

Capacity at Unsignalized Two-Stage Priority Intersections

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Abstract

This paper deals with the capacity of minor traffic movements across major divided four-lane roadways (also other roads with two separate carriageways) at unsignalized intersections. The center of the intersection, corresponding to the width of the median, often provides spaces in which the drivers who crossed the first half of the major road may stop before proceeding across the second major traffic stream. This situation which is common at multilane major streets is called two-stage priority. Here the capacity for minor through traffic is larger than at intersections without such a central storage space. The additional capacity being provided by these wider intersections can not be evaluated by conventional capacity calculation models.

This paper presents an analytical theory for the estimation under two-stage priority conditions. It is based on a former approach by Harders. However, a set of major improvements were necessary to match the results with realistic conditions. In addition to analytical theory, simulations have been performed which enable an analysis under more realistic conditions. As a result a set of equations is presented which compute the capacity for a minor through traffic movement in the two-stage priority situation. These equations are completed by two sets of graphs which enable an easy application of the theory in practice.

keywords:

capacity, unsignalized intersection, two-stage priority, median central reserve.

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At many unsignalized intersections there is a space in the center of the major street available where several minor street vehicles can be stored between the traffic flows of the two directions of the major street, especially in the case of multilane major traffic. This storage space within the intersection enables the minor street driver to pass each of the major streams at a time. This behaviour can contribute to an increased capacity.

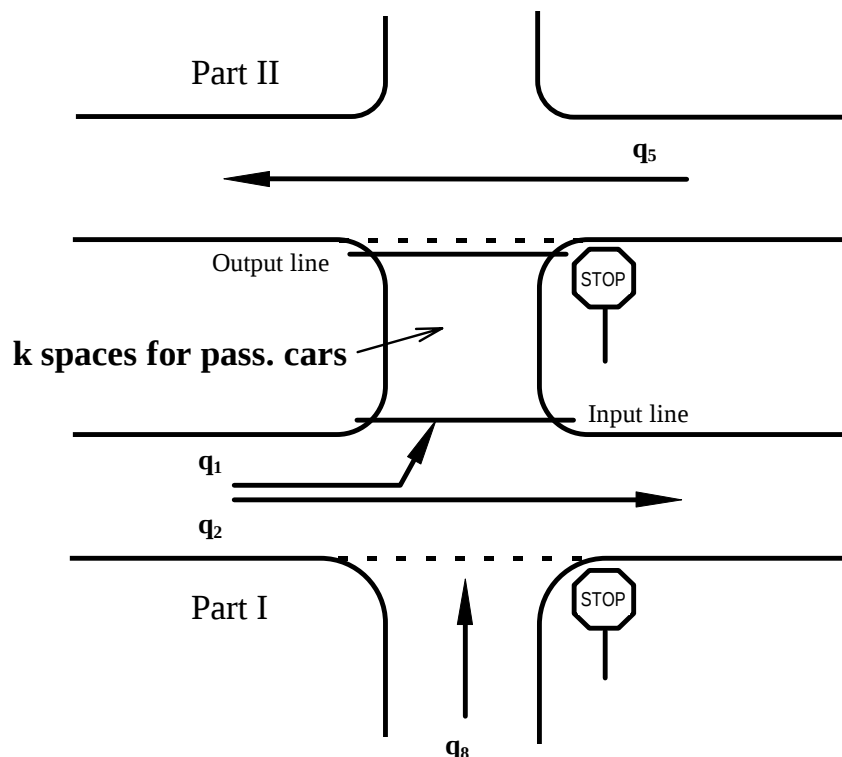


Figure 1: Minor street through traffic (movement 8) crossing the major street in 2 phases. The theory discussed here is also available if the major street provides more or less than 2 lanes per direction.

Therefore, a model is needed which can describe this behaviour and its implication on the intersection capacity. A model of this type has been developed by Harders (1968). His concept has been used here as a basis and it is described in the following derivations. However, some major amplifications as well as a correction and an adjustment to reality have been made to achieve better correspondance to realistic conditions.

For our derivations we look at an intersection consisting of two parts according to Fig. 1. Between the partial intersections I and II there is a storage space for k vehicles. This area has to be passed by the left turner from the major street (movement 1) and the minor through traffic (movement 8). Also the minor left turner (movement 7) has to pass through this area. We will see that movement 7 can be treated like movement 8. Therefore, for our derivations we concentrate on the minor through traffic (movement 8) crossing both parts of the major street. The enumeration of movements has been chosen in accordance with chapter 10 of the HCM (1994). We assume that the usual rules for unsignalized intersections from the highway code are applied by drivers at the intersections. Thus movements 2 and 5 (major through traffic) have priority over each other movement. Movement 1 vehicles have to obey the priority of movement 5 whereas movement 8 has to give the right of way to each of the movements shown in Fig. 1. In our derivations movement 5 stands for all major traffic streams at part II of the intersection. These, depending on the layout of the intersection, could include through traffic (movement 5), left turners (movement 4) and right turners (movement 6).

2. ANALYTICAL MODEL FOR THE DETERMINATION OF THE CAPACITY

To determine the capacity of the whole intersection we assume a constant queue on the minor approach (movement 8) to part I.

Let w_i be the probability for a queue of i vehicles queueing in the storage space within the central reserve. Then the probabilities w_i for all of the possible queue lengths i must sum up to 1 with $0 \leq i \leq k$, i.e.:

$$\sum_{i=0}^k w_i = 1 \quad (1)$$

where k is the number of spaces in the storage space within the central reserve

Now we consider the central area of the intersection as a closed storage system, which is limited by the input line and output line (cf. Fig 1). The capacity properties of the storage system are restricted due to the aspects of maximum input and maximum output. We now have to distinguish between different states of the system:

1. State 1 :

We first consider part I of the intersection which decides on the input to the storage area. Under state 1 we consider situations during which the number i of vehicles in the storage area is less than the maximum possible queue length k , i.e. $i < k$. During this state a minor street vehicle from movement 8 can enter the storage space if the major streams (volume q_1 and q_2) provide sufficient gaps. In this case the capacity of part I (possible input from movement 8) characterizes the capacity, i.e.:

$$c_1 = c(q_1 + q_2) \quad (2)$$

where

$c(q_1 + q_2)$ = capacity of part I in case of no obstruction by the subsequent part II ,
which is the capacity of an isolated unsignalized cross intersection for
through minor traffic with major traffic volume $q_1 + q_2$.

