Capacity of Shared-Short Lanes at Unsignalised Intersections

by Ning Wu

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ABSTRACT

The calculation procedures in recent highway capacity manuals do not exactly treat shared - short lanes at intersections without traffic signals. The capacity of individual streams (left turn, straight ahead and right turn) are calculated separately. If the streams share a common traffic lane, the capacity of the shared lane is then calculated according to the shared lane procedure from Harders (1968). That means, the lengths of the short lanes are considered either as infinite or as zero. The exact lengths of the separate short lanes cannot be take in account. Therefore, the capacity computed from conventional methods is overestimated, whereas that from the shared lanes formula - like in chapter 10 of the HCM, 1994, is underestimated.

This paper presents an analytical theory for estimating the capacity of this combination of shared and short lanes. It is based on probability theory. This theory combines the existing procedures for estimating the capacity of shared and short lanes. It was checked by simulations in the style of the KNOSIMO-simulation. This theory can be used for arbitrary lane configurations. For the simple shared-short lane configurations, explicit equations are derived for estimating the capacity. For complicated shared-short lane configurations, iteration procedures are given. For practical applications a graph, which should facilitate the iterations needed by calculating capacity of complicated shared-short lanes, is prepared.

As a special case, the so-called flared minor approaches are treated according to the theory derived.

keywords: capacity, unsignalised intersection, short lanes, shared lanes, flared lanes.

1. INTRODUCTION

Intersections (cross-roads and T-junctions) without traffic signals are the mostly used intersections in traffic management. The traffic controls at these intersections are regulated by traffic signs.

The right of way regulated by traffic signs presupposes that a driver makes the decision for passing through if he is at the first waiting position directly at the stop line or if in front of him no other vehicle is waiting. The calculation procedures developed for this situation, which are also used in numerous manuals[1,2,3], are standard for calculating the capacity of unsignalised intersections. The two most known and simplest procedures are these from Harders[4] and Siegloch[5].

The calculation procedures in recent manuals[1,2,3] assume that, firstly, the traffic streams, which have to give way, possess their own traffic lanes at the intersection. The capacity of the individual streams (left turn, straight ahead and right turn) are calculated separately. If the streams share a common traffic lane, the capacity of the shared lane is then calculated according to the shared lane procedure from Harders[4].

The procedures for considering the lane distribution at intersections without traffic signals are: for the left turn and/or right turn streams either there are infinitely long exclusive lanes or there are no exclusive lanes at all. However, the length of the turn lanes in reality cannot be considered, i.e., if a approach with short traffic lanes (cf. Fig.1a) for the left turn and/or right turn streams is calculated, the capacity is either overestimated (length of the exclusive lanes as infinite) or underestimated (length of the exclusive lanes as zero).

In this paper a procedure is derived, with which the length of the turn lanes can be considered exactly for calculating the capacity of the shared lane. The precision of this calculation procedure is examined by simulations.

In this paper the following symbols and indices are used:

Symbols:

m	=	number of sub - streams	[-]
L	=	capacity	[veh/h]
q	=	traffic flow	[veh/h]
x	=	saturation degree	[-]
n	=	length of queue space in number of vehicles	[veh]
P_s	=	probability that a point on the street is occupied by traffic	[-]
k	=	factor for estimating the capacity of shared lane = 1 / $x_{sh,real}$	[-]
$x_{sh,real}$	=	real saturation degree of shared lane = q_{sh} / L_{sh}	[-]
L_{sh}	=	capacity of shared lane	[veh/h]
q_{sh}	=	traffic flow of shared lane	[veh/h]
x_{sh}	=	apparent saturation degree of shared lane	[-]

Indices for systems with arbitrarily many sub-streams:

i	=	index for the i-th sub-stream
$i1$	=	index for the i-th sub-stream of the level 1
$i2$	=	index for the i-th sub-stream of the level 2
j	=	index for the j-th step of iterations
sh	=	index for shared lane
$sh1$	=	index for shared lane of the level 1
$sh2$	=	index for shared lane of the level 2

Indices for systems with three sub-streams:

L	=	index for left turn streams and their traffic lanes
G	=	index for crossing streams and their traffic lanes
R	=	index for right turn streams and their traffic lanes

LG = index for shared streams consisting of a left turn and a right turn stream and their traffic lanes

GR = index for shared streams consisting of a crossing and a right turn stream and their traffic lanes

L,H = index for left turn streams and their traffic lanes on the major street

G,H = index for through streams and their traffic lanes on the major street

R,H = index for right turn streams and their traffic lanes on the major street

LG,H = index for shared streams consisting of a left turn and a through stream and their traffic lanes on the major street

GR,H = index for shared streams consisting of a through and a right turn stream and their traffic lanes on the major street

Indices for systems with two sub-streams:

I = index for stream I

II = index for stream II

2. MATHEMATICAL DERIVATIONS

In Fig.1a the possible combinations of short traffic lanes are presented. The short traffic lanes at intersections without traffic signals have usually two basic forms:

A) All three direction streams divide at a point (cf. Fig.1b, type 1and 4)

B) The streams divide one after another at two points (cf. Fig.1b, type 2 and 3)

For both basic forms of short traffic lanes mathematical derivations are given in this paper.

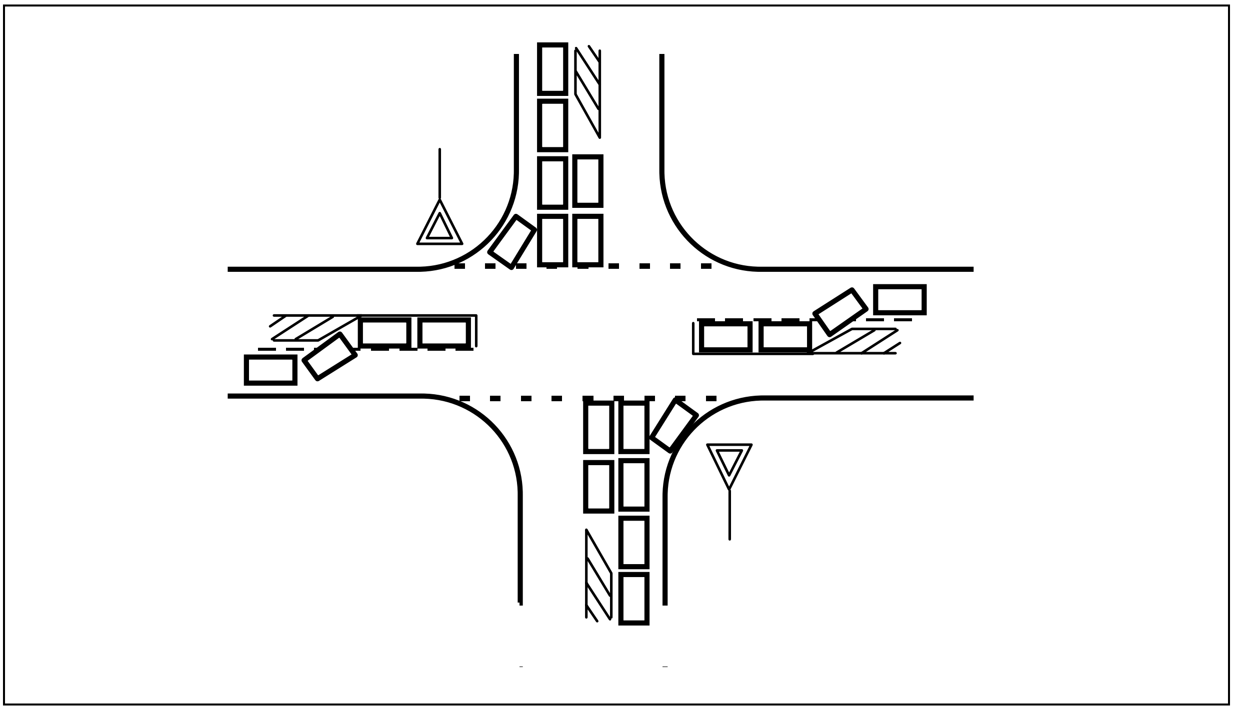


Fig.1a - Possible queues at the approaches of unsignalised intersections

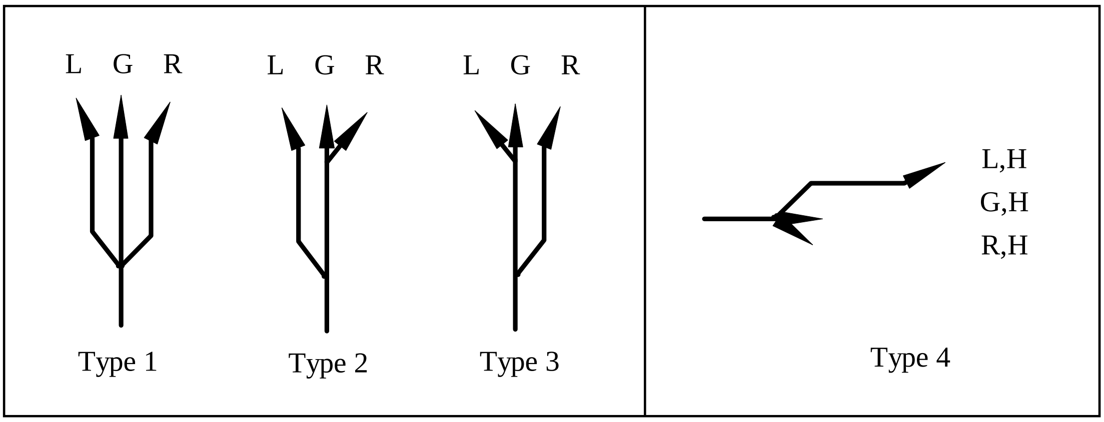


Fig.1b - Combination forms of short traffic lanes

Firstly, we consider a generalized system with m sub-streams, which all develop at the point A from one shared lane (cf. Fig.2). The sub-stream i is described by the parameters q_i (traffic flow), L_i (capacity) and x_i (saturation degree). The capacity L_i and the saturation degree $x_i = q_i / L_i$ are considered under the assumption that there are infinitely many queue places for the subject stream i. Accordingly, the shared lane has the parameters q_{sh}, L_{sh} and x_{sh}.

