehicle.

Thus, the total contribution of stage 3 to the movement-8-capacity is

$$C_3 = (1 - w_0) \cdot C_{II} \qquad \text{[veh/h]} \qquad (3)$$

where:

C_3	=	capacity for movement 8 provided during stage 3 at part II	[veh/h]
C_{II}	=	capacity for movement 8 when crossing part II of the intersection. C_{II} is a function of volumes v_4 , v_5 , and v_6 .	[veh/h]
w_0	=	probability that none of the spaces of the central storage area is occupied,	[-]

Stage 4 (part I and II): Also, if the storage area is free from any vehicles, part II can contribute to the capacity of movement 8 when an 8-vehicle drives through the whole intersection in one movement.

g) The probability for this stage is $p_g = w_0$.

h) During this stage, the capacity is identical with the capacity $C_{m,x}$ of an undivided intersection.

Thus, the contribution of stage 4 to the capacity of part II for movement 8 is

$$C_4 = w_0 \cdot C_{m,x} \qquad \text{[veh/h]} \qquad (4)$$

where:

C_4	=	capacity for movement 8 provided during stage 4	[veh/h]
$C_{m,x,}$	=	capacity for movement 8 when crossing part I and part II of the intersection in one step without stopping in the central storage area.	[veh/h]
w_0	=	probability that the central storage is empty	[-]

At capacity, flows through parts I and II of the intersection must be equal to maintain the total number of cars passing through the system.

$$C_1 + C_2 + v_1 = C_3 + C_4 \qquad \text{[veh/h]} \qquad (5)$$

From this equation, using the relations in eq.(1) through (4) as well as other mathematical derivations *(2),* we obtain the total capacity C_T which the intersection provides for movement 8 :

$$C_T = \begin{cases} \text{not defined} & \text{for } y < 0 \\ \dfrac{a}{m+1} \cdot \left[m \cdot (c_{II} - v_1) + C_{m,x} \right] & \text{for } y = 1 \\ \dfrac{a}{y^{m+1} - 1} \cdot \left[y \cdot (y^m - 1) \cdot (c_{II} - v_1) + (y-1) \cdot C_{m,x} \right] & \text{else} \end{cases} \qquad \text{[veh/h]} \qquad (6)$$

where:

C_T	=	total capacity of the intersection for movement 8 expressed as the sum of all 4 stages	[veh/h]
a, y	=	auxiliary variables (see below)	[-]

$$y = \frac{C_I - C_{m,x}}{C_{II} - v_1 - C_{m,x}} \qquad \text{for } C_I \geq C_{m,x} \text{ and } C_{II} - v_1 \geq C_{m,x} \qquad [-] \qquad (7)$$

(= eq. 17-31 in HCM 2000)

$$a = 1 - 0.32 \cdot e^{-1.3 \cdot \sqrt{m}} \qquad \text{for } m > 0 \qquad [-] \qquad (8)$$

(= eq. 17-30 in HCM 2000)

The probability that none of the spaces in the central storage area is occupied is

$$w_0 = \begin{cases} \text{not defined} & \text{for } y < 0 \\ \dfrac{1}{m+1} & \text{for } y = 1 \\ \dfrac{y-1}{y^{m+1} - 1} & \text{else} \end{cases} \qquad [-] \qquad (9)$$

Using eq.(9), we obtain w_0 as the probability of an empty storage area being available. This is also the proportion of minor-road vehicles, which do not need to do a 2-stage crossing, passing the entire intersection in one stage.

It is easy to see that the lower line in eq.(6) is not defined for $y = 1$ (division by 0). A short mathematical derivation *(2)* shows that for $y = 1$, we obtain the equation in the middle of eq.(6).

Capacities C_I, C_{II}, and $C_{m,x}$, which are input quantities for eq.(6), can be determined by any useful capacity calculation formula such as Harders' formula (which is used in the HCM *(1)*, eq. 17-3). At the same time, other capacity estimation techniques for unsignalized intersections may be combined with these 2-stage priority calculations. It should be pointed out, however, that the impedance effect of the major left-turn stream v_1 must be taken into account when calculating $C_{m,x}$. The impedance factor is $p_{0,1} = 1 - v_1/C_{II}$.

