

MODELING RELIABILITY IN FREEWAY NETWORKS

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ABSTRACT

Hitherto, the stochastic nature of freeway capacity was mainly analyzed at specific points which are considered as bottlenecks. The stochastic relationship between the adjacent bottlenecks was not taken into account. The paper introduces a concept for the stochastic interpretation of capacity and breakdown probability within a larger freeway network consisting of several combined bottlenecks. The stochastic methodology presented delivers a theoretical average capacity and the probability of breakdown for freeway segments with different lengths. The methodology can also be used to identify the effects of consecutive freeway segments and bottlenecks such as on-ramps, off-ramps, and weaving areas with different characteristics. Using the proposed method, it is possible to determine the breakdown probability as a function of the average volume or density. Hence, the risk of traffic flow disturbance along a freeway segment or within a freeway network can be analyzed.

INTRODUCTION

The capacity of a freeway is traditionally considered as a constant value in traffic engineering guidelines like the Highway Capacity Manual HCM (1). Recent investigations show that even under constant external conditions, different capacities can be observed on freeways (2, 3, 4, 5, 6, 7, 8). Most of these authors only observed traffic breakdowns at different flow volumes to demonstrate the variability of flows preceding a breakdown.

A theoretical concept for a stochastic capacity analysis was proposed by Brilon et al. (9, 10) based on ideas from Minderhoud et al. (3) and van Toorenburg (11). Geistefeldt and Brilon (12) demonstrated that this approach, which is based on statistical methods for censored data, delivers consistent stochastic capacity estimations. The approach has meanwhile been applied in a number of circumstances. Dong and Mahmassani (8, 13) used this concept to improve travel time predictions for route choice models with real-time traveler information. Elefteriadou et al. (14) applied probabilities for flow breakdown on freeways to develop pro-active ramp metering strategies. Brilon et al. (15) implemented a program system for large scale freeway network performance assessment applying the stochastic capacity concept.

Thus, the stochastic understanding of capacity and the corresponding concept for the reliability of freeways becomes an important topic in the area of theoretical freeway capacity analysis including applications in practice. Here, capacity is understood as the traffic volume below which the traffic is fluid and above which – if exceeded by the demand volume – the flow breaks down into congested (stop-and-go or even standing) traffic conditions. The demand flow volume that causes breakdowns varies depending on driver behavior in conjunction with the specific local conditions on the freeway. The breakdown flow volume, i.e. the pre-breakdown capacity, is a random variable. Empirical analyses for German freeways show that this pre-breakdown capacity can be treated as Weibull distributed with a nearly constant shape parameter representing the variance (10). The distribution of the pre-breakdown capacity can be identified using the product limit method (PLM) or by a maximum likelihood estimation (9, 10, 12). Using the distribution function of pre-breakdown capacities, the probability of traffic breakdowns and thus the reliability of the freeway can be estimated.

The stochastic pre-breakdown capacity was mainly analyzed at specific points along the freeway which are considered as bottlenecks. The stochastic relationship between the adjacent bottlenecks cannot be taken into account. Furthermore, if a long segment of a freeway without clearly defined bottlenecks is analyzed, no methods are available for estimating the distribution of pre-breakdown capacities of combined bottlenecks along a freeway. Thus, a stochastic capacity analysis in a freeway network consisting of several freeway segments and series of bottlenecks is not possible. In order to overcome this problem, this paper introduces a model dealing with a stochastic interpretation of pre-breakdown capacity and breakdown probability in a freeway network with long freeway segments and series of bottlenecks.

The model is based on the theory of continuity. Using the fundamental relationship of traffic flow (volume = density times speed), the probability distribution function of breakdowns from free flow into congested flow at a given traffic density can be estimated if the probability distribution functions of the pre-breakdown capacity and the pre-breakdown critical speed are given. The breakdown probability distribution as a function of the pre-breakdown traffic density can be estimated numerically for an arbitrarily distributed pre-breakdown capacity and critical pre-breakdown speed.

Similar to the derivation of a theoretical transformation between bottleneck-related breakdown probabilities for different interval durations, a transformation between link-related breakdown probabilities for different lengths of freeway segments can be constructed. It can be derived that the average pre-breakdown capacity and the breakdown probability are functions of the length L of the freeway segment under consideration. The average pre-breakdown capacity of the freeway segment decreases with an increasing length of the freeway segment under consideration. This decrease is not linear.

At first, a summary of the methods for stochastic capacity analysis at a single bottleneck is presented. The bottleneck-related model of stochastic capacity is extended to link-related models for freeway segments. Next, an approach for estimating reliability of large freeway networks over a longer period is presented and a discussion regarding the temporary and spatial independences of the breakdown probabilities is given. Finally, the main findings and results of the paper are presented in the conclusion.