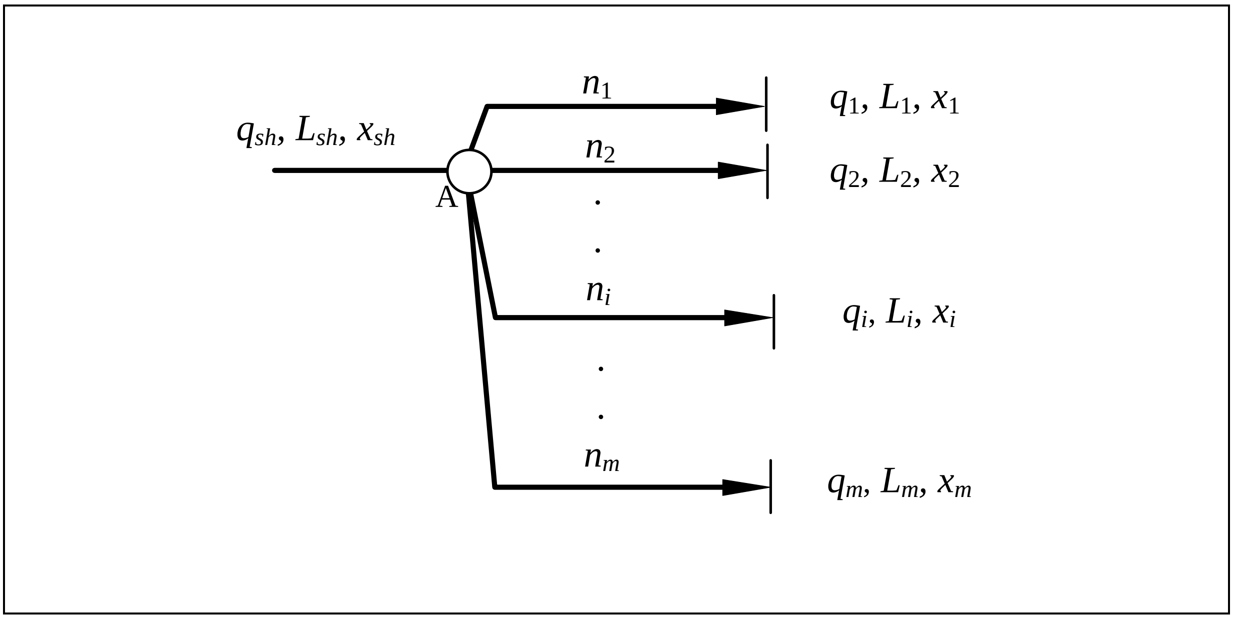


Fig.2 - Relationship between a shared lane and its sub - streams

For the point *A* the following fundamental state condition holds:

The point A is equally occupied from left (shared lane) and from right (all sub-streams) by waiting vehicles

i.e.: the probability that the point *A* is occupied on the side of the shared lane, is equal to the probability that the point *A* is occupied on the side of the sub-streams. It follows that

$$P_{s,sh} = P_{s,1} + P_{s,2} + ... + P_{s,i} + ... + P_{s,m} = \sum_{i=1}^{m} P_{s,i} \tag{1}$$

The probability that the point *A* is occupied by a sub-stream, is equal to the probability that the queue length in this sub-stream is larger than the length of the queue space (section from the stop line to point *A*), i.e., for the sub-stream *i*,

$$P_{s,i} = \Pr(N > n_i) \tag{2}$$

The distribution function of queue lengths in a waiting stream at intersections without traffic signals can be represented approximately by the following equation (cf. Wu[6)]):

$$F(n_i) = \Pr(N \le n_i) = 1 - x_i^{\,a(bn_i+1)} \tag{3}$$

with

$$x_i = \frac{q_i}{L_i}$$

a, b = parameters

Accordingly one obtains

$$P_{s,i} = \Pr(N > n_i) = 1 - F(n_i) = x_i^{a(bn_i+1)} \tag{4}$$

Also the M/M/1-queuing system is a good approximation for the queuing system at intersections without traffic signals (cf. Wu[6)]). In this case we have $a = 1$ and $b = 1$. Thus,

$$P_{s,i} = \Pr(N > n_i) = x_i^{n_i+1} \tag{5}$$

For the further derivations the queuing system at intersections without traffic signals is considered as an M/M/1-queuing system. The resulting deviation can be considered as negligible (cf. Wu[6)]).

If one considers the point A as a counter in the sense of queuing system, then the probability that the point A is occupied on the side of the shared lane is equal to the saturation degree of the shared lane, i.e.,

$$P_{s,sh} = \Pr(N > 0) = x_{sh}^{0+1} = x_{sh} \tag{6}$$

Inserting eq.(6) and eq.(5) in the eq.(1), one obtains

$$P_{s,sh} = \sum_{i=1}^{m} x_i^{n_i+1} = x_{sh} \tag{7}$$

However, here x_{sh} is only the apparent saturation degree of the shared lane. That means,

$$x_{sh} \neq \frac{q_{sh}}{L_{sh}}$$

and accordingly one has also

$$L_{sh} \neq \frac{q_{sh}}{x_{sh}} = \frac{\sum_{i=1}^{m} q_i}{\sum_{i=1}^{m} x_i^{n_i+1}}$$

The establishment of these inequalities lies therein, that no linear relationship exits between the traffic flow and the saturation degree in the shared lane due to the exponents of x_i. The capacity of the shared lane can only be determined by other ways.

For estimating the capacity of the shared lane, the following definition is made:

The capacity of the shared lane is the traffic flow, at which the merge point A on both sides is occupied 100 percent ($P_{s,sh,\max} = x_{sh,\max} = \sum x_i = 1$).

As a rule, the traffic flows q_i (existing or predicted) do not describe the complete saturation of the shared lane. The capacity of the shared lane lies generally over the sum of q_i (in case of under-saturation by existing q_i). In this case the traffic flows at the subject traffic stream would approach the limit of the capacity, if the q_i-values increase. In general, each q_i-value could have another increase. It is assumed however, that for these fictional increases of existing traffic flows equal increase factor k can be applied. k is thus that factor, by which all traffic flows on the subject approach has to increase, for reaching just the maximal possible traffic flow: the capacity.

Multiplying the saturation degree of all sub-streams by this factor k and postulating

$$P_{s,sh,\max} = x_{sh,\max} = \sum_{i=1}^{m} (k \cdot x_i)^{n_i+1} \stackrel{!}{=} 1 \quad , \tag{8}$$

one obtains the capacity of the subject shared lane

$$L_{sh} = k \cdot q_{sh} = k \cdot \sum_{i=1}^{m} q_i \tag{9}$$

Accordingly , the real saturation degree of the shared lane becomes

$$x_{sh,real} = \frac{q_{sh}}{L_{sh}} = \frac{1}{k} \tag{10}$$

Thereby k is determined implicitly by eq.(8). For $n_1 = n_2 =. .. = n_i =. .. = n_m = n$, i.e., all sub-streams have the same length of queue space, one gets,

$$k\big|_{all\ n_i=n} = \frac{1}{\sqrt[n+1]{\sum_{i=1}^{m} x_i^{n+1}}} \tag{11}$$

and

$$L_{sh}\big|_{all\ n_i=n} = \frac{\sum_{i=1}^{m} q_i}{\sqrt[n+1]{\sum_{i=1}^{m} x_i^{n+1}}} \tag{12}$$

For n_i with general values the eq.(8) cannot be solved explicitly for k. The solution for k can be found however according to the Newton-Method iteractively and numerically. The procedure of the iterations is

$$k_{j+1} = k_j - \frac{f(k_j)}{f'(k_j)} \quad (j = 0, 1, 2,...; k_0=1)$$

with (13)

$$f(k) = \sum_{i=1}^{m} (k \cdot x_i)^{n_i+1} - 1$$

This corresponds to

$$k_{j+1} = k_j - \frac{\sum_{i=1}^{m} (k_j \cdot x_i)^{n_i+1} - 1}{\sum_{i=1}^{m} \left[(n_i + 1) \cdot (k_j \cdot x_i)^{n_i} \cdot x_i \right]} \quad (j = 0, 1, 2,...; k_0=1) \tag{14}$$

The iterations are convergent for all $k > 0$.