Regarding the HCM, it must be noted that to calculate capacities C_I and C_{II} as well as $C_{m,x}$ from eq. 17-2, critical gap values must be reduced. The reduction according to eq. 17-1 is *1 sec* (see: $t_{c,T} = 1$ *sec* in eq. 17-1 of the HCM), with a remaining t_c -value of $t_c = 6.5 - 1 = 5.5$ *sec* for the minor through movement 8. This reduction is due to the fact that at such intersections, visibility is usually quite good, and fewer priority movements need to be observed simultaneously, compared to a normal intersection. This has also been observed in a limited number of real-life cases *(4)*. Of course, these reduced critical gaps contribute to the increase in capacity resulting from the procedure. At this point, it should be noted that the follow-up time in movement 8 according to exh. 17-4 is $t_f = 4$ *sec.*

Because of the correction of the critical gaps, capacities C_I and C_{II} for the two-stage case can not be derived from the graphs in the HCM (like exh. 17- 6 or 17-7). Thus, it is a bit unfortunate that these capacities must be evaluated from the original eq. 17-3 in the HCM (Harders' formula *(3)*). For greater ease, Fig. 2 indicates the potential capacities C_I and C_{II} as well as $C_{m,x}$ for the two-stage priority case. They can be introduced into eq.(6) to calculate the total capacity C_T .

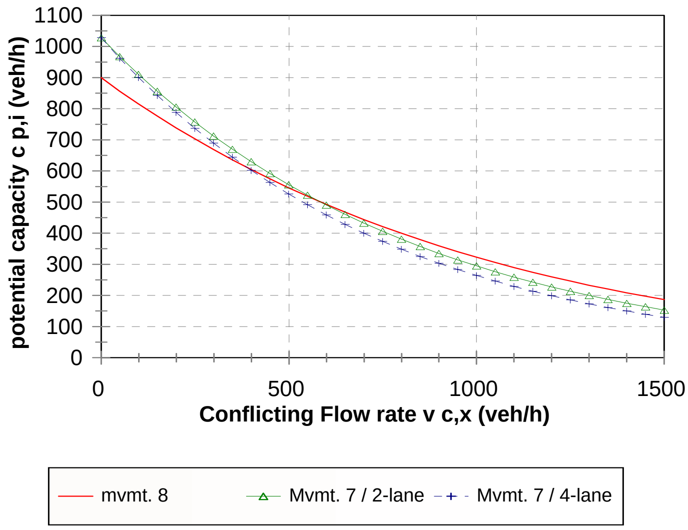

Figure 2: Potential Capacities for C_I , C_{II} , and $C_{m,\,x}$ may be obtained from this graph by adding the conflicting flows according to exh. 17-4 of the HCM. For calculating the value of $C_{m,\,x}$, the value read from the graph should multiplied by the impedance factor $p_{0,1}$ = 1-v_1/C_{II}.

The calculation of capacities $C_{I,}$, C_{II}, and $C_{m,\,x}$ is predicated on a clear understanding of the conflicting (i.e. priority movements) volumes. This is clearly indicated by the 4 lower lines of exh. 17-4 in the HCM, a table precisely tailored for two-stage priority. Similar tables might be of much help in standard single-stage cases with a future correction of the HCM.

The variable *a* is an adjustment factor used to overcome some shortcomings in the theoretical derivation. Thus, some of the statistical interdependencies between parameters were neglected. A useful function for *a* (eq.(8)) has been developed in *(2)* using Monte Carlo simulations. As an alternative to eq.(8), the original description in *(2)* provides another function which is more precise and can be used instead of eq.(8) to improve computation results.