The probability for this state 1 is $p_1 = 1 - w_k$. Thus, the contribution of state 1 to the
capacity of part I for movement 8 is

$$c_{1,1} = (1 - w_k) \cdot c(q_1 + q_2) \quad (3)$$

Of course, during state 1 also vehicles from movement 1 can enter the storage space.

2. State 2 :

For this state we assume that the storage area is occupied; i.e. k vehicles are queueing in the
storage space. In this case normally no minor vehicle from movement 8 or vehicles from
movement 1 can get into the storage area. If, however, a sufficient gap for the passage of one
minor street vehicle can be accommodated at both parts (I and II) of the intersection
simultaneously then also a vehicle can get into the storage area. The capacity for q_8 (possible
input from movement 8) during this stage is

$$c_2 = c(q_1 + q_2 + q_5) \quad (4)$$

where

$c(q_1 + q_2 + q_5)$ = capacity of an isolated cross intersection for through traffic with major
traffic volume $q_1 + q_2 + q_5$:

Thus, the contribution of state 2 to the capacity of part I is

$$c_{1,2} = w_k \cdot c(q_1 + q_2 + q_5) \quad (5)$$

where w_k = probability that k vehicles are in the storage space

State 1 and state 2 exclude each other. The capacity of part I is the total maximum input to the storage area. Here the volume q_1 of movement 1 in addition to the partial capacities mentioned above has to be included. Therefore, the total maximum input to the storage area is

$$\begin{aligned} Input &= c_{I,1} + c_{I,2} + (1-w_k) \cdot q_1 \\ &= (1-w_k) \cdot [c(q_1 + q_2) + q_1] + w_k \cdot c(q_1 + q_2 + q_5) \end{aligned} \quad (6)$$

3. State 3 :

We now consider the output of the storage area. Here we concentrate on part II of the intersection. For $i > 0$ each possibility for a departure from the storage area provided by the major stream of volume q_5 can be utilized. The capacity (maximum output of the storage area) of part II in this case is

$$c_3 = c(q_5) \quad (7)$$

where $c(q_5)$ = capacity of part II in case of no obstruction by the upstream part I
which is the capacity of an isolated unsignalized cross intersection for
through minor traffic with major traffic volume q_5 .

The probability for this state is $p_3 = 1 - w_0$.

Thus the contribution of state 3 to the capacity of part II is

$$c_{II,3} = (1 - w_0) \cdot c(q_5) \quad (8)$$

where w_0 = probability that 0 vehicles are in the storage space

No vehicles from movement 1 (volume q_1) can directly (i.e. without being impeded by movement 5) pass through the storage area in this state.

4. State 4 :

For $i = 0$ (i.e. an empty storage area) no vehicle can depart the storage area even if the major stream of volume q_5 would allow a departure. If, however, a sufficient gap is provided in the major streams of both parts of the intersection simultaneously, a minor street vehicle from movement 8 can pass the whole intersection without being queued somewhere in the storage area. The possible output of the storage area from movement 8 vehicles during this state is

$$c_4 = c(q_1 + q_2 + q_5) \quad (9)$$

Thus, the contribution of state 4 to the capacity of part II is

$$c_{II,4} = w_0 \cdot c(q_1 + q_2 + q_5) \quad (10)$$

Also vehicles from movement 1 can pass through the storage area in this state. The number of vehicles from movement 1 which pass through the storage area in this state is

$$c_{II,4,q1} = w_0 \cdot q_1 \quad (11)$$

Here, $c_{II,4,q1}$ does not mean the capacity for q_1 , but the demand on the capacity. The traffic intensity of q_1 should be less than the capacity of the part II $c(q_5)$. i.e. q_1 is subject to the restriction $q_1 < c(q_5)$. Otherwise, the intersection is overloaded and due to this non-stationarity no solution can be derived.

State 3 and 4 exclude each other. Therefore, the total maximum output of the storage area is

$$\begin{aligned} output &= c_{II,3} + c_{II,4} + c_{II,4,q1} \\ &= (1-w_0) \cdot c(q_5) + w_0 \cdot c(q_1 + q_2 + q_5) + w_0 \cdot q_1 \\ &= (1-w_0) \cdot c(q_5) + w_0 \cdot [c(q_1 + q_2 + q_5) + q_1] \end{aligned} \quad (12)$$

One might argue that the derivations of $c_{I,2}$ and $c_{II,4}$ neglect the travel time of the vehicles from part I up to part II. This, however, is justified: The probability that a minor street vehicle will meet a sufficient gap in part I and part II at time t_I and time t_{II} (with $t_{II} = t_I + \Delta t$) is independent of the travel time Δt if $\Delta t = \text{constant}$ for all vehicles and if the two arrival processes in the major streams are independent of each other. Therefore, the result is the same if Δt has a realistic positive value or if Δt is assumed to be 0.

During times when the whole intersection is operating at capacity, due to reasons of continuity, the maximum input and output of the storage area must be equal.