BOTTLENECK-RELATED MODEL OF STOCHASTIC CAPACITIES

Pre-breakdown capacity for an isolated bottleneck

To describe the capacity as a random variable, its distribution function is required. However, the pre-breakdown capacity cannot be directly obtained from field measurements, which deliver only pairs of values of traffic volumes and average speeds during predetermined intervals. According to the definition of pre-breakdown capacity, the observed flow volume will be below the pre-breakdown capacity if the average speed is above a certain threshold value (e.g. 70 km/h). When the average speed is lower than this threshold value, the traffic flow is congested. Thus, the flow volume must have exceeded the pre-breakdown capacity during the time between two such intervals. Higher flow volumes are less likely to be measured in the field since a breakdown is likely to have happened before. Both effects make it difficult to estimate the pre-breakdown capacity distribution function, which is defined as (9, 10):

$$F_c(q) = P(c \leq q) \quad (1)$$

where $F_c(q)$ = pre-breakdown capacity distribution function

c = pre-breakdown capacity (veh/h)

q = traffic volume (veh/h)

A practicable method for estimating $F_c(q)$ was first presented by van Toorenburg (11), see also Minderhoud et al. (3), and extended by Brilon et al. (9, 10). The method is based on the statistics of censored data, which is commonly used in lifetime analysis and renewal theory. Lifetime distributions are often estimated by experiments of limited durations. Thus, lifetimes of individuals in the population that exceed the duration of the experiment cannot be measured. It is only possible to state that these lifetimes are longer than the duration of the measurement. Those data are called 'censored data' (16). The 'uncensored data' are directly measured lifetimes.

If a traffic breakdown is considered as a failure event, the statistics for censored data can be used to estimate the pre-breakdown capacity c , which is the analogue of the lifetime. Here, the 'censored data' are the measurements where the capacity c is greater than the observed traffic demand q . The 'uncensored data' are pre-breakdown capacities that can be observed directly.

To estimate distribution functions based on samples that include censored data, both non-parametric and parametric methods can be used. A non-parametric method to estimate lifetime function is the so-called ‘product limit method’ PLM (17). This method can also be adapted for estimating the pre-breakdown capacity distribution function. For details of the method, readers are referred to Brilon et al. (9, 10).

The PLM does not need a specific type of distribution function. However, if the type of the distribution is given, the parameters of the distribution can be estimated with the maximum likelihood method. Here it is necessary to know the mathematical expression of the distribution function $F_c(q)$. By comparing different types of functions based on the maximum value of the likelihood function, the Weibull distribution turned out to be the function that best fits the observations on all freeway segments under investigation (9, 10). The Weibull distribution function for the pre-breakdown capacity c can be expressed as

$$F_c(q) = P(q \leq c) = 1 - e^{-(q/\beta_c)^{\alpha_c}} \quad \text{for } q \geq 0 \quad (2)$$

where

α_c = shape parameter of the Weibull distribution

β_c = scale parameter of the Weibull distribution (veh/h)

The function

$$S_c(q) = P(q > c) = 1 - F_c(q) = e^{-\left(\frac{q}{\beta_c}\right)^{\alpha_c}} \quad \text{for } q \geq 0 \quad (3)$$

is called the survival function which describes the probability that the random variable q is larger than a given threshold c .

The mean value of the Weibull distribution is

$$E(c) = \beta_c \cdot \Gamma\left(1 + \frac{1}{\alpha_c}\right) \quad (4)$$

and the variance is

$$\sigma^2(c) = \beta_c^2 \cdot \left[\Gamma\left(1 + \frac{2}{\alpha_c}\right) - \Gamma^2\left(1 + \frac{1}{\alpha_c}\right) \right] \quad (5)$$

The median value of the distribution is

$$c_{\text{median}} = \beta_c [-\ln(0.5)]^{1/\alpha_c} = \beta_c \cdot 0.693^{1/\alpha_c} \quad (6)$$

In Fig. 1, the pre-breakdown capacity distributions are illustrated for two freeways in Germany (9, 10). The shape parameter α_c in the Weibull distribution typically ranges between 10 and 20. An average value of $\alpha_c = 13$ was recommended for all types of freeways in order to ease mathematical derivations (9). This value, in the subsequent context, is used as an example to demonstrate consequences of this parameter α_c on other characteristic variables.

Considering the shape parameter α_c as a constant, we can transform the pre-breakdown capacity distribution function for different interval durations T , cf. (9). According to Eq. (1), we define $F_{c,5}(q)$ as the probability of a breakdown during $\Delta = 5$ minutes at flow volume q . Hence, $p_{5,nbr} = 1 - F_{c,5}(q)$ is the probability of no breakdown occurring in this interval. Assuming an independence between traffic breakdowns in the 12 succeeding 5-minute intervals within an hour yields:

$$P_{60,nbr} = (p_{5,nbr})^{12} = [1 - F_{c,5}(q)]^{12} \quad (7)$$

Using the Weibull distribution, i.e. Eq. (2), yields

$$P_{60,nbr} = \left[e^{-\left(\frac{q}{\beta_{c,5}}\right)^{\alpha_c}} \right]^{12} = e^{-12\left(\frac{q}{\beta_{c,5}}\right)^{\alpha_c}} \quad (8)$$

and

$$F_{c,60}(q) = 1 - P_{60,nbr} = 1 - e^{-12\left(\frac{q}{\beta_{c,5}}\right)^{\alpha_c}} = 1 - e^{-\left(\frac{q}{\frac{\beta_{c,5}}{12^{1/\alpha_c}}}\right)^{\alpha_c}} \quad (9)$$

which is again a Weibull distribution with an unchanged shape parameter α_c and a scale parameter $\beta_{c,60} = r \cdot \beta_{c,5}$, where $r = 12^{(-1/\alpha_c)}$. In general we have the transformation

$$F_{c,T|\Delta}(q) = 1 - e^{-\frac{T}{\Delta}\left(\frac{q}{\beta_{c,\Delta}}\right)^{\alpha_c}} \quad (10)$$

where T is the duration of the output interval and Δ the is duration of the input interval (9).