Some comments on the significance of the auxiliary variable *y* appear in order here. One basic condition for the validity of the method is that for *y*, only positive values are allowed. Otherwise, eq.(6) would yield negative values for the total capacity C_T . How could *y* become negative ? The upper part of eq.(7) (C_I - $C_{m,x,}$) must always be greater than 0, since $C_{m,x,}$ is calculated from the same priority volumes as C_I (i.e. v_1 , v_2) plus the priority volumes at part II (v_4 , v_5, v_6) . Thus, *y* can only become negative if the denominator in eq.(7) is negative. It is easy to understand that if C_{II} - v_1 < 0 , then the whole capacity of part II is absorbed by the left turner at part I and there is no capacity left for movement 8 . Therefore, in this case $C_T = 0$ is the right solution. However, within a range of C_{II} of $v_1 < C_{II} < v_1 + C_{m,\,x}$ the constraint imposed by eq.(6) and (7) is less plausible. This will have to be considered further at some future time.

So far, all derivations apply to movement 8 only. The minor left movement 7 may be treated similarly. Special attention, however, must be paid to the fact that this movement needs its own critical gap:

$t_c = 7.1 – 1 = 6.1$ *sec* for a 2-lane and $t_c = 7.5 – 1 = 6.5$ *sec* for a 4-lane major street

with special attention to be paid to a further reduction by *0.7 sec* for a 2-stage T-junction and a shorter follow-up time *(t_f = 3.5 sec)* at part II as it is indicated by exh. 17-5 of the HCM. These special rules should only apply to part II since at part I (at an intersection like that shown in Fig. 1) both the minor left turn as well as the minor through movement must follow the same pattern. Thus, the corresponding lines (left turn) in Fig. 2 should only be applied to part II of the intersection.

4. GRAPHIC SOLUTIONS

Eqs.(6) – (9) are rather difficult to solve manually, so that graphic representations should be helpful. When working with these graphs, the following points should be noted:

- It is useful always to treat C_{II} - v_1 together in one term since eq.(6) and (7) also contain C_{II} only in this combination.
- $C_{m,x}$ can be recalculated if C_I ,C_{II} , *and* v_1 are given. As a very close approximation, we can use

$$C_{m,x} \approx \frac{C_I \cdot (C_{II} - v_1) \cdot t_f}{3600} \qquad \text{[veh/h]} \qquad (10)$$

where:

t_f = follow-up time of the movement under consideration
(= 4 s in the relevant case according to exh. 17-5 of the HCM) [s]

Equation (10) is recommended in the German guideline HBS *(6)*. Using eq.(10), the auxiliary variable y can never be smaller than 0. Thus, eqs.(6) and (9) will always be defined.