Therefore

$$\text{input} = \text{output} \quad (\text{cf. eq. 6 and eq. 12})$$

i.e.:

$$(1-w_k) \cdot [c(q_1+q_2) + q_1] + w_k \cdot c(q_1+q_2+q_5) = (1-w_0) \cdot c(q_5) + w_0 \cdot [c(q_1+q_2+q_5) + q_1] \quad (13)$$

The total capacity c_T for minor through traffic (movement 8) regarding the whole intersection is identical to both sides of this equation minus q_1 . In addition, since negative traffic volumes are not possible c_T must fulfill the restriction:

$$c_T = \max \begin{cases} \text{output} - q_1 = c_{II,3} + c_{II,4} + c_{II,4,q1} - q_1 \\ 0 \end{cases} \quad (14)$$

For the easiest case of $k = 1$ we get

$$w_0 + w_1 = 1 \quad (15)$$

Together with eq. 13 and the subsequent explanation we get

$$w_0 = [c(q_5) - c(q_1+q_2+q_5)] / [c(q_1+q_2) - 2 c(q_1+q_2+q_5) + c(q_5)]$$

$$w_k = 1 - w_0 = [c(q_1+q_2) - c(q_1+q_2+q_5)] / [c(q_1+q_2) - 2 c(q_1+q_2+q_5) + c(q_5)] \quad (16)$$

For $k > 1$ some more general derivations are necessary. For these derivations we assume the following simplifying conditions:

- a) Let q_2 and q_5 be constant over time. Then also $c(q_2)$, $c(q_5)$, and $c(q_2 + q_5)$ are constant over time.
- b) We divide the continuous time scale into intervals of duration t_f = follow-up time = average time interval between the departure of two subsequent minor vehicles which enter into the same gap of the major flow. It is also assumed that the minimum gap between two vehicles of movement 1 is of the same size as t_f .

Let a = probability that a vehicle enters the central storage area from intersection part I during a time interval of duration t_f .

b = probability that a vehicle can pass intersection part II during a time interval of duration t_f .

a and b are variables which are only introduced for the following derivations. They need not to be evaluated later for the application of the theory. Both a and b are looked at for the fictitious case that part I and part II would be independent intersections. The follow-up time t_f for part I and part II should be of similar duration for this derivation. We now treat the process of the number of vehicles in the storage space as a statistical process with Markow-properties. We then can say

$$w_0(t) = w_0(1 - a) + w_0 \cdot a \cdot b + w_1 \cdot b \cdot (1 - a) \quad (17)$$

= probability that no vehicle is queueing
in the storage area at time t

This is valid because the case of an empty queue at time t can be achieved by the following possibilities:

- either: no queue at time $t - t_f$ (prob. = w_0) and no arrival (prob. = $1 - a$) during t_f
- or: no queue at time $t - t_f$ (prob. = w_0)
and one arrival (prob. = a) and one departure (prob. = b) during t_f
- or: one vehicle queued at time $t - t_f$ (prob. = w_1)
and no arrival (prob. = $1 - a$) and one departure (prob. = b) during t_f .

By similar considerations we get an expression for the probability of i vehicles queueing in the storage space at time t :

$$\begin{aligned}
 w_i(t) = & w_{i-1} \cdot a \cdot (1-b) \\
 & + w_i \cdot a \cdot b \\
 & + w_i \cdot (1-a) \cdot (1-b) \\
 & + w_{i+1} \cdot (1-a) \cdot b
 \end{aligned} \tag{18}$$

Since k is the maximum number of vehicles in the storage space we get

$$\begin{aligned}
 w_k(t) = & w_k \cdot (1-b) \\
 & + w_k \cdot a \cdot b \\
 & + w_{k-1} \cdot a \cdot (1-b)
 \end{aligned} \tag{19}$$

Due to the assumed stationarity of the process w_0 , w_i and w_k do not depend on each other at time t .

Equations 17 through 19 form a system of $k + 1$ equations which can be written as

$$-w_0 \cdot (a - ab) + w_1(b - ab) = 0 \tag{20}$$

$$w_{i-1}(a - ab) - w_i[(a - ab) + (b - ab)] + w_{i+1}(b - ab) = 0 \tag{21}$$

$$w_{k-1}(a - ab) - w_k(b - ab) = 0 \tag{22}$$

For abbreviation we use

$$A = a - a \cdot b \tag{23}$$

$$B = b - a \cdot b \tag{24}$$

Our system of equations 20, 21 and 22 then is written as:

$$\begin{aligned}
 (0) \quad & -A \cdot w_0 \quad + B \cdot w_1 \quad = 0 \\
 (1) \quad & A \cdot w_0 \quad -(A+B) \cdot w_1 \quad + B \cdot w_2 \quad = 0 \\
 (2) \quad & \quad A \cdot w_1 \quad -(A+B) \cdot w_2 \quad + B \cdot w_3 \quad = 0 \\
 & \dots \\
 (i) \quad & \quad \quad A \cdot w_{i-1} \quad -(A+B) \cdot w_i \quad + B \cdot w_{i+1} \quad = 0 \quad (25) \\
 & \dots \\
 (k-2) \quad & \quad \quad A \cdot w_{k-3} \quad -(A+B) \cdot w_{k-2} \quad + B \cdot w_{k-1} \quad = 0 \\
 (k-1) \quad & \quad \quad A \cdot w_{k-2} \quad -(A+B) \cdot w_{k-1} \quad + B \cdot w_k \quad = 0 \\
 (k) \quad & \quad \quad A \cdot w_{k-1} \quad - B \cdot w_k \quad = 0
 \end{aligned}$$

From the first equation we get

$$\begin{aligned}
 A \cdot w_0 &= B \cdot w_1 \\
 w_1 &= \frac{A}{B} \cdot w_0 \quad (26)
 \end{aligned}$$

From the last equation we get:

$$\begin{aligned}
 A \cdot w_{k-1} &= B \cdot w_k \\
 w_k &= \frac{A}{B} \cdot w_{k-1} \quad (27)
 \end{aligned}$$

If we sum up all our equations (0) through (i) we get:

$$\begin{aligned}
 -A \cdot w_i + B \cdot w_{i+1} &= 0 \\
 w_{i+1} &= \frac{A}{B} \cdot w_i \quad (28)
 \end{aligned}$$

The sequence of the probabilities, therefore, is forming a geometric series where each subsequent term is resulting from the prior term by a multiplication with the factor $y = A/B$.

$$y = \frac{A}{B} = \frac{a - ab}{b - ab} \quad (29)$$

i.e.:

$$w_{i+1} = y \cdot w_i \quad (30)$$

or:

$$w_i = y^i \cdot w_0 \quad (31)$$

According to equation 1 the w_i ($i = 0, \dots, k$) are subject to the restriction

$$\begin{aligned} \sum_{i=0}^k w_i &= 1 \\ \sum_{i=0}^k y^i \cdot w_0 &= 1 \\ w_0 \sum_{i=0}^k y^i &= 1 \end{aligned}$$