Sequences of bottlenecks

The breakdown probability distribution function for a sequence of freeway segments can be derived for the case of a constant shape parameter α_c of the capacity distribution function of each sub-segment (9). In general, the survival function for an isolated point i which is treated as a bottleneck is

$$S_c(q_i) = 1 - F_c(q_i) = e^{-\left(\frac{q_i}{\beta_{c,i}}\right)^{\alpha_{c,i}}} \quad (11)$$

For successive sub-segments along the freeway where each sub-segment is treated as one bottleneck, the survival function for this chain of n bottlenecks is

$$S_{c,n} = \prod_{i=1}^n S_c(q_i) = \prod_{i=1}^n e^{-\left(\frac{q_i}{\beta_{c,i}}\right)^{\alpha_{c,i}}} = e^{-\sum_{i=1}^n \left(\frac{q_i}{\beta_{c,i}}\right)^{\alpha_{c,i}}} \quad (12)$$

This equation describes the probability that no breakdown occurs at any of the n bottlenecks. The combined survival functions can be used for defining the reliability of a freeway network. Here we assume that the distribution functions and thus the survival functions at different bottlenecks are independent of each other. This assumption is not critical if the bottlenecks are located far enough from each other.

Generally, the mean volumes q_i at different bottlenecks can have different values. For the special case that all $q_i = q$, for example along a single freeway segment, we obtain

$$S_{c,n}(q) = \prod_{i=1}^n S_c(q) = e^{-\sum_{i=1}^n \left(\frac{q}{\beta_{c,i}} \right)^{\alpha_{c,i}}} \quad (13)$$

However, the resulting distribution function $F_{c,n}(q) = 1 - S_{c,n}(q)$ is no longer a Weibull function as long as α_c and β_c are specific for each bottleneck i , but has always a Weibull-like shape.

Normally, the shape parameters α_c can be approximately considered as constant for all bottlenecks (9). That is $\alpha_{c,i} = \alpha_c$ and

$$S_{c,n}(q) = \prod_{i=1}^n S_c(q) = e^{-\sum_{i=1}^n \left(\frac{q}{\beta_{c,i}} \right)^{\alpha_c}} = e^{-\left(q \cdot \sqrt[n]{\sum_{i=1}^n \left(\frac{1}{\beta_{c,i}} \right)^{\alpha_c}} \right)^{\alpha_c}} = e^{-\left(\frac{q}{\beta_{c,n}} \right)^{\alpha_c}} \quad (14)$$

This combined survival function and the corresponding distribution function have the same shape parameter α_c as for the single bottlenecks. The scale parameter β_c of the corresponding distribution function $F_{c,n}(q) = 1 - S_{c,n}(q)$ is

$$\beta_{c,n} = \frac{1}{\sqrt[n]{\sum_{i=1}^n \left(\frac{1}{\beta_{c,i}} \right)^{\alpha_c}}} \quad (15)$$

For a homogeneous freeway segment one can assume additionally $\beta_{c,i} = \beta_c$ and thus,

$$S_{c,n}(q) = \prod_{i=1}^n S_c(q) = e^{-\left(\frac{q \cdot \sqrt[n]{n}}{\beta_c} \right)^{\alpha_c}} \quad (16)$$

and

$$\beta_{c,n} = \frac{\beta_c}{\alpha_c \sqrt[n]{n}} = \beta_c \cdot f_{c,n} \quad (17)$$

where

$$f_{c,n} = \frac{\beta_{c,n}}{\beta_c} = \frac{1}{\alpha_c \sqrt[n]{n}} \quad (18)$$

is the capacity reduction factor for n consecutive identical bottlenecks.

As mentioned above we consider the stochastic flow processes at different bottlenecks as independent of each other. This assumption is not always realistic. For two closely adjacent sub-segments, the stochastic flow processes are expected to be highly dependent on each other. For statistically positive-dependent bottlenecks, the scale parameter of the combined distribution generally has a larger value, i.e. the combined distribution has a large mean value. If two bottlenecks are totally positive-dependent, the combined scale parameter is always equal to the scale parameters of the single bottlenecks. In this extreme case, the combination of two bottlenecks can be considered as a joint bottleneck. For statistically negative-dependent single bottlenecks, the effect might be the opposite. Those dependences can eventually be estimated by measurements under real-world conditions.

LINK-RELATED MODEL OF STOCHASTIC CAPACITIES

Pre-breakdown capacity of a single freeway segment

According to the capacity estimation methods described in the previous section, the distribution function of pre-breakdown capacities at a single bottleneck can be determined. The capacity estimation technique can also be applied to traffic densities k instead of traffic volumes q . However, the traffic density cannot easily be observed in the field. For a homogeneous freeway segment under steady-state condition, the distribution function of the critical (pre-breakdown) density k_c corresponds to the distribution function of the pre-breakdown capacity c . The fundamental relationship of traffic flow, $q = k \cdot v$, is also valid for $q = c$; thus $c = k_c \cdot v_c$ or $k_c = c / v_c$ where k_c and v_c are the corresponding critical density and critical speed at the pre-breakdown capacity c . Therefore, the distribution function of the critical density $F_{k_c}(k)$ can be estimated if the distribution function of the pre-breakdown capacity $F_c(q)$ and the probability distribution function of the critical speed $F_{v_c}(v)$ is given. That is, the density-related (link-related) breakdown distribution function for a freeway segment, $P_{br}(k_c \leq k) = F_{k_c}(k)$, can be transformed from the flow-related (bottleneck-related) probability distribution function of breakdowns at an isolated bottleneck, $P_{br}(c \leq q) = F_c(q)$. The density-related breakdown probability distribution function then is