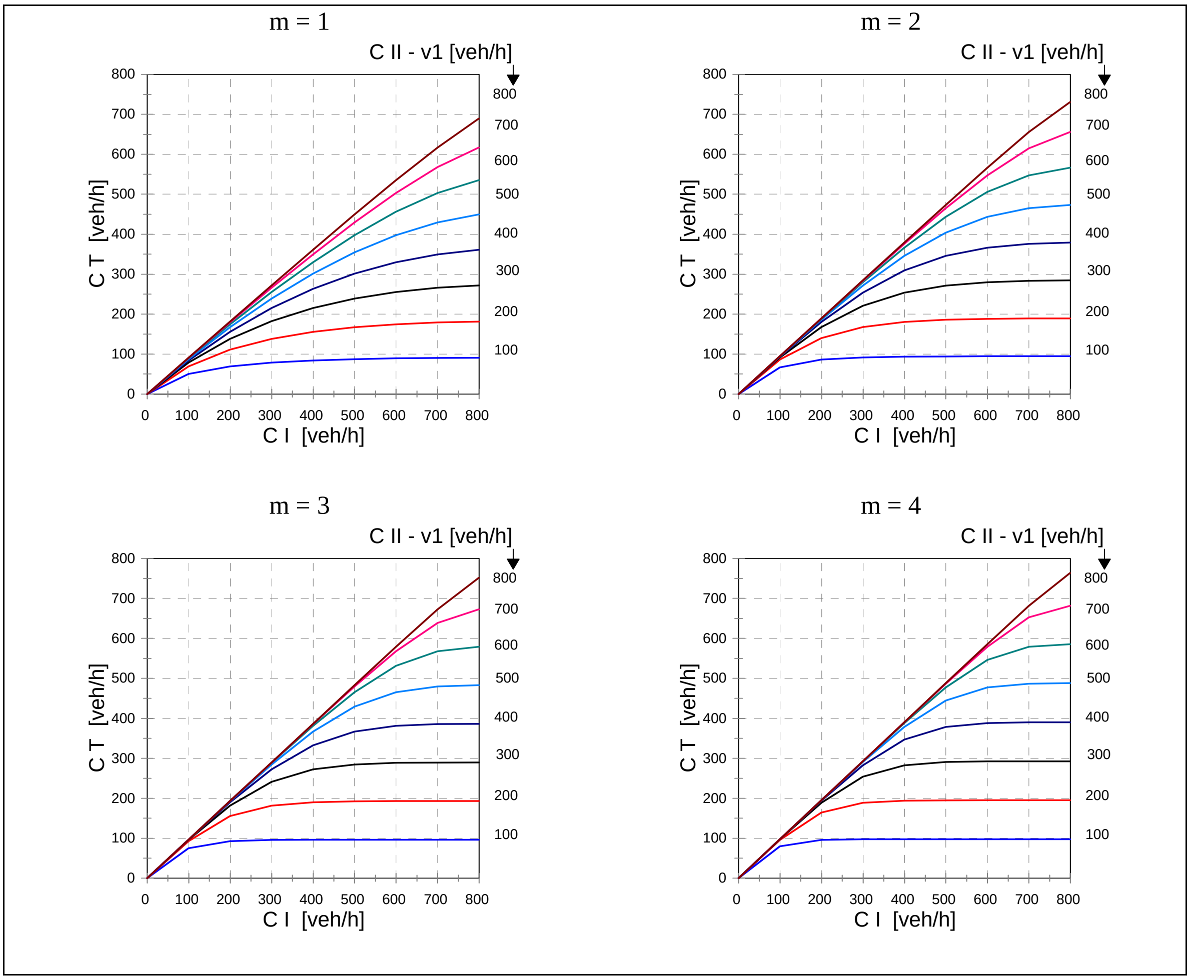

Figure 3: Capacity C_T as a function of C_I and $C_{II} - v_1$ according to eq.(6) for m= 1, 2, 3 , and 4

With these points, we can establish C_T and w_0 as functions mainly of the two parameters C_I and $(C_{II} - v_1)$ with the additional parameter m. v_1 is eliminated as an input parameter. The functions of C_T and w_0 are shown as graphs in Fig. 3 and 4. The graphs can be entered on the horizontal axis with C_I and on the curves with $C_{II} - v_1$. Then C_T and w_0 can be obtained on the left vertical axis. The graphs allow for easy interpolation.