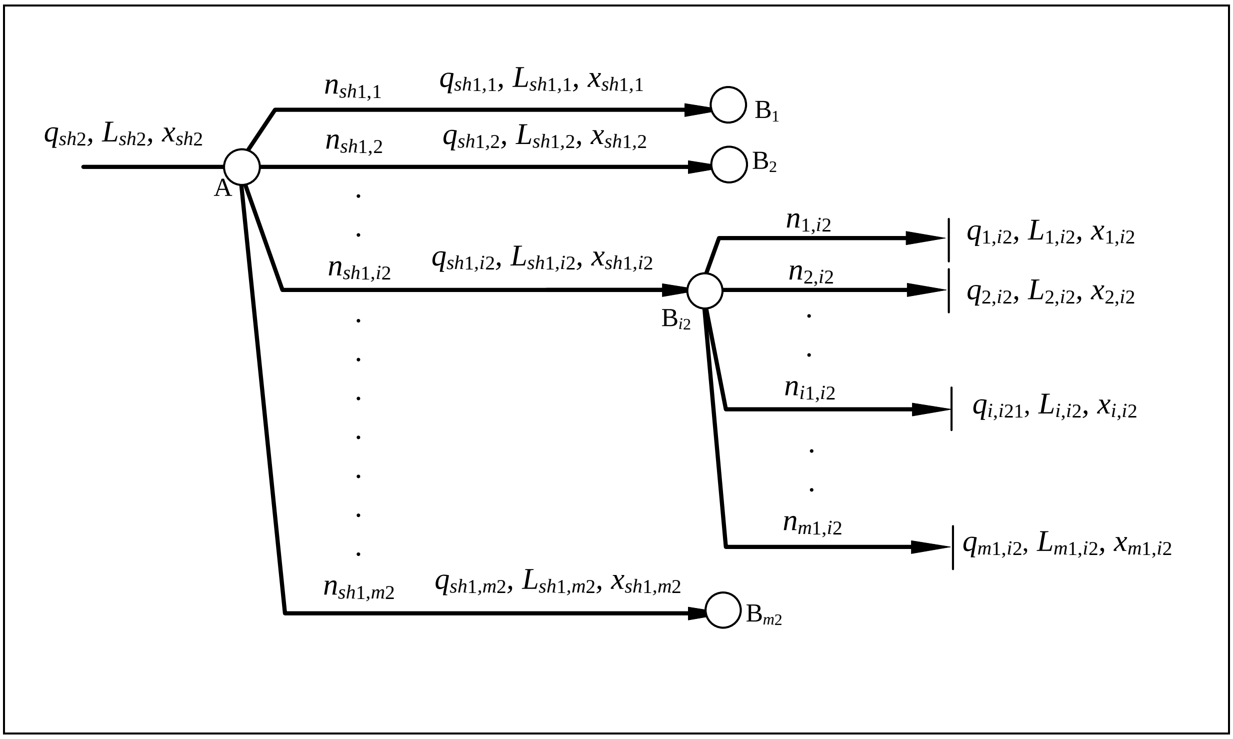


Fig.3 - Relationship between shared lanes and their sub- and sub-sub streams

If a sub-stream again consists of several sub-sub-streams, this sub-stream must be considered as a shared stream itself. One gets accordingly, in analog to the eq.(1), for the merge point *A*:

$$P_{s,sh2} = \sum_{i2=1}^{m2} P_{s,i2} \tag{15}$$

And for the sub merge points B_{i2}, one obtains:

$$P_{s,sh1,i2} = \sum_{i1=1}^{m1_{i2}} P_{s,i1,i2} \tag{16}$$

If one considers the queuing systems in all sub-and sub-sub-streams as M/M/1-queuing systems respectively, then one gets for the sub-sub-stream with index *i*1, *i*2

$$P_{s,i1,i2} = x_{i1,i2}{}^{n_{i1,i2}+1} \quad , \tag{17}$$

for the sub merge point with the index B_{i2} (section between point *A* and B_{i2})

$$P_{s,sh1,i2} = x_{sh1,i2} = \sum_{i1=1}^{m1_{i2}} P_{s,i1,i2} = \sum_{i1=1}^{m1_{i2}} x_{i1,i2}{}^{n_{i1,i2}+1} \quad , \tag{18}$$

for the sub-stream with the index $i2$

$$P_{s,i2} = x_{sh1,i2}{}^{n_{sh1,i2}+1} = P_{s,sh1,i2}{}^{n_{sh1,i2}+1} = \left(\sum_{i1=1}^{m1_{i2}} P_{s,i1,i2}\right)^{n_{sh1,i2}+1} = \left(\sum_{i1=1}^{m1_{i2}} x_{i1,i2}{}^{n_{i1,i2}+1}\right)^{n_{sh1,i2}+1} , \qquad (19)$$

and for the merge point A

$$P_{s,sh2} = x_{sh2} = \sum_{i2=1}^{m2} P_{s,i2} = \sum_{i2=1}^{m2}\left(\sum_{i1=1}^{m1_{i2}} P_{s,i1,i2}\right)^{n_{sh1,i2}+1} = \sum_{i2=1}^{m2}\left(\sum_{i1=1}^{m1_{i2}} x_{i1,i2}{}^{n_{i1,i2}+1}\right)^{n_{sh1,i2}+1} . \qquad (20)$$

Multiplying the saturation degree of all sub-sub-streams by a factor k and postulating

$$P_{s,sh2,\max} = x_{sh2,\max} = \sum_{i2=1}^{m2}\left[\sum_{i1=1}^{m1_{i2}}\left(k \cdot x_{i1,i2}\right)^{n_{i1,i2}+1}\right]^{n_{sh1,i2}+1} \overset{!}{=} 1 , \qquad (21)$$

the capacity of the total shared stream becomes

$$L_{sh} = L_{sh2} = k \cdot q_{sh2} = k \cdot \sum_{i2=1}^{m2}\sum_{i1=1}^{m1_{i2}} q_{i1,i2} \qquad (22)$$

The eqs.(20), (21), and (22) are the generalized forms of eqs.(7), (8) and (9). Setting $m1_{i2}$ or $m2$ equal to 1, one obtains here the eqs.(7), (8) and (9) again.

For $n_{i1,i2}$ and $n_{sh1,i2}$ with general values the iteration procedure for solving k becomes

$$k_{j+1} = k_j - \frac{f(k_j)}{f'(k_j)} \quad (j = 0, 1, 2,...; k_0=1)$$

with (23)

$$f(k) = \sum_{i2=1}^{m2}\left[\sum_{i1=1}^{m1_{i2}}\left(k \cdot x_{i1,i2}\right)^{n_{i1,i2}+1}\right]^{n_{sh1,i2}+1} - 1$$

This corresponds to

$$k_{j+1} = k_j - \frac{\sum_{i2=1}^{m2}\left[\sum_{i1=1}^{m1_{i2}}\left(k_j \cdot x_{i1,i2}\right)^{n_{i1,i2}+1}\right]^{n_{sh1,i2}+1} - 1}{\sum_{i2=1}^{m2}\left\{\left(n_{sh1,i2}+1\right)\cdot\left[\sum_{i1=1}^{m1_{i2}}\left(k_j \cdot x_{i1,i2}\right)^{n_{i1,i2}+1}\right]^{n_{sh1,i2}} \cdot \sum_{i1=1}^{m1_{i2}}\left[\left(n_{i1,i2}+1\right)\cdot\left(k_j \cdot x_{i1,i2}\right)^{n_{i1,i2}} \cdot x_{i1,i2}\right]\right\}}$$

Analogously we can also treat systems with arbitrarily many levels of sub-sub-streams.