$$\begin{aligned}
 F_{k_c}(k) &= P(k_c \leq k) = \oint_{c \leq k_c v_c} f_c(q) f_{v_c}(v) dq dv \\
 &= \int_0^\infty \left(\int_0^{kv} f_c(q) dq \right) f_{v_c}(v) dv = \int_0^\infty F_c(kv) f_{v_c}(v) dv
 \end{aligned} \tag{19}$$

The flow-related pre-breakdown capacity distribution $F_c(q)$ is assumed to be a Weibull function. Here, any reasonable distribution for the critical speed v_c can be used. For simplicity and in order to ease the derivation, we assume the critical speed v_c to be Weibull distributed as an approximation. This assumption seems to be more reasonable than the usual assumption of a Normal distribution for speeds since Weibull is only defined for positive values. Moreover, Weibull reveals significant probabilities only for a narrow range of speed values with a rather sharp lower limit which seems to be an important attribute especially for the critical speed. Thus,

$$\begin{aligned}
F_{k_c}(k) &= \int_0^\infty F_c(kv) f_{v_c}(v) dv = \int_0^\infty \left(1 - e^{-\left(\frac{kv}{\beta_c}\right)^{\alpha_c}}\right) \cdot \frac{\alpha_{v_c}}{\beta_{v_c}} \left(\frac{v}{\beta_{v_c}}\right)^{\alpha_{v_c}-1} e^{-\left(\frac{v}{\beta_{v_c}}\right)^{\alpha_{v_c}}} dv \\
&= 1 - \frac{\alpha_{v_c}}{\beta_{v_c}} \int_0^\infty \left(\frac{v}{\beta_{v_c}}\right)^{\alpha_{v_c}-1} \cdot e^{-\left[\left(\frac{kv}{\beta_c}\right)^{\alpha_c} + \left(\frac{v}{\beta_{v_c}}\right)^{\alpha_{v_c}}\right]} dv
\end{aligned} \tag{20}$$

Using a constant critical speed v_c , the transformation can be carried out analytically. The resulting density-related distribution function $F_{k_c}(k)$ is also a Weibull distribution. That is,

$$F_{k_c}(k)|_{v=\text{const}} = F_c(kv) = 1 - e^{-\left(\frac{kv}{\beta_c}\right)^{\alpha_c}} = 1 - e^{-\left(\frac{k}{\beta_c/v}\right)^{\alpha_c}} \tag{21}$$

The density-related distribution function $F_{k_c}(k)$ can only be estimated numerically for a arbitrarily distributed critical speed v_c . Using a mean value of $v_c = 80$ km/h for the critical speed and a standard deviation $\sigma(v_c) = 5$ km/h, which are common values in reality, we obtain the parameters $\beta_{v_c} = 82$ km/h and $\alpha_{v_c} = 20$ for a Weibull distributed critical speed. Using $\beta_c = 4532$ veh/h (from the example freeway, cf. Fig. 1a) and $\alpha_c = 13$ for a two-lane freeway segment, Eq. (20) yields a Weibull-like but not exactly a Weibull distribution. This distribution can be approximated to a Weibull distribution with the parameter $\beta_{k_c} = 57$ veh/km and $\alpha_{k_c} = 10.7$ (Fig. 2a). For a three-lane freeway segment with $\beta_c = 7170$ veh/h (from the example freeway, cf. Fig. 1b) and $\alpha_c = 13$, Eq. (20) yields a Weibull-like distribution with parameters $\beta_{k_c} = 89$ veh/km and $\alpha_{k_c} = 10.7$ (Fig. 2b).

It can be proven that the shape parameter of the critical density α_{k_c} only depends on the shape parameters α_c and α_{v_c} . The parameter α_{k_c} is independent of the scale parameters β_{k_c} , β_c , and β_{v_c} . In Table 1, the parameters α_{k_c} resulting from different combinations of α_c and α_{v_c} values are illustrated. Because the shape parameters α_c and α_{v_c} can be approximately assumed to be constant values (e.g. $\alpha_c = 13$ and $\alpha_{v_c} = 20$) for all types of freeway segments, the shape parameter for the critical density is also a constant (e.g. $\alpha_{k_c} = 10.7$).

This result can be verified by real world measurements. Regler (18) conducted a field measurement using data from three-lane freeway segments. The median of the critical densities ranged from 70 to 90 veh/km with Weibull parameters $\alpha_{k_c} = 8.4$ through 13.2 and $\beta_{k_c} = 72$ through 92 veh/km for the analysis of 5-minute intervals.

As a result we can state that the distribution of critical densities k_c is approximately Weibull distributed with a shape parameter $\alpha_{k_c} = 10.7$. That is,

$$F_{k_c}(k) = 1 - e^{-\left(\frac{k}{\beta_c/\bar{v}_c}\right)^{\alpha_{k_c}}} = 1 - e^{-\left(\frac{k}{\beta_{k_c}}\right)^{\alpha_{k_c}}} \tag{22}$$

where $\beta_{k_c} = \beta_c / \bar{v}_c$, $\alpha_{k_c} = 10.7$, and \bar{v}_c is the mean value of the critical speed v_c .