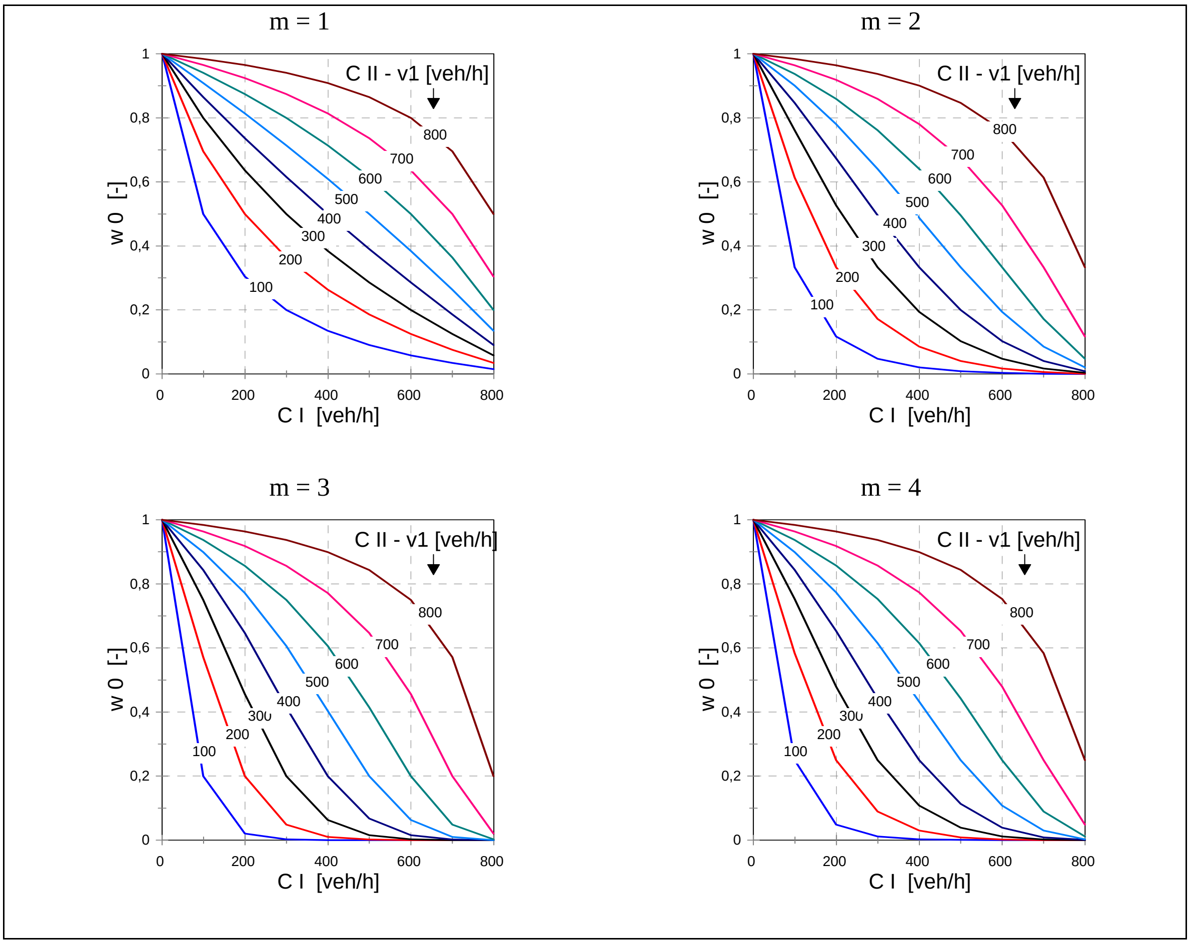

Figure 4: Proportion of minor-road vehicles which do not need to do a 2-stage crossing and drive though the whole intersection in one **s**tage, w_0 , as a function of C_I and C_{II} according to eq.(9) for m= 1, 2, 3 , and 4

5. LIMITATIONS OF THE METHOD

Due to the nature of the derivations and of the two-stage priority process, we have to be aware of certain limitations and peculiarities of the results:

- The total capacity for movement 8 cannot exceed both C_I or $C_{II} - v_1$; i.e. $C_T < \mathrm{Min}\{C_I, C_{II} - v_1\}$.
- The first space *(m = 1)* in the central area has a much bigger effect on increasing the capacity than the second or the third space could have (see e.g. Fig. 5). Usually, more than half the maximum possible increase in capacity (compared to an undivided major street) may be achieved with no more than one storage space in the center of the intersection.
- The theory has only been developed for a situation in which all vehicles in the central storage area are queuing in one lane one behind the other in both minor-street directions. Multi-lane queuing in this area is not precisely

described by the theory. The method may, however, be used as a first approach even for intersections where the vehicles are standing side by side in the center of the intersection. A solution to this problem is offered in *(5)*.