3. PRACTICAL APPLICATIONS OF THE THEORY

3.1. Type 1 of lane combinations (cf. Fig.1b and 4)

The right and left turn streams divide at a point from the crossing stream

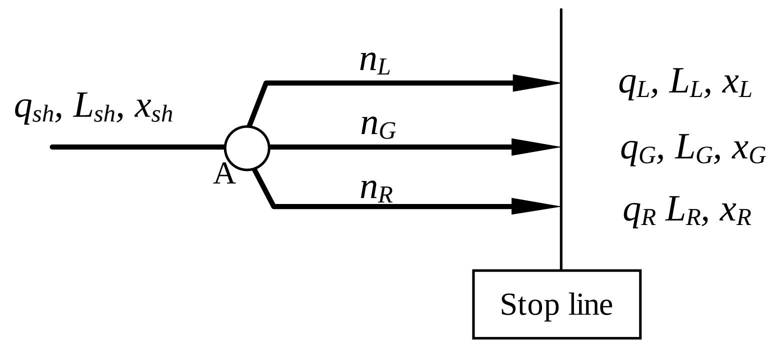


Fig.4 - Parameters for Type 1 of short traffic lanes

Setting in eqs.(20), (21), and (22) $m2 = 1$ and $i1 = L, G, R$, one obtains for Type 1 of short lanes at intersections without traffic signals the equations for estimating the capacity of the shared lane:

$$P_{s,sh}\big|_{type1} = \sum P_{s,i} = \sum P(N > n_i) = \sum x_i^{n_i+1} = x_L^{\,n_L+1} + x_G^{\,n_G+1} + x_R^{\,n_R+1} \tag{25}$$

$$P_{s,sh,\max}\big|_{type1} = x_{sh,\max} = \sum (k \cdot x_i)^{n_i+1} = (k \cdot x_L)^{n_L+1} + (k \cdot x_G)^{n_G+1} + (k \cdot x_R)^{n_R+1} \overset{!}{=} 1 \tag{26}$$

$$L_{sh}\big|_{type1} = k \cdot q_{sh} = k \cdot (q_L + q_G + q_R) \tag{27}$$

In general, the three streams (L, G, and R) must stop and wait at the same stop line. This means, that the numbers of the available queue places are equal for all three streams. Setting in this case $n_L = n_G = n_R = n$, one obtains

$$k|_{type1} = \frac{1}{\sqrt[n+1]{\sum x_i^{n+1}}} = \frac{1}{\sqrt[n+1]{x_L^{\ n+1} + x_G^{\ n+1} + x_R^{\ n+1}}} \tag{28}$$

and

$$L_{sh}|_{type1} = k|_{type1} \cdot (q_L + q_G + q_R) = \frac{q_L + q_G + q_R}{\sqrt[n+1]{x_L^{\ n+1} + x_G^{\ n+1} + x_R^{\ n+1}}} \tag{29}$$

At $n = 0$ one gets

$$L_{sh}|_{type1,n=0} = \frac{q_L + q_G + q_R}{x_L + x_G + x_R} \tag{30}$$

That is exactly the well-known shared lane formula from Harders[4)].

For n_L, n_G, and n_R with general values the iteration procedure for solving k (cf. eq.(14)) yields

$$k_{j+1}|_{type1} = k_j - \frac{(k_j \cdot x_L)^{n_L+1} + (k_j \cdot x_G)^{n_G+1} + (k_j \cdot x_R)^{n_R+1} - 1}{(n_L+1)\cdot(k_j \cdot x_L)^{n_L} \cdot x_L + (n_G+1)\cdot(k_j \cdot x_G)^{n_G} \cdot x_G + (n_R+1)\cdot(k_j \cdot x_R)^{n_R} \cdot x_R}$$

$$(j = 0, 1, 2, \ldots; k_0 = 1) \tag{31}$$

With this equation, the Newton-Iteration-Procedure is to be used for determining the subject k. The capacity of the whole approach can then be obtained according to eq.(9).

3.2. Type 2 of lane combinations (cf. Fig.1b and 5):

Right turn stream merge in the crossing stream before the left turn stream

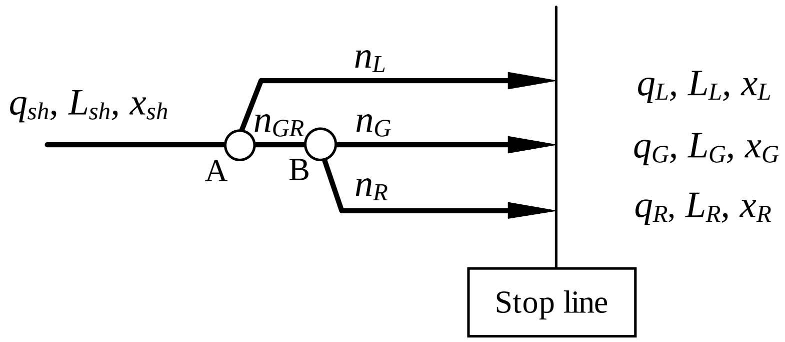


Fig. 5 - Parameters for Type 2 of short traffic lanes

Setting in eqs.(20), (21), and (22) $i2 = L$, GR and $i1 = G$, R, one obtains for Type 2 of short lanes at intersections without traffic signals for estimation the capacity of the shared lane:

$$P_{s,sh}\big|_{type2} = x_L^{\,n_L+1} + (x_G^{\,n_G+1} + x_R^{\,n_R+1})^{n_{GR}+1} \tag{32}$$

$$P_{s,sh,\max}\big|_{type2} = x_{sh,\max} = (k\cdot x_L)^{n_L+1} + \left[(k\cdot x_G)^{n_G+1} + (k\cdot x_R)^{n_R+1}\right]^{n_{GR}+1} \overset{!}{=} 1 \tag{33}$$

$$L_{sh}\big|_{type2} = k\cdot q_{sh} = k\cdot(q_L + q_G + q_R) \tag{34}$$

Also here the three streams must generally stop and wait at the same stop line. The following relationships exist between the available queue places:

$$n_G = n_R = n_{G,R}$$

$$n_L = n_{GR} + n_{G,R}$$

where $n_{G,R}$ is the common number of queue spaces for left turn and crossing streams.

According to these relationships one gets

$$P_{s,sh}\big|_{type2,n_G=n_R,n_L=n_{GR}+n_{G,R}} = x_L^{\,n_L+1} + (x_G^{\,n_{G,R}+1} + x_R^{\,n_{G,R}+1})^{n_{GR}+1} \tag{35}$$

$$
\begin{aligned}
P_{s,sh,\max}\big|_{type2,n_G=n_R,n_L=n_{GR}+n_{G,R}} = x_{sh,\max} \\
&= (k\cdot x_L)^{n_L+1} + \left[(k\cdot x_G)^{n_{G,R}+1} + (k\cdot x_R)^{n_{G,R}+1}\right]^{n_{GR}+1} \\
&= (k\cdot x_L)^{n_L+1} + \left[\left(x_G{}^{n_{G,R}+1} + x_R{}^{n_{G,R}+1}\right)^{\frac{1}{n_{G,R}+1}} \cdot k\right]^{n_{GR}\cdot n_{G,R}+n_{GR}+n_{G,R}+1} \\
&= (k\cdot x_L)^{n_L+1} + \left[\left(x_G{}^{n_{G,R}+1} + x_R{}^{n_{G,R}+1}\right)^{\frac{1}{n_{G,R}+1}} \cdot k\right]^{n_{GR}\cdot n_{G,R}+n_L+1} \\
&\overset{!}{=} 1
\end{aligned}
\tag{36}
$$

Setting

$$
\begin{aligned}
x_I &= x_L \\
n_I &= n_L \\
x_{II} &= \left(x_G{}^{n_{G,R}+1} + x_R{}^{n_{G,R}+1}\right)^{\frac{1}{n_{G,R}+1}} \\
n_{II} &= n_{GR}\cdot n_{G,R} + n_{GR} + n_{G,R} \\
&= n_{GR}\cdot n_{G,R} + n_L
\end{aligned}
\tag{37}
$$

one obtains for the postulate (eq.(36))

$$
P_{s,sh,\max}\big|_{type2,n_G=n_R,n_L=n_{GR}+n_{G,R}} = x_{sh,\max} = (k\cdot x_I)^{n_I+1} + (k\cdot x_{II})^{n_{II}+1} \overset{!}{=} 1 \tag{38}
$$

This means: under the marginal condition that all three sub-streams stop and wait at the same stop line, the shared lane-system with three sub-streams can be simplified in a shared lane-system with only two sub-streams.

Furthermore, the capacity under this condition is

$$
L_{sh}\big|_{type2,n_G=n_R,n_L=n_{GR}+n_{G,R}} = k\cdot(q_L + q_G + q_R) = \frac{(q_L + q_G + q_R)}{x_{sh,real}} \tag{39}
$$

For n_L, n_G, and n_R with general values the iteration procedure for estimating k yields

$$k_{j+1} = k_j - \frac{a}{b + c \cdot d} \quad , \qquad (j = 0,1,2,...;\ k_0=1) \tag{40}$$

with

$$a = (k_j \cdot x_L)^{n_L+1} + \left[(k_j \cdot x_G)^{n_G+1} + (k_j \cdot x_R)^{n_R+1}\right]^{n_{GR}+1} - 1$$

$$b = (n_L + 1) \cdot (k_j^{\,n_L} \cdot x_L)^{n_L} \cdot x_L$$

$$c = (n_{GR} + 1) \cdot \left[(k_j \cdot x_G)^{n_G+1} + (k_j \cdot x_R)^{n_R+1}\right]^{n_{GR}}$$

$$d = \left[(n_G + 1) \cdot (k_j \cdot x_G)^{n_G} \cdot x_G + (n_R + 1) \cdot (k_j \cdot x_R)^{n_R} \cdot x_R\right]$$