This transformed distribution function is only valid for a length L_Δ of a freeway segment that corresponds to the analysis interval Δ for the pre-breakdown capacity. For example, if the scale parameter $\beta_{c,5}$ for the pre-breakdown capacity is obtained for 5-minute intervals, the resulting distribution function is only valid for a segment length of $L_\Delta = \bar{v}_c \cdot \Delta = 80 \text{ km/h} \cdot 5/60 = 6.67 \text{ km}$.

Similarly to the derivation of the theoretical transformation between bottleneck-related breakdown probabilities for different interval durations, a transformation between link-related breakdown probabilities for different lengths of freeway segments can be constructed. The probability function of breakdowns for a freeway segment of length L can be expressed as

$$F_{k_c,L}(k) = 1 - e^{-\left(\frac{k}{\beta_{k_c,L}}\right)^{\alpha_{k_c}}} = 1 - e^{-\frac{L}{L_\Delta} \left(\frac{k}{\beta_{k_c,L_\Delta}}\right)^{\alpha_{k_c}}} = 1 - e^{-\frac{L}{\bar{v}_c \cdot \Delta} \left(\frac{k}{\beta_{c,\Delta}/\bar{v}_c}\right)^{\alpha_{k_c}}} = 1 - e^{-\left(\frac{k}{\frac{\beta_{c,\Delta}/\bar{v}_c}{\alpha_{k_c} \sqrt{L/L_\Delta}}}\right)^{\alpha_{k_c}}} \quad (23)$$

with the scale parameter

$$\beta_{k_c,L} = \frac{\beta_{k_c,L_\Delta}}{\alpha_{k_c} \sqrt{L/L_\Delta}} = \frac{\beta_{c,\Delta}/\bar{v}_c}{\alpha_{k_c} \sqrt{L/L_\Delta}} \quad (24)$$

Eq. (23) describes the probability that no breakdown occurs on a freeway segment of length L within a time interval Δ . The parameter $\beta_{c,\Delta}$ is the scale parameter of the Weibull distributed pre-breakdown capacities estimated in Δ -minute intervals.

For $\Delta = 5 \text{ min} = 1/12 \text{ h}$, $\bar{v}_c = 80 \text{ km/h}$, $L_\Delta = 6.67 \text{ km}$, and $\alpha_{k_c} = 10.7$, we get

$$F_{k_c,L}(k) = 1 - e^{-\frac{L}{6.67} \left(\frac{k}{\beta_{c,5}/80}\right)^{10.7}} \quad (25)$$

This equation can be transformed into

$$F_{k_c,L}(k) = 1 - e^{-\frac{L}{6.67} \left(\frac{k}{\beta_{c,5}/80}\right)^{10.7}} = 1 - e^{-\left(\frac{k}{\frac{\beta_{c,5}/80}{10.7 \sqrt{L/6.67}}}\right)^{10.7}} = 1 - e^{-\left(\frac{k}{\frac{\beta_{c,5,L}}{80}}\right)^{10.7}} = 1 - e^{-\left(\frac{k}{\beta_{k_c,L}}\right)^{10.7}} \quad (26)$$

For two freeway segments of lengths L_1 and L_2 , we obtain the relationship

$$\frac{\beta_{k_c,L_1}}{\beta_{k_c,L_2}} = \frac{\beta_{c,5}/80}{10.7 \sqrt{L_1/6.67}} \bigg/ \frac{\beta_{c,5}/80}{10.7 \sqrt{L_2/6.67}} = 10.7 \sqrt{\frac{L_2}{L_1}} \quad (27)$$

This means e.g.: if the length L_2 of the freeway segment is double the length of L_1 , the scale parameter β_{k_c} of the density (and capacity) is reduced by the factor $1/10.7\sqrt{2} = 0.937$.

Fig. 3 shows the parameters $\beta_{k_c,L}$ and $\beta_{c,L}$, which indicate the critical density and the corresponding pre-breakdown capacity as a function of length L for a two-lane freeway segment. It can be seen that the scale parameter β for the critical density k_c and the corresponding scale parameter β for the pre-breakdown capacity c of the freeway segments decrease with increasing length of the freeway segment.

Sequence of consecutive freeway segments

The survival function for a single freeway segment j is

$$S_{k_c}(k_j, L_j) = 1 - F_{k_c, L_j}(k_j) = e^{-\frac{L_j}{L_{\Delta,j}} \left(\frac{k_j}{\beta_{c,\Delta,j} / \bar{v}_{c,j}} \right)^{\alpha_{k_c,j}}} = e^{-\left(\frac{\frac{k_j}{\beta_{c,\Delta,j} / \bar{v}_{c,j}}}{\alpha_{k_c,j} \sqrt{\frac{L_j}{L_{\Delta}}}} \right)^{\alpha_{k_c,j}}} \quad (28)$$

For $\Delta = 5 \text{ min} = 1/12 \text{ h}$, $\bar{v}_{c,j} = \bar{v}_c = 80 \text{ km/h}$, $L_{\Delta,j} = L_{\Delta} = 6.67 \text{ km}$, and $\alpha_{k_c,j} = \alpha_{k_c} = 10.7$, we get

$$S_{k_c}(k_j, L_j) = e^{-\frac{L_j}{6.67} \left(\frac{k_j}{\beta_{c,5,j} / 80} \right)^{10.7}} \quad (29)$$