- The model assumes that in part II, movement 1 (major left turn) must obey the priority of the opposing left turn movement 4. This is realistic at intersections designed rather like the sketch in Fig. 1. In reality, however, many large unsignalized intersections allow a tangential left turn where the two major left turns do not intersect or impede each other. It has to be stated that the HCM procedures were not made to cover this case at two-stage priority intersections. To resolve this contingency, an extended theory has recently been developed *(5)* which still needs some refinement before it can be issued for practical use.

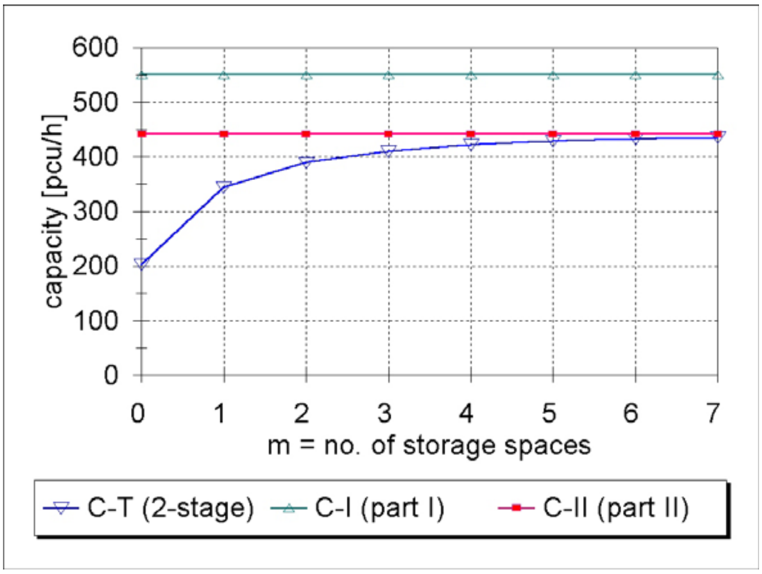

Figure 5: Influence of the number *m* of storage spaces in the median on the total capacity for one example.

6. EMPIRICAL EVIDENCE

The whole 2-stage priority method is mainly based on theory. Some real-world evidence was provided by NCHRP Project 3-46 *(4)*. Another example has been studied by the Authors in Germany. Here, a rural and rather conventional intersection between two 2-lane rural highways had been slightly overloaded during peak hours, so much so that setting up a traffic signal was being considered. To sidestep this option, the highway administration converted the site into a two-stage priority intersection by adding two islands in the middle of the major highway (Fig. 6). Between these islands there was a 10-m-wide area offering enough space for one passenger car from both directions of the minor street. The space was even wide enough for two vehicles (one from each direction) not to impede each other's visibility. Detailed before-and-after observations showed that the resultant capacity increase was within the range predicted by the calculation. Queuing on the minor approaches was significantly reduced to an acceptable level, so that the traffic signal was no longer needed. Moreover, safety at this point remained at an acceptable level. However, drivers in off-peak periods were rather reluctant to accept the crossing of the major street in two stages. They seemed to consider it risky to wait in the middle of the major street between the two fast moving major streams. During the design of the construction project, it was found that the road surface presented some problems. The lateral slopes had to be built to ensure comprehensive drainage for the whole intersection, which led to the design of a roof-shaped cross section (highest point in the center of the storage area). On the other hand, this made it difficult for drivers to discern the exact margins of the opposite major street roadway. Thus, the point for stopping in the center area was difficult to detect.