3.3. Type 3 of lane combinations (cf. Fig.1b and 6):

Left turn stream merges before the right turn stream in the crossing stream

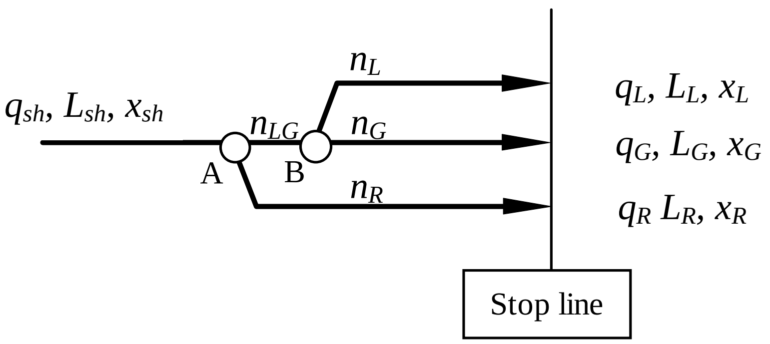


Fig. 6 - Parameters for Type 3 of short traffic lanes

Type 3 of short lanes is symmetrically to Type 2. Analogously to eqs.(32), (33), and (34) one obtains here

$$P_{s,sh}\big|_{type3} = \left(x_L^{\,n_L+1} + x_G^{\,n_G+1}\right)^{n_{LG}+1} + x_R^{\,n_R+1} \tag{41}$$

$$P_{s,sh,\max}\big|_{type3} = x_{sh,\max} = \left[(k \cdot x_L)^{n_L+1} + (k \cdot x_G)^{n_G+1}\right]^{n_{LG}+1} + (k \cdot x_R)^{n_R+1} \overset{!}{=} 1 \tag{42}$$

$$L_{sh}\big|_{type3} = k \cdot q_{sh} = k \cdot (q_L + q_G + q_R) \tag{43}$$

Under the conditions ($n_{L,G}$ is the common number of queue places for crossing and left turn streams)

$$n_L = n_G = n_{L,G}$$

$$n_R = n_{LG} + n_{L,G}$$

one obtains

$$P_{s,sh}\big|_{type3, n_L = n_G, n_R = n_{LG} + n_{L,G}} = (x_L{}^{n_{L,G}+1} + x_G{}^{n_{L,G}+1})^{n_{LG}+1} + x_R{}^{n_R+1} \tag{44}$$

and

$$P_{s,sh,\max}\big|_{type3, n_G = n_R, n_L = n_{GR} + n_{G,R}} = x_{sh,\max} = (k \cdot x_I)^{n_I+1} + (k \cdot x_{II})^{n_{II}+1} \overset{!}{=} 1 \tag{45}$$

with

$$\begin{aligned} x_I &= \left(x_L{}^{n_{L,G}+1} + x_G{}^{n_{L,G}+1}\right)^{\frac{1}{n_{L,G}+1}} \\ n_I &= n_{LG} \cdot n_{L,G} + n_{LG} + n_{L,G} \\ &= n_{LG} \cdot n_{L,G} + n_R \\ x_{II} &= x_R \\ n_{II} &= n_R \end{aligned} \tag{46}$$

For n_L, n_G, and n_R with general values the procedure of the iterations for estimating k yields

$$k_{j+1} = k_j - \frac{a}{b \cdot c + d} \quad , \qquad (j = 0, 1, 2, ...; k_0 = 1) \tag{47}$$

with

$$a = \left[(k_j \cdot x_L)^{n_L+1} + (k_j \cdot x_G)^{n_G+1}\right]^{n_{LG}+1} + (k_j \cdot x_R)^{n_R+1} - 1$$

$$b = (n_{LG} + 1) \cdot \left[(k_j \cdot x_L)^{n_L+1} + (k_j \cdot x_G)^{n_G+1}\right]^{n_{LG}}$$

$$c = \left[(n_L + 1) \cdot (k_j \cdot x_L)^{n_L} \cdot x_L + (n_G + 1) \cdot (k_j \cdot x_G)^{n_G} \cdot x_G\right]$$

$$d = (n_R + 1) \cdot (k_j \cdot x_R)^{n_R} x_R$$

3.4. Type 4 of lane combinations (cf. Fig.1a and 7)

Left turn stream on the major street

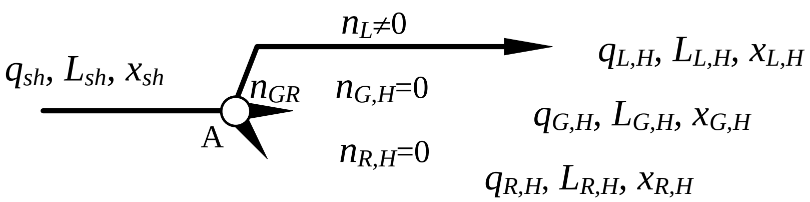


Fig. 7 - Parameters for Type 4 of short lanes on major streets

Setting in the eqs.(20), (21), and (22) $m2 = 1$ and $i1 = L,H,\ G,H,\ R,H$, with $n_{G,H} = 0$ and $n_{R,H} = 0$, one obtains here:

$$\begin{aligned} P_{s,sh}\big|_{type4} &= \sum P_{s,i} = \sum P(N > n_i) = \sum x_i^{n_i+1} \\ &= x_{L,H}{}^{n_{L,H}+1} + x_{G,H}{}^{n_{G,H}+1} + x_{R,H}{}^{n_{R,H}+1} \\ &= x_{L,H}{}^{n_{L,H}+1} + x_{G,H} + x_{R,H} \end{aligned} \tag{48}$$

$$\begin{aligned} P_{s,sh,\max}\big|_{type4} = x_{sh,\max} &= (k \cdot x_{L,H})^{n_{L,H}+1} + k \cdot x_{G,H} + k \cdot x_{R,H} \\ &= (k \cdot x_{L,H})^{n_{L,H}+1} + k \cdot (x_{G,H} + x_{R,H}) \\ &\overset{!}{=} 1 \end{aligned} \tag{49}$$

$$L_{sh}\big|_{type4} = k \cdot q_{sh} = k \cdot (q_L + q_G + q_R) \tag{50}$$

Setting

$$\begin{aligned} x_I &= x_{L,H} \\ n_I &= n_{L,H} \\ x_{II} &= x_{G,H} + x_{R,H} \\ n_{II} &= 0 \end{aligned} \tag{51}$$

one obtains again for the postulate (eq.(49))

$$P_{s,sh,\max}\big|_{type4} = x_{sh,\max} = (k \cdot x_I)^{n_I+1} + k \cdot x_{II} \overset{!}{=} 1 \tag{52}$$

For $n_{L,H}$ with general values the iteration procedure for estimating k becomes

$$k_{j+1}\big|_{type4} = k_j - \frac{(k_j \cdot x_{L,H})^{n_{L,H}+1} + k_j \cdot x_{G,H} + k_j \cdot x_{R,H} - 1}{(n_L + 1)\cdot(k_j \cdot x_{L,H})^{n_{L,H}} \cdot x_{L,H} + x_{G,H} + x_{R,H}}$$

$$(j = 0, 1, 2,...; k_0=1) \qquad (53)$$

For the system with two sub-streams, a graph (see appendix) for estimating the value of $x_{sh,real} = 1 / k$ is prepared. With this graph the iterations for estimating $x_{sh,real} = 1 / k$ can be done without computers. The input data of the graph are the saturation degrees of the sub-streams x_I, x_{II}, and the numbers of queue places for the sub-streams n_I, n_{II}.

The usage of this graph is explained using an example.

Example:

Estimate of the capacity of a shared lane on the major street (cf. Fig. 7)

Given:

Capacity of the left turn stream:	$L_{L,H}$ =	500	[veh/h]
Capacity of the through traffic stream:	$L_{G,H}$ =	1800	[veh/h]
Capacity of the right turn stream :	$L_{R,H}$ =	1600	[veh/h]
Traffic flow of the left turn stream:	$q_{L,H}$ =	500	[veh/h]
Traffic flow of the through traffic stream:	$q_{G,H}$ =	450	[veh/h]
Traffic flow of the right turn stream:	$q_{R,H}$ =	80	[veh/h]
Number of queue places for the left turn stream:	$n_{L,H}$ =	2	[veh]
Number of queue places for the through traffic stream:	$n_{G,H}$	0	[veh]
Number of queue places for the right turn stream:	$n_{R,H}$ =	0	[veh]

Solution:

According to the eq.(51) one obtains $x_I = x_{G,H} + x_{R,H}$

$= 450 / 1800 + 80 / 1600$

$= 0.25 + 0.05$

$= 0.3$

$x_{II} = x_{L,H}$

$= 250 / 500$

$= 0.5$

$n_I = x_{G,H} = x_{R,H}$

$= 0$

$n_{II} = x_{L,H}$

$= 2$

(Note, that the order of *I* and *II* is unimportant here)

Estimating $x_{sh,real}$ from the graph (see appendix):

Step 1: Draw two vertical lines through $x_I = 0.3$ and $x_{II} = 0.5$

Step 2: Go along the line of the smaller x (here $x_I = 0.3$, important for the convergence) upwards to the line of the initial value of $x_{sh,real|\text{loop }0}$ of the iterations: Point A (here with $x_{sh,real|\text{loop }0} = 1$)

Step 3: Go horizontally to the right until to the line with the value n_I: Point 1 (here with $n_I = 0$)

Step 4: Go vertically upwards to the line with the value n_{II}: Point 2 (here with $n_{II} = 2$)

Step 5: Go horizontally again to the left to the line with the value x_{II}: Point 3 (here with $x_{II} = 0.5$).