Therefore:

$$w_0 = \frac{1}{1 + y^1 + y^2 + \dots + y^k} \quad (32)$$

The sum in the denominator is the sum of a finite geometric series which is

$$\sum_{i=0}^k y^i = \frac{y^{k+1} - 1}{y - 1} \quad (33)$$

Thus, and with eq. 30 and 29 we get:

$$w_0 = \frac{y - 1}{y^{k+1} - 1} \quad (34)$$

$$w_k = \frac{y^{k+1} - y^k}{y^{k+1} - 1} \quad (35)$$

Let us now recall eq. 13 and 14 and combine those with eq. 34 and 35 . Then we get

$$\begin{aligned} & \left(1 - \frac{y^{k+1} - y^k}{y^{k+1} - 1}\right) \cdot [c(q_1 + q_2) + q_1] + \frac{y^{k+1} - y^k}{y^{k+1} - 1} \cdot c(q_1 + q_2 + q_5) \\ &= \left(1 - \frac{y-1}{y^{k+1} - 1}\right) \cdot c(q_5) + \frac{y-1}{y^{k+1} - 1} \cdot [c(q_1 + q_2 + q_5) + q_1] \end{aligned} \quad (36)$$

Note that in this equation the capacities $c(q_2)$, $c(q_5)$ and $c(q_2+q_5)$ as well as k are treated to be known whereas the variable y has to be obtained from the equation. As a result we get:

$$y = \frac{c(q_1 + q_2) - c(q_1 + q_2 + q_5)}{c(q_5) - q_1 - c(q_1 + q_2 + q_5)} \quad (37)$$

Using this result for y we can now calculate the total capacity c_T for the minor movement 8 using eq.14 .

$$\begin{aligned} c_T &= \left(1 - \frac{y-1}{y^{k+1} - 1}\right) \cdot c(q_5) + \frac{y-1}{y^{k+1} - 1} \cdot [c(q_1 + q_2 + q_5) + q_1] - q_1 \\ c_T &= \left(1 - \frac{y-1}{y^{k+1} - 1}\right) \cdot [c(q_5) - q_1] + \frac{y-1}{y^{k+1} - 1} \cdot c(q_1 + q_2 + q_5) \end{aligned} \quad (38)$$

It should be noted that for the special case of $k = 1$ using some algebra we get the solution of eq. 16 which might give some confirmation for the above derivations.

For $y = 1$ (i.e. $c(q_1 + q_2) = c(q_5) - q_1$) this expression is not defined. By developing the limiting case for $y \rightarrow 1$ we get

$$c_T = \frac{1}{k+1} \cdot [k \cdot (c(q_5) - q_1) + c(q_1 + q_2 + q_5)] \quad (39)$$

At this point it should be noted that the capacities $c(q_1 + q_2 + q_5)$ and $c(q_5)$ can be calculated by any useful procedure, e.g. by formulas from gap acceptance theory. But also solutions from the linear regression method or Kyte's method (for details cf. Brilon, Troutbeck, Tracz, 1995) could be used.

3. CAPACITY ACCORDING TO GAP ACCEPTANCE THEORY

The most simple formula for the capacity of an unsignalized intersection with one minor and one major traffic stream is Siegloch's (1973) formula. Several authors (cf. Brilon, Troutbeck, Tracz, 1995) have shown that this formula produces also realistic results if the basic assumptions for the formula are not fulfilled. The Siegloch's formula is shown as following:

$$c(q) = \frac{1}{t_f} \cdot e^{-q \cdot t_0} \quad (40)$$

where $c(q)$ = capacity for the minor movement (veh/s)

t_f = follow-up time (s)

= average gap between two successive minor flow vehicle entering into the same major stream gap

t_0 = $t_c - t_f / 2$ (s)

t_c = critical gap (s)

= average gap between two successive major flow vehicle which - as a minimum - is accepted by the minor stream vehicles to cross the intersection

We have now to distinguish between different t_c - and t_f - values:

- a) t_c - and t_f - values for part I of the intersection (state 1 and state 2)
- b) t_c - and t_f - values for part II of the intersection (state 3 and state 4)
- c) t_c - and t_f - values for crossing part I and part II of the intersection simultaneously in the case of $k = 0$. It is realistic to assume that a driver who has to cross the whole major street at one time without having a central storage area needs longer t_c - and t_f - values than in case a) or b).

It is justified to assume that the t_c - and t_f - values in case a) and b) are of the same magnitude and that especially the t_f - values between both cases are nearly identical. This assumption is important for the following derivations.

Realistic values for the t_c - and t_f - values can be obtained from table 1. The given critical gaps t_c and follow-up times t_f are of realistic magnitude compared with the measurement results worked out by the NCHRP-project 3-46 (Kyte e.a., 1995, working paper #16). Here the critical gap and the follow-up time for the case without central reserve ($k = 0$) are larger then for the two-stage priority case which seems to be more realistic.

	$k = 0$ i.e. no central reserve case c)	$k \geq 1$ i.e. a central reserve of variable (with k) width	
		part I case a)	part II case b)
t_c	7.0 s	6.0 s	6.0 s
t_f	3.8 s	3.8 s	3.8 s

Table 1: Typical t_c - and t_f - values for two-stage priority situations within multilane major streets under US-conditions

Based on eq. 40 with the assumption that all of the t_f - values are nearly identical we can say:

$$\frac{c(q_1 + q_2 + q_5)}{c_0} = \frac{c(q_1 + q_2)}{c_0} \cdot \frac{c(q_5) - q_1}{c_0} \quad (41)$$

$$\text{where } c_0 = \frac{1}{t_f} \quad (\text{veh/s})$$

= maximal capacity, for the case of no cross traffic

This relation makes it possible to standardize all of the capacity terms by c_0 . If c_0 is used in units of veh/s also the other capacity terms must have this unit. Of course, also the unit veh/h could be used for all of the capacity terms. Then it is useful to standardize also c_T in eq. 38/39 :

$$\text{where } \hat{c}_T = \frac{c_T}{c_0} \quad (\text{veh/s}) \quad (42)$$

$\hat{c}_T$ (which has to be obtained from eq. 38/39) then can be expressed as a function of $c(q_1 + q_2)/c_0$ and $[c(q_5) - q_1] / c_0$. Thus it is possible to indicate the results of these derivations by graphs (cf. Fig. 2).