The survival function for m combined freeway segments then is

$$S_{k_c,m} = \prod_{j=1}^m S_{k_c}(k_j, L_j) = \prod_{j=1}^m e^{-\frac{L_j}{6.67} \left(\frac{k_j}{\beta_{c,5,j} / 80} \right)^{10.7}} = e^{-\sum_{j=1}^m \frac{L_j}{6.67} \left(\frac{k_j}{\beta_{c,5,j} / 80} \right)^{10.7}} \quad (30)$$

This equation describes the probability that no breakdown occurs on any of the m freeway segments during a 5-minute interval. The freeway segments can have different values of the capacity scale parameter β_c (or β_{k_c}), density k , and length L . This can also be used for defining the reliability of a network. Here we assume again the distribution functions and thus also the survival functions at different freeway segments are independent of each other. Normally, for long freeway segments, this independence is given.

ANALYSIS OF LARGE FREEWAY NETWORKS OVER LONG TIME PERIODS

Using the approach for sequences of freeway segments, the reliability of a larger freeway network can be estimated over a long time period. All parameters used in this section are link-related parameters, i.e. they are parameters for the freeway segments according to the previous section. However, these link-related parameters can be transformed from bottleneck-related parameters.

The reliability of a larger freeway network can be defined as the probability that on any freeway segment within the network and at any time no breakdown occurs. According to this definition, the reliability can be expressed as the combined survival function of the pre-breakdown capacity over time and space.

The survival function of the pre-breakdown capacity for a single freeway segment j over a time period i of duration T_i and a space-link of length L_j is (cf. Eqs. (10) and (23))

$$S_{T_i+L_j}(k_{ij}(q_{ij}), L_j, T_i) = e^{-\frac{L_j}{L_{\Delta,j}} \cdot \frac{T_i}{\Delta} \left(\frac{k_{ij}}{\beta_{c,\Delta,j} / \bar{v}_{c,ij}} \right)^{\alpha_{kc,j}}} = e^{-\frac{L_j}{L_{\Delta,j}} \cdot \frac{T_i}{\Delta} \left(\frac{q_{ij}}{\beta_{c,\Delta,j}} \right)^{\alpha_{kc,j}}} \quad (31)$$

For $\Delta = 5 \text{ min} = 1/12 \text{ h}$, $\bar{v}_{c,ij} = \bar{v}_c = 80 \text{ km/h}$, $L_{\Delta,j} = L_{\Delta} = 6.67 \text{ km}$, and $\alpha_{kc,j} = \alpha_{kc} = 10.7$ we get

$$\begin{aligned} S_{T_i+L_j}(k_{ij}(q_{ij}), L_j, T_i) &= e^{-\frac{L_j}{6.67} \cdot \frac{T_i}{1/12} \left(\frac{k_{ij}}{\beta_{c,5,ij}/80} \right)^{10.7}} = e^{-1.8 \cdot L_j T_i \left(\frac{k_{ij}}{\beta_{c,5,ij}/80} \right)^{10.7}} \\ &= e^{-\frac{L_j}{6.67} \cdot \frac{T_i}{1/12} \left(\frac{q_{ij}}{\beta_{c,5,ij}} \right)^{10.7}} = e^{-1.8 \cdot L_j T_i \left(\frac{q_{ij}}{\beta_{c,5,ij}} \right)^{10.7}} \end{aligned} \quad (32)$$

The survival function for m combined freeway segments and n intervals is

$$S_{T+L, m \times n} = \prod_{j=1}^m \prod_{i=1}^n e^{-1.8 \cdot L_j T_i \left(\frac{k_{ij}}{\beta_{c,5,ij}/80} \right)^{10.7}} = e^{-\sum_{j=1}^m \sum_{i=1}^n 1.8 \cdot L_j T_i \left(\frac{q_{ij}}{\beta_{c,5,ij}} \right)^{10.7}} \quad (33)$$

This equation describes the probability that in the time period of duration $T = \sum T_i$ and within a network of a total length $L = \sum L_j$ no breakdown occurs. The freeway segments j can have different values of scale parameter β for capacity, density k , and length L_j for different time period T_i . According to this formulation, a quantitative assessment of the reliability in a large network over a long period can be conducted.

DISCUSSIONS

The temporary and spatial independence of the breakdown probabilities are pre-assumptions for the derivations in this paper. Those assumptions are only approximations and they are not always realistic. Nevertheless, those assumptions are reasonable under certain pre-conditions, which is discussed in the following.

Temporary independence of breakdown probabilities in succeeding intervals

Generally, the assumption about independence between breakdowns (actually the probabilities of no-breakdowns are needed) in succeeding intervals (i.e. within one hour) can be considered as reasonable because:

1) According to the definition, a breakdown can only occur in free-flow conditions. Normally, the traffic flow under free-flow conditions can approximately be considered as independent of each other.

2) From the theoretical point of view, the probability of breakdowns (or no-breakdowns) is defined by the capacity distribution function at a cross-section. The capacity is then defined by the average time headway within consecutive vehicles thus by the car-following behaviors of the vehicles. Since the car-flow behaviors of a given driver population are generally considered as

time-independent, the capacity distribution is also time-independent and thus the probability of breakdowns is time-independent as well.