Step 6: Draw a line through point 3 and the origin (corresponding to the value of $x_{sh,real|\text{loop }1}$) and go along this line downwards to the line with the value x_I: Point 4 (here with $x_I = 0.3$)

Step 7: Repeat the steps 3 through 7 until the line drawn in the step 6 can no longer be distinguished from the preceding loop or if the precision expected is reached

Step 8: Read the final value on the axis of $x_{sh,real}$: Point B (here with $x_{sh,real} = 0.625$)

Thus, the capacity of the shared lane yields

$$
\begin{aligned}
L_{sh} &= \frac{q_{sh}}{x_{sh,real}} \\
&= \frac{q_L + q_G + q_R}{x_{sh,real}} \\
&= \frac{250 + 450 + 80}{0.625} \\
&= 1248 \quad [\text{veh / h}]
\end{aligned}
$$

so that the shared lane has a total capacity of 1248 [veh/h].

3.5. Flared lane at minor approaches

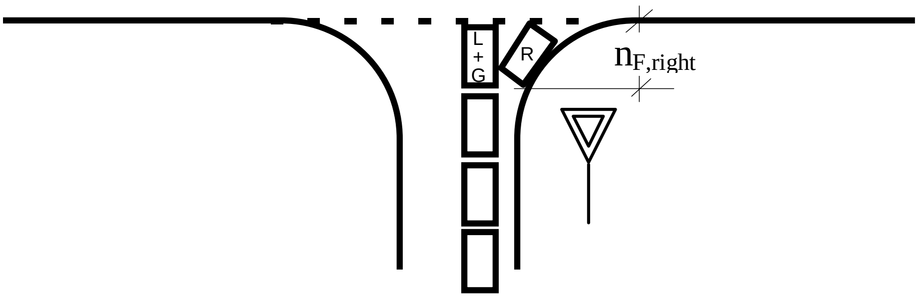


Fig.8 - Right flared approach

3.5.1. Right flared lane at minor approaches

A special application of eqs.(33) and (42) is the so-called flared lanes (cf. Fig.8). For the right flared approach (right turn stream passes by the left + crossing stream) the following relationships are valid:

$$n_L = n_G = 0$$

$$n_{LG} = n_R = n_{F,right}$$

Accordingly, one gets the postulate

$$P_{s,sh}\big|_{F,right} = x_{sh.\max} = \left[(k_{F,right} \cdot x_L) + (k_{F,right} \cdot x_G)\right]^{n_{F,right}+1} + (k_{F,right} \cdot x_R)^{n_{F,right}+1} \stackrel{!}{=} 1$$

Solving this equation for $k_{F,right}$, one obtains

$$k_{F,right} = \frac{1}{\sqrt[n_{F,right}+1]{(x_L + x_G)^{n_{F,right}+1} + x_R^{n_{F,right}+1}}} \tag{54}$$

and

$$L_{F,right} = k_{F,right} \cdot \sum q_i = \frac{q_L + q_G + q_R}{\sqrt[n_{F,right}+1]{(x_L + x_G)^{n_{F,right}+1} + x_R^{n_{F,right}+1}}} \tag{55}$$

For $n_{F,right} = 1$ it becomes

$$L_{F,rechts}\big|_{n_{F,rechts}=1} = \frac{q_L + q_G + q_R}{\sqrt[1+1]{(x_L + x_G)^{1+1} + x_R^{1+1}}} = \frac{q_L + q_G + q_R}{\sqrt[2]{(x_L + x_G)^2 + x_R^2}} \tag{56}$$

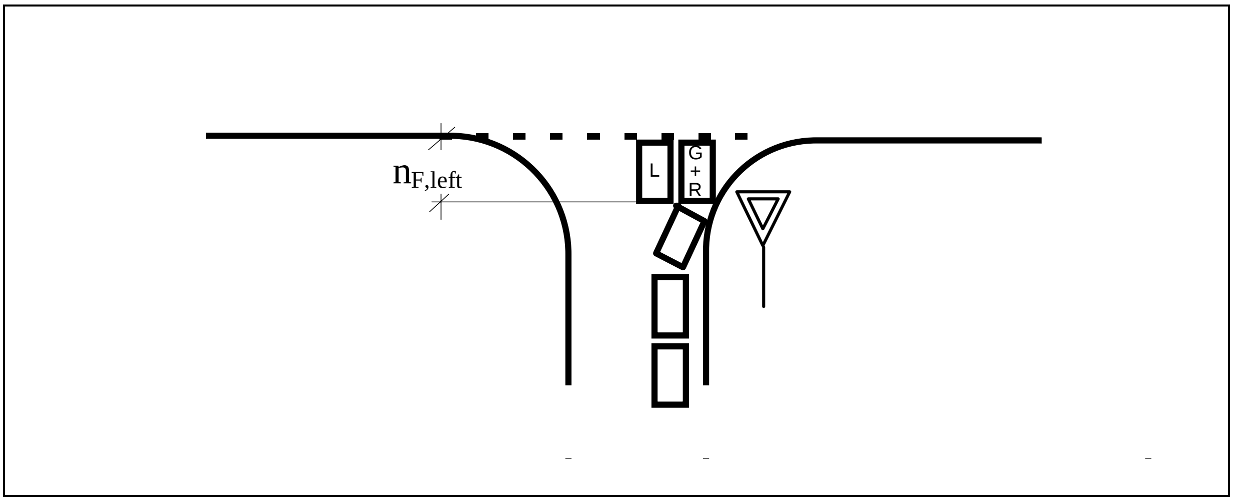


Fig.9 - Left flared approach

3.5.2. Left flared lane at minor approaches

Analogously, one obtains for the left flared approach (left turn stream passes by the right + crossing stream, Fig.9)

$$L_{F,left} = \frac{q_L + q_G + q_R}{\sqrt[n_{F,left}+1]{x_L^{n_{F,left}+1} + (x_G + x_R)^{n_{F,left}+1}}} \tag{57}$$

and for $n_{F,left} = 1$

$$L_{F,left}\big|_{n_{F,left}=1} = \frac{q_L + q_G + q_R}{\sqrt[2]{x_L^2 + (x_G + x_R)^2}} \tag{58}$$

For n_F = 0 the eqs.(55) and (57) yield

$$L_{F,right}\big|_{n_{F,right}=0} = L_{F,left}\big|_{n_{F,left}=0} = \frac{q_L + q_G + q_R}{\sqrt[0+1]{(x_L + x_G)^{0+1} + x_R^{\ 0+1}}} = \frac{q_L + q_G + q_R}{\sqrt[0+1]{x_L^{\ 0+1} + (x_G + x_R)^{0+1}}} = \frac{q_L + q_G + q_R}{x_L + x_G + x_R}$$

One gets again the shared lane formula from Harders[4)].

3.5.3. Mixed flared lane at minor approaches

Figs.8 and 9 show the two possibilities, how a flared approach can be used by vehicles. However, it is not easy to forecast, how the vehicles in reality would use the flared approach. Here, only the driver behavior of the crossing vehicles is decisive for the calculation, because the right and left turn vehicles always pass by each other at a flared approach. The decision of a crossing driver, whether he passes by a waiting left turn vehicle or by a waiting right turn vehicle, determines the configuration of the flared approach. If a crossing driver passes on the left of a waiting right turn vehicle, the approach is a right flared approach (because the left turn vehicles must pass by the waiting right turn vehicle also). If a crossing driver passes on the right of a waiting left turn vehicle, the approach is a left flared approach (because the right turn vehicles must pass by the waiting left turn vehicle also). If a crossing driver arrives while another crossing vehicle is waiting alone on the stop line, the approach could also be considered as a right flared approach (because in this case only the right turn vehicles may drive by on the right. As an approximation one can assume, that the probabilities, whether the approach is used as a left or right flared approach, are proportional to the corresponding saturation degrees of the traffic streams. According to this consideration an equation for estimating the capacity of the flared approach with mixed configuration, which treats the approach both as a left flared approach and a right flared approach, can be represented by

$$L_{F,mix} = L_{F,left} \cdot \frac{x_L}{x_L + x_G + x_R} + L_{F,right} \cdot \frac{x_G + x_R}{x_L + x_G + x_R} \qquad (59)$$

according to the saturation degrees, respectively.