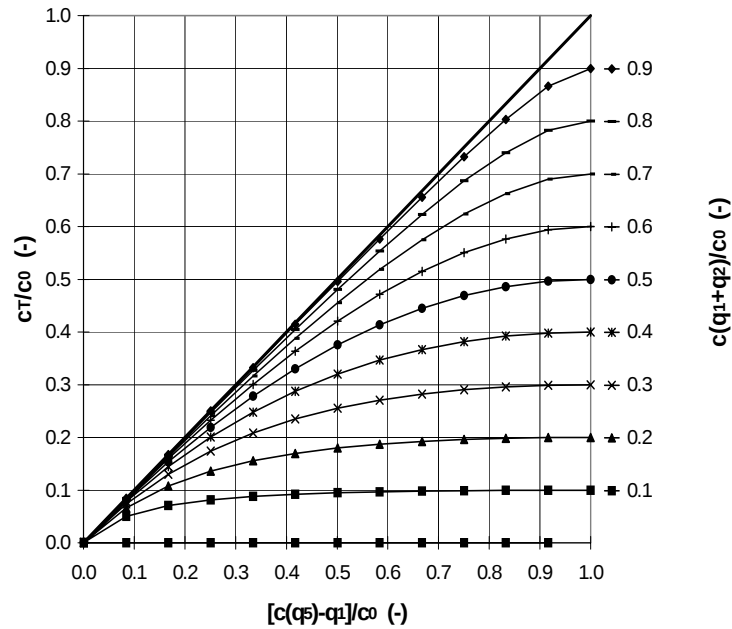


Figure 2: Total capacity $\hat{c}_T = c_T/c_0$ as a result of eq. 42 (in combination with eq. 38) in dependence of $c(q_1 + q_2)/c_0$ and $[c(q_5) - q_1] / c_0$ for $k = 1$.

It is further justified to use graphs of this type with sufficient approximation also under circumstances which differ from the conditions of gap acceptance theory, e.g.

- if capacities $c(q_1 + q_2)$ and $c(q_5)$ are computed from other theories than gap acceptance or even if they should be measured,
- if within gap acceptance theory the critical gaps t_c are different for each part of the intersection.

The only necessary condition for the application of these graphs is that the follow-up times t_f are of nearly identical magnitude.

4. LIMITATIONS OF THE THEORY

With a critical view on the theory which lead to eq. 38 we see that this theoretical concept has to be treated with care. The concept would be true if we could estimate the capacities $c(q_1 + q_2 + q_5)$ and $c(q_5)$ completely according to the fact that at a time, as a maximum, only k vehicles can enter one major stream gap both in part I and part II of the intersection due to the restricted storage space within the median reserve of the two-stage situation. This restriction applies especially for $c(q_1 + q_2)$ and $c(q_5)$. (state 1 and 3, cf. above) . This restriction does not apply for $c(q_1 + q_2 + q_5)$ since during state 2 and state 4 (definition see above) the number of minor stream vehicles departing during one large gap (being provided simultaneously in major streams 1 and 2 as well as 5) is not limited. Each of the conventional formulas for the capacity $c(q)$ (e.g. the Siegloch-formula eq. 40) are , however, based on the assumption that during large major stream gaps a greater number of minor stream vehicles can be accommodated, which is not true in the two-stage gap-acceptance situation (state 1 and 3) since here the number of minor vehicles per gap is limited to k in both parts of the intersection.

To take account of this limited validity of eq. 38 different approaches have been tested. The derivation of an analytical formula which takes into account these effects seemed not to be possible. Only a partial approach to the complete realistic truth was possible (cf. Brilon, Wu, Lemke, 1995). Therefore, some approximations were necessary.

5. SIMULATION STUDIES

Therefore, and for the test of the theory leading to eq. 38, the solution has been further investigated based on simulations. For this purpose a simulation model has especially been developed (Lemke, 1995). The basic structure of the model is closely related to the ideas of KNOSIMO (cf. Grossmann, 1992). The important features can be characterised as follows:

- The headways in the major streams are distributed according to a hyperlang-distribution (cf. Dawson, 1969 ; Grossmann, 1991).
- The critical gaps and the follow-up times are distributed according to an Erlang-distribution with the parameters given by Grossmann (1991) which are also used in KNOSIMO.

Both these assumptions together relate the model closer to reality than the theoretical derivations mentioned above. On the other side, the following assumptions are a simplification compared to reality. They do, however, correspond to the assumptions of the theory described above.

- No delays due to limited acceleration or deceleration of the vehicles are taken into account.
 - The travel time Δt between the two parts of the intersection has not been regarded; i.e. $\Delta t = 0$. (cf. argumentation following eq. 12).
 - Each minor street driver has a minimum delay of t_f at the first part of the intersection, also if no major stream vehicle is nearby. This simulates the time which a driver needs to realise the traffic situation on the major street when he is first approaching the intersection. This time margin is also necessary for the driver to decide if he can enter the intersection. Such an orientation time is not applied for vehicles entering the second part of the intersection since here a better visibility is assumed.
 - All traffic volumes are kept constant over time.
-

- The program is organised such that a constant queue in front of the first stop line of movement 8 is always maintained. Thus, the maximum number of vehicles which can enter the intersection can be evaluated.

This number is the representation of the capacity for movement 8 . A comprehensive set of simulation runs has been performed for different parameters q_1 , q_2 , q_5 .