3) From the empirical point of view, the probabilities of breakdowns in different time intervals are independent of each other because the observed breakdowns are always isolated events. The observation procedure requires free-flow pre-breakdown and congested post-breakdown traffic conditions. The process of breakdowns is a renewal process and hence time-independent (9).

Spatial independence of breakdown probabilities in series of bottlenecks

The breakdown probabilities at different bottlenecks and on different freeway segments cannot generally be considered as independent of each other. However, if only bottlenecks with long distances in between or only long freeway segments are investigated, the breakdown probabilities can be considered as nearly independent of each other.

The breakdowns can occur at any possible locations along a freeway segment. Certainly, the breakdown probability at a location with lower capacity is higher than at a location with higher capacity. Sometimes, those capacity bottlenecks can be more decisive for the traffic flow under consideration. However, also the spontaneous traffic breakdowns along a long, homogenous freeway segment have to be investigated. The probabilities of breakdowns at particular bottlenecks and along freeway segments can be combined in order to investigate the total breakdown probability of a network.

Applicability

The stochastic concept of breakdown capacity is, meanwhile, also a basis of applications for practice. E.g. Elefteriadou et al. (14) derived innovative proactive ramp metering algorithms based on breakdown probabilities. Their simulations revealed improvements for freeway flow performance. Within this concept, the effects of breakdown probabilities combined over successive bottlenecks along the freeway, including ramps and weaving sections, might allow further insights.

Brilon et al. (15) developed a macroscopic simulation concept on behalf of the German Federal State of Hesse to assess the statewide freeway network performance over longer periods, e.g. one year. Here, also breakdown probability distribution functions were calibrated for those parts of the network which suffer frequent congestions. These stochastic descriptions of the capacity have been contrasted to locally specified patterns of traffic demand over the year. This project was the starting point for the considerations described in this paper. For this project the lengths of freeway segments and the interference between successive bottlenecks had to be modeled by stochastic approaches. Within this concept, the influence of freeway segment length, as pointed out in this paper, constitutes a necessary input.

Also the context of the macroscopic network-wide parameters derived in this paper is important for comparisons of network performance over the years and between several parts of the network. These comparisons in practice are requested from a superior view of transportation policy.

In addition, for traffic control considerations, the paper offers the message that activities – either by technical devices or by driver education – to influence the breakdown distribution function could help to improve the reliability of freeways. Here it is not only an increase of the

average capacity – which usually is difficult to attain – but also a reduction of the variance which helps to improve network reliability.

CONCLUSIONS

Using a theoretical approach, a methodology for the assessment of reliability within a freeway network was introduced. The stochastic methodology presented allows for a derivation of a theoretical average pre-breakdown capacity and breakdown probability for freeway segments with different lengths. This link-related methodology can also be used to identify the effects of consecutive freeway segments and bottlenecks such as on-ramps, off-ramps, and weaving areas with different characteristics. As a result, the stochastic relationship between several adjacent bottlenecks can be taken into account. Furthermore, a long segment of a freeway without clearly defined bottlenecks can be analyzed.

Using this method, it is possible to determine the probability distribution function of breakdowns from free flow into congested flow conditions for a freeway segment as a function of the average pre-breakdown density. This link-related breakdown probability distribution can be estimated by transforming the distribution function of pre-breakdown capacities measured at isolated bottlenecks. It turns out that the link-related pre-breakdown capacity distribution (a Weibull-like distribution) has a smaller scale parameter and, thus, a larger variance than the bottleneck-related capacity distribution.

Using the methodology presented in this paper, the risk of disturbance of traffic flow (breakdowns from free flow into congested flow) along a freeway segment and within a freeway network can be estimated and analyzed. The reliability of a freeway network can be estimated quantitatively. The paper demonstrates basic probabilistic considerations which – for practical application – must be based on breakdown probability functions calibrated for the important parts of the network.

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FIGURE 2 Numerically estimated distribution function of critical densities ($F_{kc}(k)$) and its approximation as a Weibull distribution ($F_{kc}(k)^*$)

FIGURE 3 Scale parameter β as a function of the length of the freeway segment L

TABLE 1 Parameters α_{kc} resulting from different combinations of α_c and α_{vc} values.

α_c	α_{vc}			
	10	15	20	25
10	6.81	8.1	8.77	9.15
13	7.65	9.56	10.66	11.32
16	8.21	10.66	12.21	13.21
σ_{vc} (km/h) ($v_c = 80$ km/h)	9.6	6.5	5.0	4.0