Inserting eqs.(55) and (57) in the eq.(59) and setting $n_{F,left} = n_{F,right} = n_F$, one obtains

$$\begin{aligned}L_{F,mix} &= \frac{q_L+q_G+q_R}{\sqrt[n_F+1]{x_L^{\,n_F+1}+(x_G+x_R)^{n_F+1}}}\cdot\frac{x_L}{x_L+x_G+x_R}+\frac{q_L+q_G+q_R}{\sqrt[n_F+1]{(x_L+x_G)^{n_F+1}+x_R^{\,n_F+1}}}\cdot\frac{x_G+x_R}{x_L+x_G+x_R}\\ &= \left(\frac{x_L}{\sqrt[n_F+1]{x_L^{\,n_F+1}+(x_G+x_R)^{n_F+1}}}+\frac{x_G+x_R}{\sqrt[n_F+1]{(x_L+x_G)^{n_F+1}+x_R^{\,n_F+1}}}\right)\cdot\frac{q_L+q_G+q_R}{x_L+x_G+x_R}\\ &= \left(\frac{x_L}{\sqrt[n_F+1]{x_L^{\,n_F+1}+(x_G+x_R)^{n_F+1}}}+\frac{x_G+x_R}{\sqrt[n_F+1]{(x_L+x_G)^{n_F+1}+x_R^{\,n_F+1}}}\right)\cdot L_{n=0}\end{aligned} \tag{60}$$

where

$$L_{n=0}=\frac{q_L+q_G+q_R}{x_L+x_G+x_R}$$

is the capacity of the shared lane for the case n = 0 (corresponding to Harders[4)] formula).

For $n_F = 1$, the eq.(60) yields

$$\begin{aligned}L_{F,mix}\big|_{n_F=1} &= \left(\frac{x_L}{\sqrt[2]{x_L^{\,2}+(x_G+x_R)^2}}+\frac{x_G+x_R}{\sqrt[2]{(x_L+x_G)^2+x_R^{\,2}}}\right)\cdot\frac{q_L+q_G+q_R}{x_L+x_G+x_R}\\ &= \left(\frac{x_L}{\sqrt[2]{x_L^{\,2}+(x_G+x_R)^2}}+\frac{x_G+x_R}{\sqrt[2]{(x_L+x_G)^2+x_R^{\,2}}}\right)\cdot L_{n=0}\end{aligned} \tag{61}$$

In Fig.11 a comparison of increases of capacity caused by the flaring of the approach is presented. For this comparison the north bound approach of a standard intersection is calculated. The capacities of the separate traffic streams are obtained according to the German Highway Capacity Manual[2)]. The traffic flow of this intersection is shown in Fig.10. The calculation yields the parameters x_L = 0.33, x_G = 0.46, and x_R = 0.05 for the subject approach. These parameters characterized qualitatively most of the real traffic conditions at approaches of intersections without traffic signals.

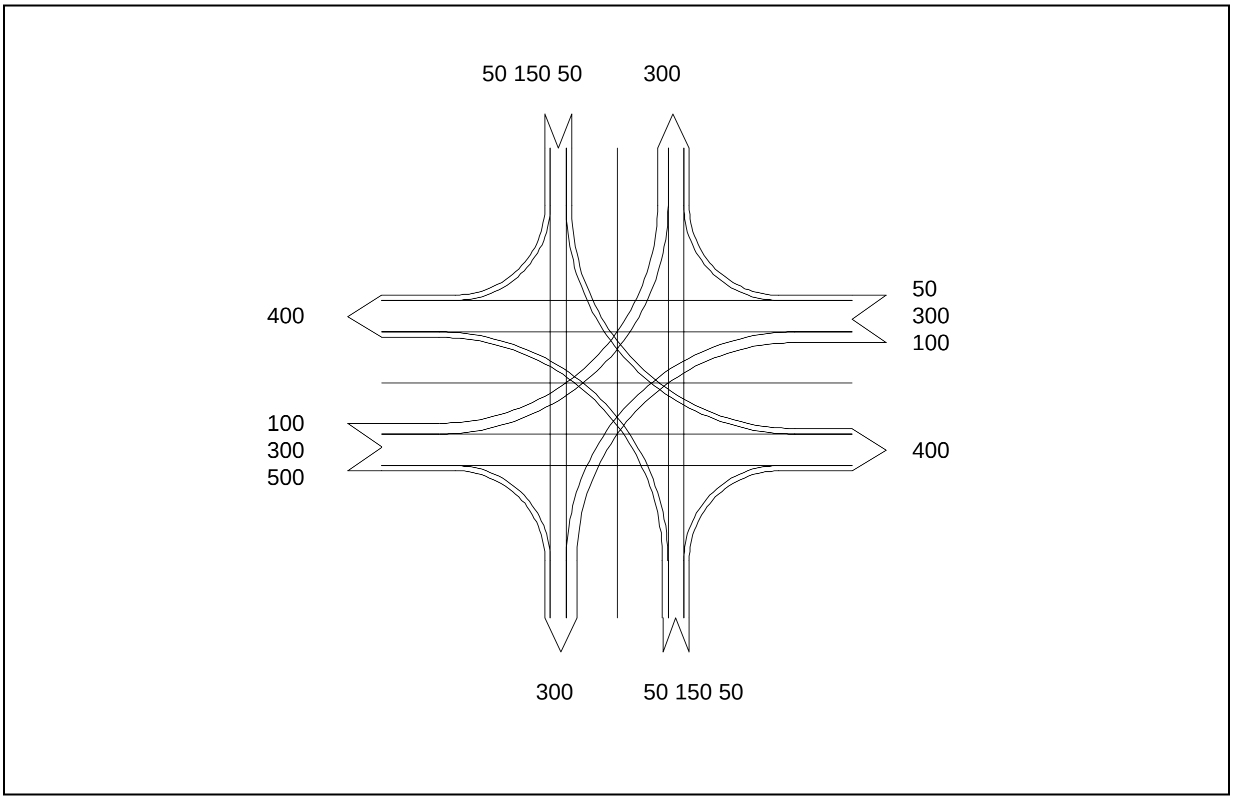


Fig.10 - Traffic flow of the test example

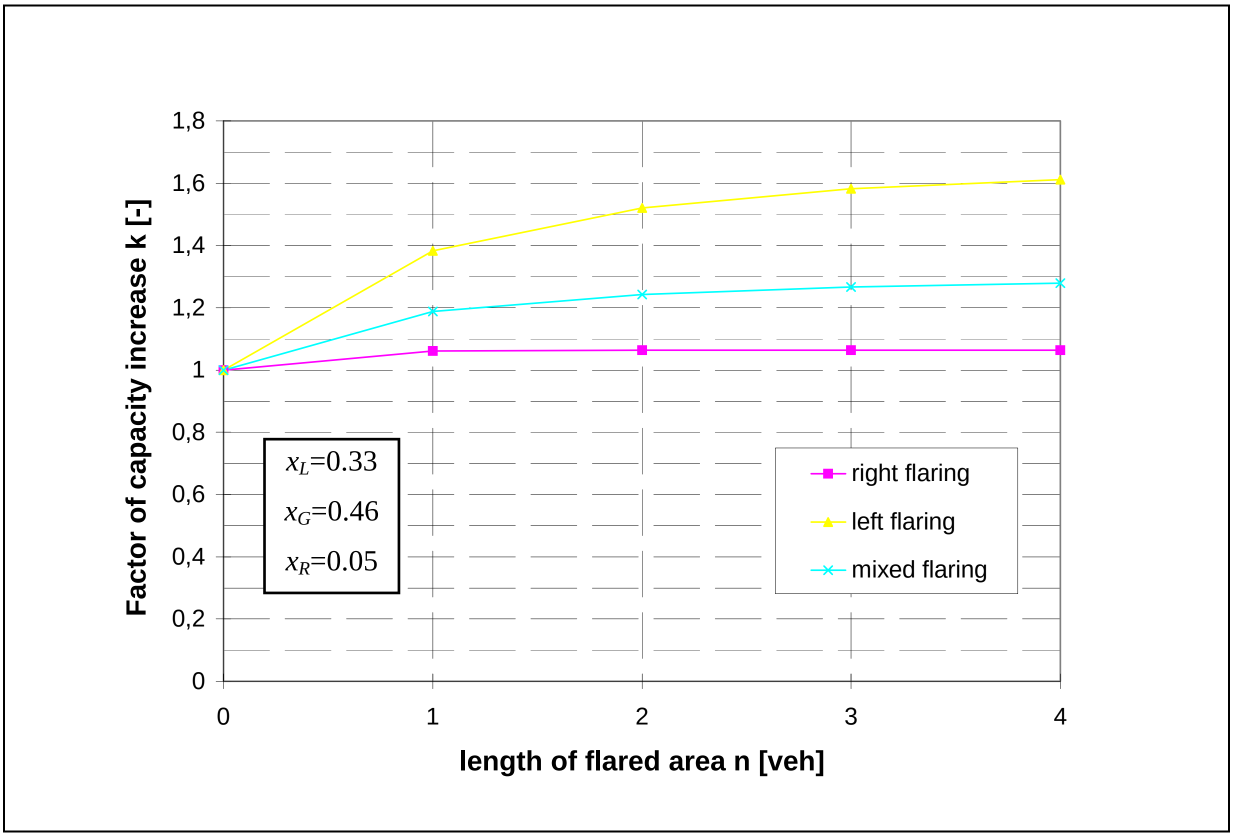


Fig. 11 - Increases of capacity caused by flaring

One can recognize clearly, that the left flaring causes a manifold increase of capacity compared to the right flaring. For the left flaring, the example has at $n_{F,left}$ = 1 a capacity increase of 38% compared to that with $n_{F,left}$ = 0. For the right flaring it is barely 6%. The mixed using of the flared area, which is more realistic than the pure left and/or right flaring, delivers approximately 18% increase of capacity. The value of the mixed flaring corresponds very well to the measurements in the technical report from Kyte et al[8] (see there, Fig.8.7).