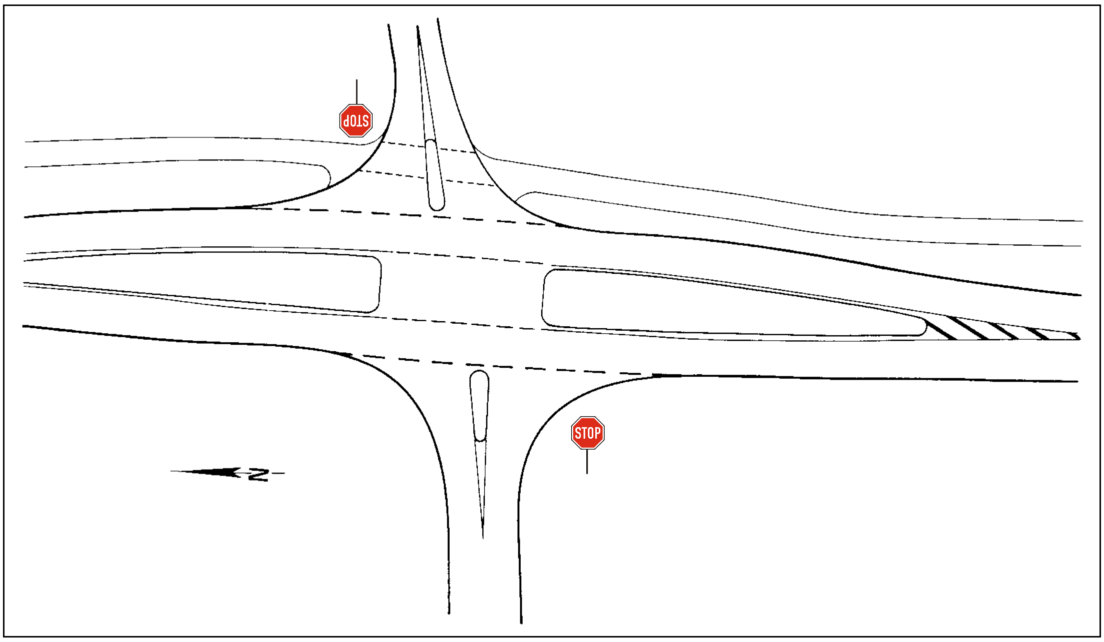

Figure 6: Example for the use of the two-stage priority process at a rural intersection in Germany

7. CONCLUSION

The two-stage priority procedure described in Chapter 17 of the HCM 2000 significantly enhances the range of possibilities for analytically evaluating capacities and flow qualities of real-world unsignalized intersections. The method demonstrates the capacity-increasing effect of a two-stage priority system. However, the method is described too briefly in the HCM for users to completely understand the method and its correct application. This paper has attempted to illustrate the analytical background of the procedure as well as the correct use of the various parameters. Moreover, the method may be represented graphically. With the aid of these graphs, the HCM procedure becomes significantly easier to use. Besides, the method has some limitations, which should be taken into account in practical application. Although based on theory, the method has a useful potential for estimating the effects of two-stage priority acceptance on capacity and quality of flow.

8. REFERENCES

1. **HCM 2000 :** *Highway Capacity Manual*, edition 2000. Transportation Research Board, Washington D.C., 2000.
2. **Brilon, W., Wu, N., Lemke, K. :** Capacity at Unsignalized Two-Stage Priority Intersections. *Transportation Research Record*, TRB, Washington DC, No. 1555, pp. 74 - 82, 1996.
3. **Harders, J. :** Die Leistungsfaehigkeit nicht signalgeregelter staedtischer Verkehrsknoten (Capacity of unsignalized urban intersections). *Series "Strassenbau und Strassenverkehrtechnik"*, vol. 76 , Bonn, 1968
4. **Kyte, M., Tian, Z., Mir, Z., Hammedansor, Z., Kittelson, W., Vandehey, M., Robinson, B., Brilon, W., Troutbeck, R. :** Capacity and Level of Service at Unsignalized Intersections. *National Cooperative Highway Research Program* Project 3-46, 1996.
5. **Wu, N. :** Capacity at unsignalized two-stage priority intersections – Generalization and extension. *Unpublished paper*, copy can be obtained from ning.wu@rub.de , 2002
6. **FGSV (editor) :** *Handbuch fuer die Bemessung von Strassenverkehrsanlagen* (German Highway Capacity Manual). FGSV Verlag GmbH, Köln, 2001.