Different attempts have been made to find an easy to be used approximative description of the results. Several of these attempts are described in Brilon, Wu, Lemke (1995) together with a statistical assessment of their precision. A good compromise between easy application and highest precision seemed to be the following solution. Instead of c_T we use a more realistic solution c_{Tr} which is obtained as a good approximation to the simulation results.

$$c_{Tr} = \alpha \cdot c_T \quad (\text{veh/s}) \quad (43)$$

where c_{Tr} = realistic total capacity for movement 8
(minor through traffic)

c_T = result from the theoretical approach obtained from
eq. 38

$$\alpha = \begin{cases} 1 & \text{for } k = 0 \\ 1 - 0.32 \cdot \exp(-1.3 \cdot \sqrt{k}) & \text{for } k > 0 \end{cases} \quad (44)$$

An even better solution for the correction term α is given by the following formula. This approach, however, has the drawback of a rather complicated use. Thus it is recommended rather for computer applications.

$$\alpha = 1 - 0.245 \cdot \frac{e_2 \cdot e_5}{k^{1.65}} \quad (45)$$

with

$$e_2 = \frac{\lambda_2 \cdot (\lambda_2 \cdot z_2)^k}{(k)!} \cdot e^{-(\lambda_2 \cdot z_2)}$$

$$\lambda_2 = a + b \cdot z_5 + c \cdot z_5^2$$

$$e_5 = \frac{\lambda_5 \cdot (\lambda_5 \cdot z_5)^k}{(k)!} \cdot e^{-(\lambda_5 \cdot z_5)}$$

$$\lambda_5 = a + b \cdot z_2 + c \cdot z_2^2$$

$$z_2 = \frac{c(q_2)}{c_0} \quad z_5 = \frac{c(q_5)}{c_0}$$

$$c_0 = \frac{1}{t_f} \text{ (veh / s)} \quad \text{or} \quad c_0 = \frac{3600}{t_f} \text{ (veh / h)}$$

Within this set of equations the following parameters should be applied:

$$a = 2.788$$

$$b = -1.259$$

$$c = -0.576$$

These solutions for the total capacity c_{Tr} of movement 8 approximate the simulated results with a standard deviation s (between results for c_T being simulated and those being estimated from eq. 42) according to table 2 . Other solutions with smaller deviations but more complicated formulas for the calculation of realistic c_{Tr} can be obtained from Brilon, Wu, Lemke (1995). The effect of the correction term α is also illustrated in Fig. 3 .

	s		
	$q_1 = 50$	$q_1 = 100$	$q_1 = 200$
$\alpha = 1$	29	30	32
eq. 44	18	18	19
eq.45	7	8	15
	veh/h	veh/h	veh/h

Table 2: Standard deviation s for computed c_T - values compared to the simulated results for two different approaches of the correction term α

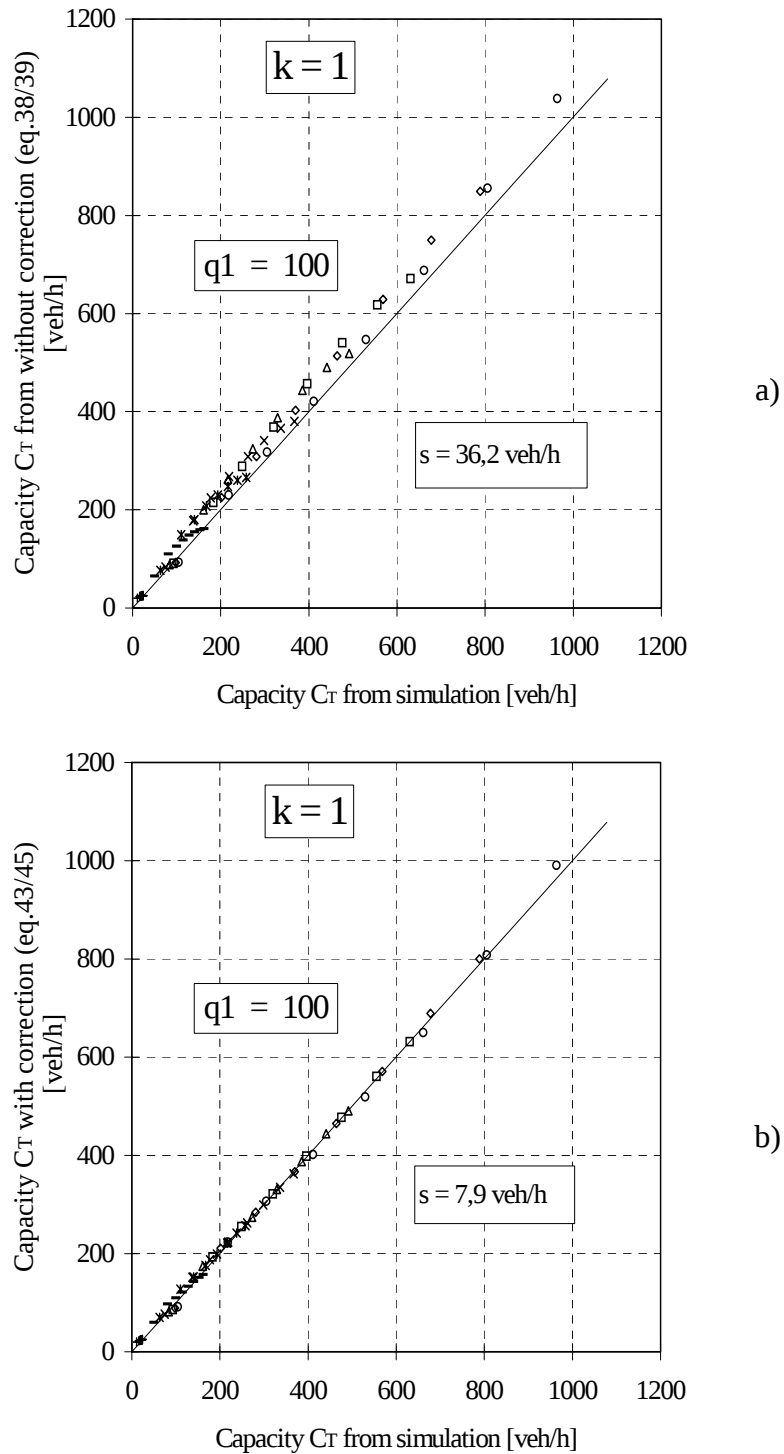


Figure 3: Comparison of simulated capacities C_T for movement 8 and calculated C_T - values . The simulated results are regarded as the true values.

a) calculation without correction term for eq. 38/39

b) calculation with correction term α (eq. 43/45).