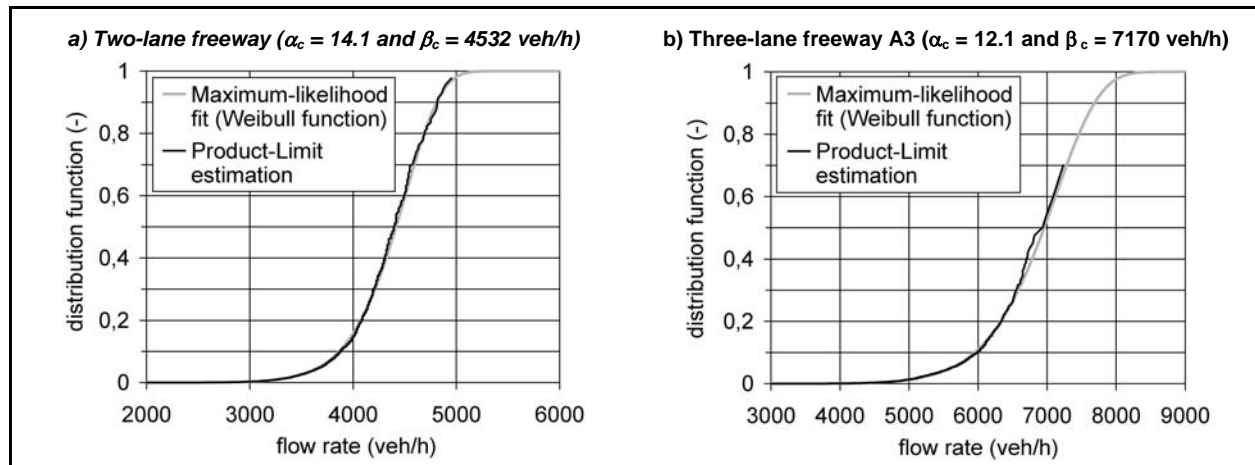


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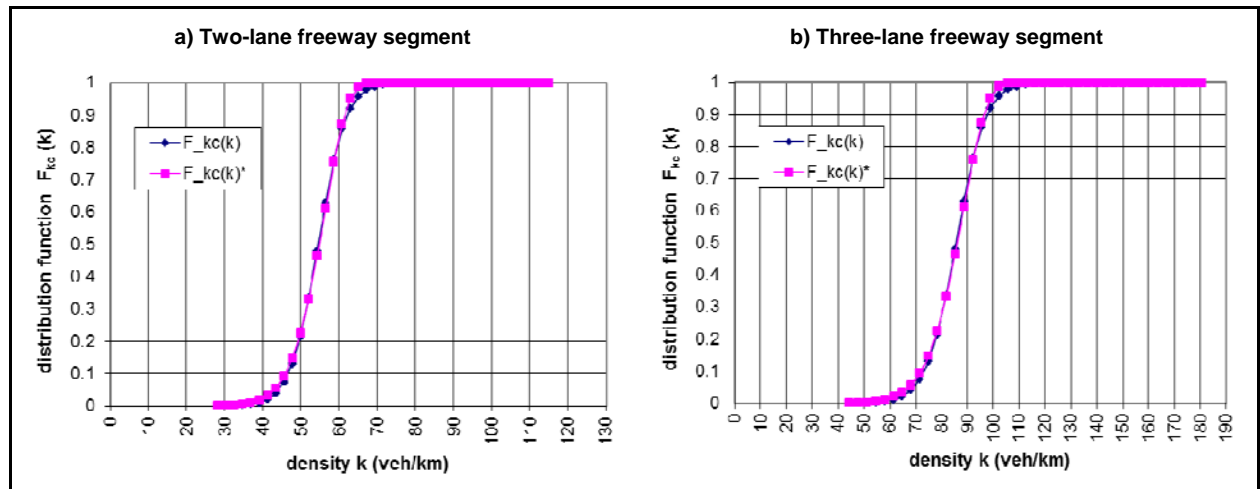


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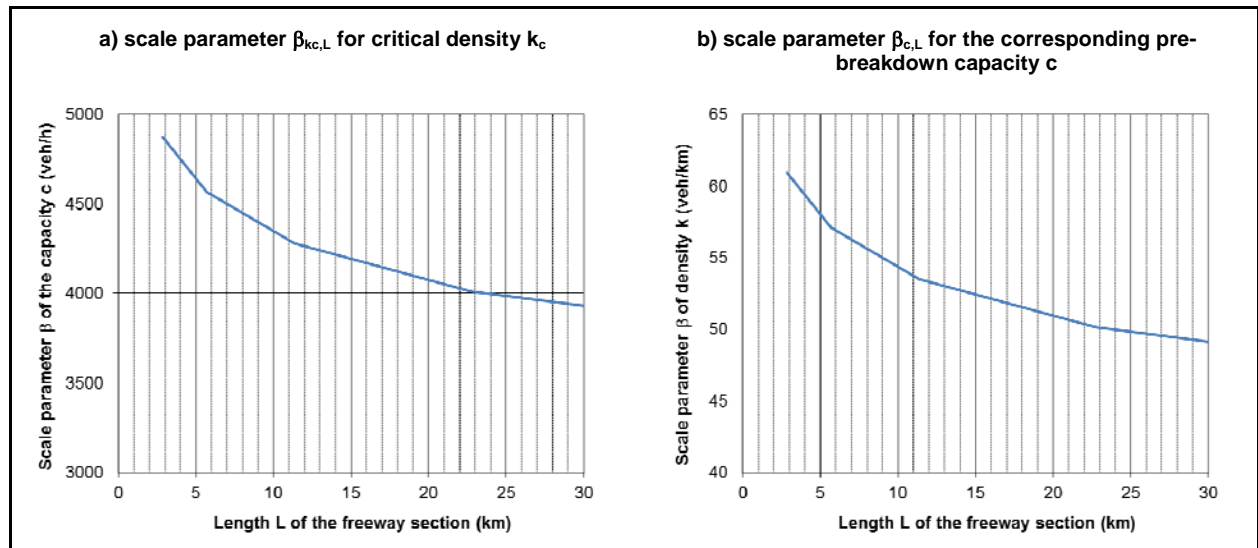


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