At this point we can conclude the steps of computation which are necessary to estimate the capacity of an unsignalized intersection where the minor movements have to cross the major street in two stages:

q_1 = volume of priority street left turning traffic at part I
 q_2 = volume of major street through traffic coming from the left at part I
 q_5 = volume of the sum of all major street flows coming from the right at part II. Of course, here the volumes of all priority movements at part II have to be included. These are: major right (6, except if this movement is guided along a triangular island separated from the through traffic), major through (5), major left (4); numbers of movements according to HCM 1994, chapter 10.

$c(q_1 + q_2)$ = capacity at part I

$c(q_5)$ = capacity at part II

$c(q_1+q_2+q_5)$ = capacity at a cross intersection for minor through traffic with a major street traffic volume of $q_1+q_2+q_5$

(all capacity terms apply for movement 8. They are to be calculated by any useful capacity formula, e.g. the Siegloch-formula, eq. 40)

$$y = \frac{c(q_1 + q_2) - c(q_1 + q_2 + q_5)}{c(q_5) - q_1 - c(q_1 + q_2 + q_5)} \quad (37)$$

$$c_T = \frac{\alpha}{y^{k+1} - 1} \left\{ y \cdot (y^k - 1) \cdot [c(q_5) - q_1] + (y - 1) \cdot c(q_1 + q_2 + q_5) \right\} \text{ for } y \neq 1 \quad (38)$$

$$c_{T(y=1)} = \frac{\alpha}{k+1} [k \cdot [c(q_5) - q_1] + c(q_1 + q_2 + q_5)] \text{ for } y = 1 \quad (39)$$

c_T = total capacity of the intersection for minor through traffic

$$\text{with } \alpha = \begin{cases} 1 & \text{for } k = 0 \\ 1 - 0.32 \cdot \exp(-1.3 \cdot \sqrt{k}) & \text{for } k > 0 \end{cases} \quad (44)$$

Equations 38, 39 and 40 are only valid for $c(q_5) - q_1 > 0$.

6. GRAPHS FOR PRACTICAL APPLICATION

The results for the theory given in this paper are illustrated in Fig. 4 for $k = 1$ and 2 . Here the capacities $c(q_1 + q_2)$ and $c(q_5)$ can be introduced independent of the type of formula from which they have been determined. Another advantage of these graphs is that they can be applied with each arbitrary value of q_1 .

For example, we look at two-stage priority intersection with the traffic volumes $q_1=100$ veh/h, $q_2=600$ veh/h and $q_5=400$ veh/h. Let there be two possible storage spaces within the central reserve (cf. also Fig. 1). The capacities for movement 8 crossing the intersection separately can be calculated from the Siegloch's formula (eq.40). The corresponding values of t_c and t_f can be obtained from table 1. Then the parameters for the entering application graph (Fig.4) can be calculated as following:

$$\begin{aligned} \text{Part I: } c(q_1 + q_2) &= \frac{1}{t_f} \cdot e^{-(q_1+q_2) \cdot t_0} = \frac{1}{3.8} \cdot e^{-\frac{(100+600)}{3600} \cdot (6 - \frac{3.8}{2})} = 0.119 \quad (\text{veh/s}) \\ \text{Part II: } c(q_5) &= \frac{1}{t_f} \cdot e^{-q_5 \cdot t_0} = \frac{1}{3.8} \cdot e^{-\frac{400}{3600} \cdot (6 - \frac{3.8}{2})} = 0.167 \quad (\text{veh/s}) \end{aligned}$$

(cf. eq.40)

And with

$$\begin{aligned} c_0 &= \frac{1}{t_f} = \frac{1}{3.8} = 0.263 \quad (\text{veh/s}) \\ q_1 &= \frac{100}{3600} = 0.028 \quad (\text{veh/s}) \end{aligned}$$

we obtain the parameters for using Fig.4:

$$\begin{aligned} \frac{c(q_1 + q_2)}{c_0} &= \frac{0.119}{0.263} = 0.45 \\ \frac{c(q_5) - q_1}{c_0} &= \frac{0.167 - 0.028}{0.263} = 0.53 \end{aligned}$$

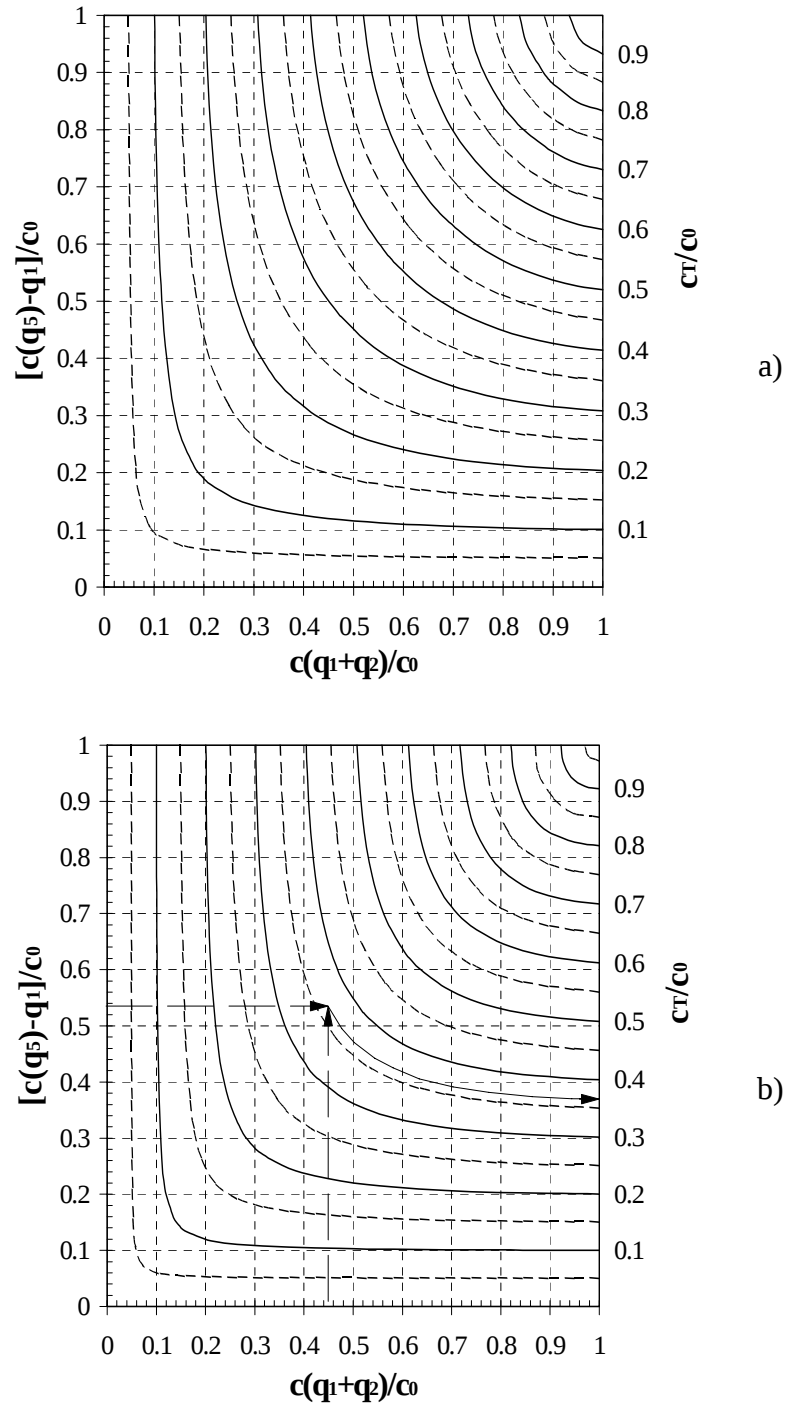


Figure 4: Capacities $\hat{c}_T = c_T / c_0$ (cf. eq. 42) for movement 8 in relation to standardized values of capacities and of q_1 (calculation with correction term α , eq. 43/45)

a) $k = 1$

b) $k = 2$

With these two parameters we obtain the relative capacity for the movement 8

$$\hat{c}_T = c_T / c_0 = 0.36 \quad (\text{Fig.4, b}).$$

Therefore, the absolute capacity for movement 8 is

$$\begin{aligned} c_T &= \hat{c}_T \cdot c_0 \\ &= 0.36 \cdot 0.263 \\ &= 0.095 \text{ veh/s} \\ &= 342 \text{ veh/h.} \end{aligned}$$

If gap acceptance theory is applied (eq. 40) to estimate the basic capacity terms ($c(q_1 + q_2)$ and $c(q_5)$) and if the t_c - and t_f - values are known, then the capacity for movement 8 can also be indicated by graphs directly depending on q_2 and q_5 . Then, however, one graph has to be indicated for each possible q_1 - value. This type of graphs using t_c - and t_f - values from table 1 (right columns) is given in Fig. 5 as one example.

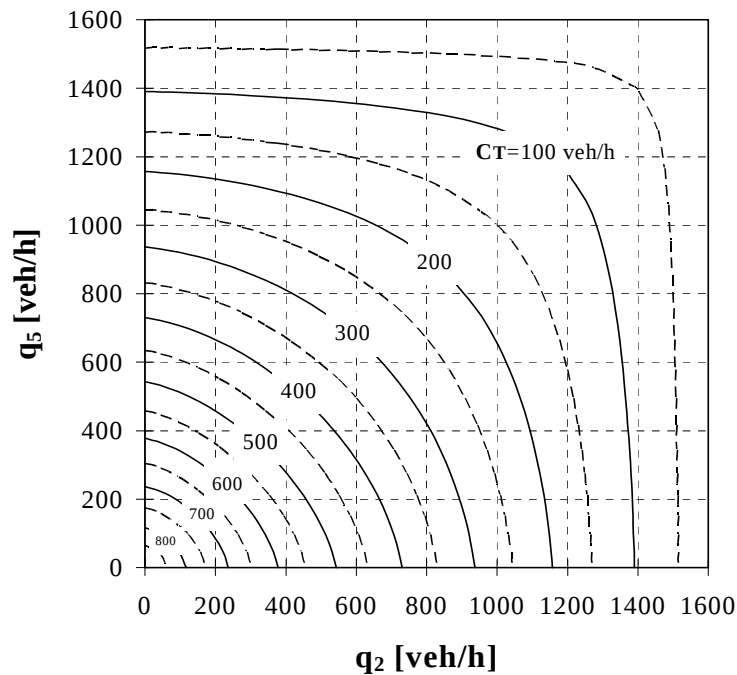


Figure 5: Capacities c_T for movement 8 depending on traffic volumes q_2 and q_5 with $q_1=0$ (calculation with correction term α , eq. 43 / 38 / 45 , $k = 1$).

Of course, the same theory as it has been described here can be used to determine the capacity of the minor left turner (movement 7) under two-stage priority conditions. If there is no separate lane for this movement in the central storage area the so-called mixed lane formula (eq. 10-9) of the HCM, 1994) has to be used to calculate the total capacity for movements 7 and 8 .

Delay estimations for the two-stage priority situation can be performed using the concept of reserve capacities (cf. Brilon, 1995) or the general delay formula by Kimber, Hollis (1979).

An easy to be understood procedure for the practical application of the theory presented here still has to be developed for the future HCM . Also some tests of this theory against measurement data which are available from field studies in NCHRP-project 3-46 are desirable.

7. CONCLUSION

The two-stage priority situation as it exists at many unsignalized intersection within multi-lane major streets provides larger capacities compared to intersections without central reserve areas. Capacity estimation procedures for this situation have not been available up to now. The paper provides an analytical solution for this problem. In addition, simulation studies lead to a correction of the theoretical results. Based on these derivations a set of graphs could be evaluated which enable an easy estimation of the capacity at an unsignalized intersection under two-stage priority. These graphs are ready to be used in practice.

Nevertheless, an empirical confirmation of this model approaches would be desirable. Also the question of the validity of the model for larger k -values should be discussed. It is questionable if the theory also applies for a grid of one-way street networks. Also if these questions should be addressed in the future, the presented theory is recommended for use at unsignalized intersection in practice.

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