4. EXAMINATION OF THE THEORY BY SIMULATION

To check the derived theory, different combinations of shared-short lanes are simulated in the style of KNOSIMO[7]. Altogether 95 traffic flow and lane variations were simulated. The simulation results are presented in Fig.12, together with the theoretical values. The key statistical values of this comparison are assembled in Table 1. It shows that the relationships between both parameters are narrowly correlated. Accordingly, one can be certain for the correctness of the derived theory.

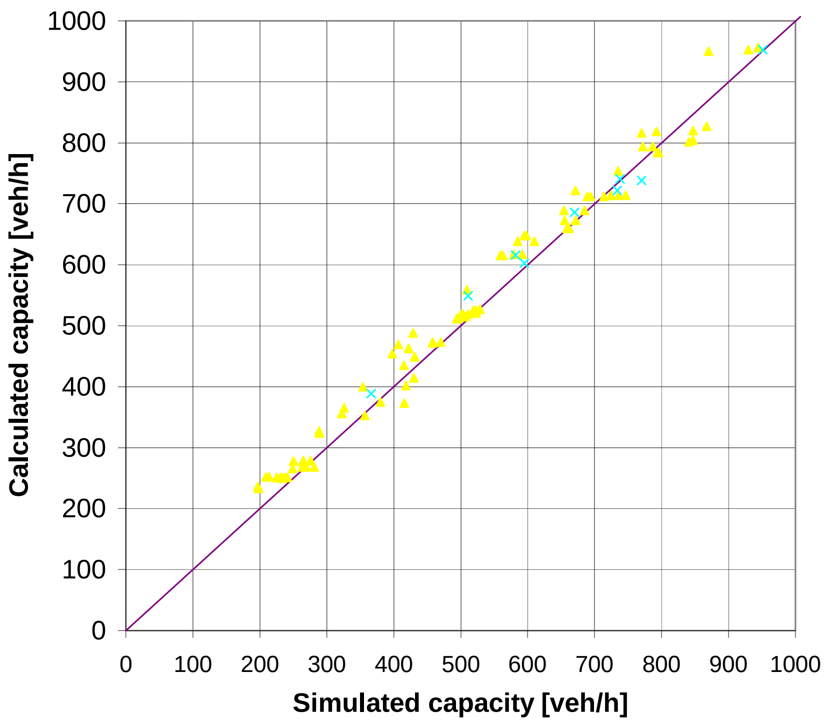


Fig. 12 - comparison of calculated and simulated capacities

Multiple correlation coefficient [-]	0.993
Certainty [-]	0.985
Adjusted certainty [-]	0.985
Standard errors [veh/h]	24.42
Observations [-]	95

Tab.1 - Key statistical values of the comparison

5. SUMMARY AND OUTLOOK

The theory derived here delivers a general approach for estimating the capacity of shared-short lanes at intersections without traffic signals. This theory considers the length of short lanes and fills out a gap in the current calculation procedures for intersections without traffic signals.

The derivation of this theory presumes, that the queuing systems at intersection without traffic signals can be approximately considered as M/M/1-queuing systems. This could lead to a minor deviation of the results from reality. The simulation results show however, that this deviation is negligibly small and statistical not significant.

For practical applications the eqs.(55), (57), and (60) are most important. With these three equations the capacity of minor approaches at intersection without traffic signals with left, right and mixed flaring can be on the simplest way and exactly determined.

It shows, that most minor approaches with three traffic streams (and lanes) can be mathematically simplified in a system with two sub-streams. For this system with two sub-streams a graph is prepared, which makes the manual calculation of the capacity possible also under complicated lane combinations.

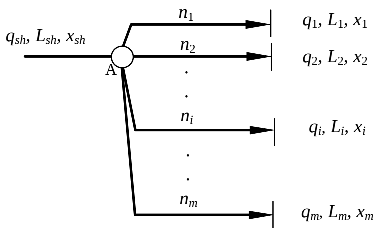
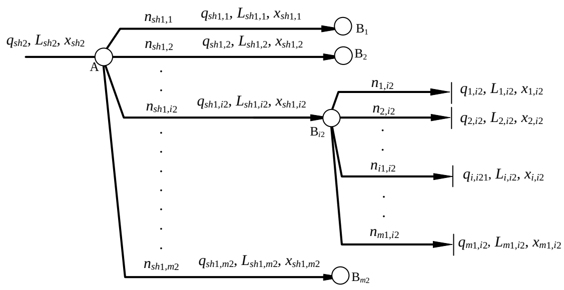
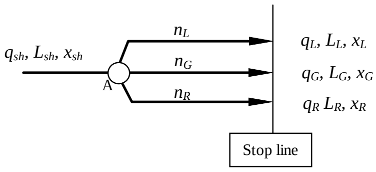
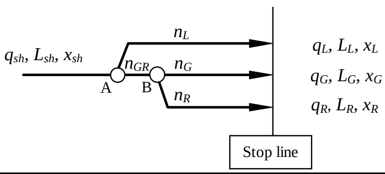
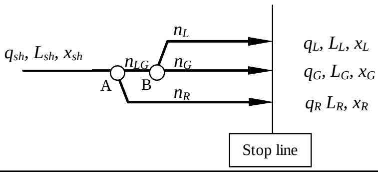
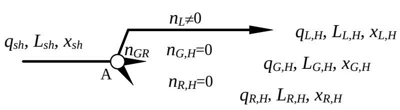
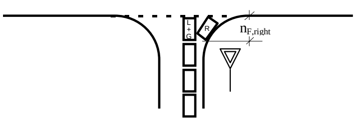
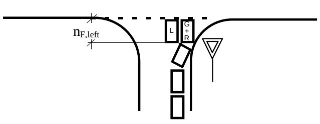
For shared-short lanes with arbitrary lane combinations a general implicit equation for estimating the capacity is given (eqs.(21) and (22)). For the solution of the implicit eq.(21) the Newton-Iteration-Method can be used (eq.(23)). The eq.(24) describes the concrete procedure of the iterations. With this procedure all possible shared-short lane combinations at intersections without traffic signals can be determined without large expenses using computers. If a worksheet program is available, one can also use the so-called SOLVER (by Excel) to solve the eq.(21).

As a summary, all possible configurations of shared-short lanes and their solutions are assembled in Table 2.

In this paper, it is presupposed for estimating the capacity of shared lanes, that the traffic flows of all sub-streams increase proportionally to their original traffic flows. All sub-streams were multiplied by the same factor *k*. It is also possible however, to determine the capacity of a certain sub-stream by using fixed traffic flows for all other sub-streams.

The theory should be expanded to intersections with traffic signals

Tab.2 - Capacity of shared - short lanes L_{sh}

Case	Figure	Equation for k[1)]	Notation
1		eqs.(13) and (14), iteration procedure. For all n_i=n, eq.(11), explicit[2)].	Generalized case with one level of sub-streams
2		eqs.(23) and (24) , iteration procedure.	Generalized case with two levels of sub-streams
3		eq.(31) , iteration procedure. For n_i=n, eq.(28), explicit[2)].	
4		eq.(40) , iteration procedure.	Graph applicable
5		eq.(47) , iteration procedure.	Graph applicable
6		eq.(53) , iteration procedure.	Graph applicable
7		eq.(54), explicit[2)].	Right flared lane
8		eq.(57), explicit[2)].	Left flared lane
9	Case 7 + Case 8	eq.(60), explicit[2)].	Mixed flared lane

[1)] $L_{sh} = k \cdot q_{sh} = k \cdot \sum q_i$, [2)]L_{sh} is directly available

REFERENCES

1) **HCM**. *Highway Capacity Manual.* Transportation Research Board, Special Report 209, new edition, Washington 1994.

2) **D-HCM (German Highway Capacity Manual).** *Verfahren für die Berechnung der Leistungsfähigkeit und Qualität des Verkehrsablaufes auf Straßen (Deutsches HCM).* Schriftenreihe "Forschung Straßenbau und Straßenverkehrstechnik", Heft 669, 1964.

3) **Worksheet for calculating capacities at unsignalised intersections.** *Merkblatt zur Berechnung der Leistungsfähigkeit von Knotenpunkten ohne Lichtsignalanlagen.* Herausgeber: Forschungsgesellschaft für Straßen- und Verkehrswesen. 1991.

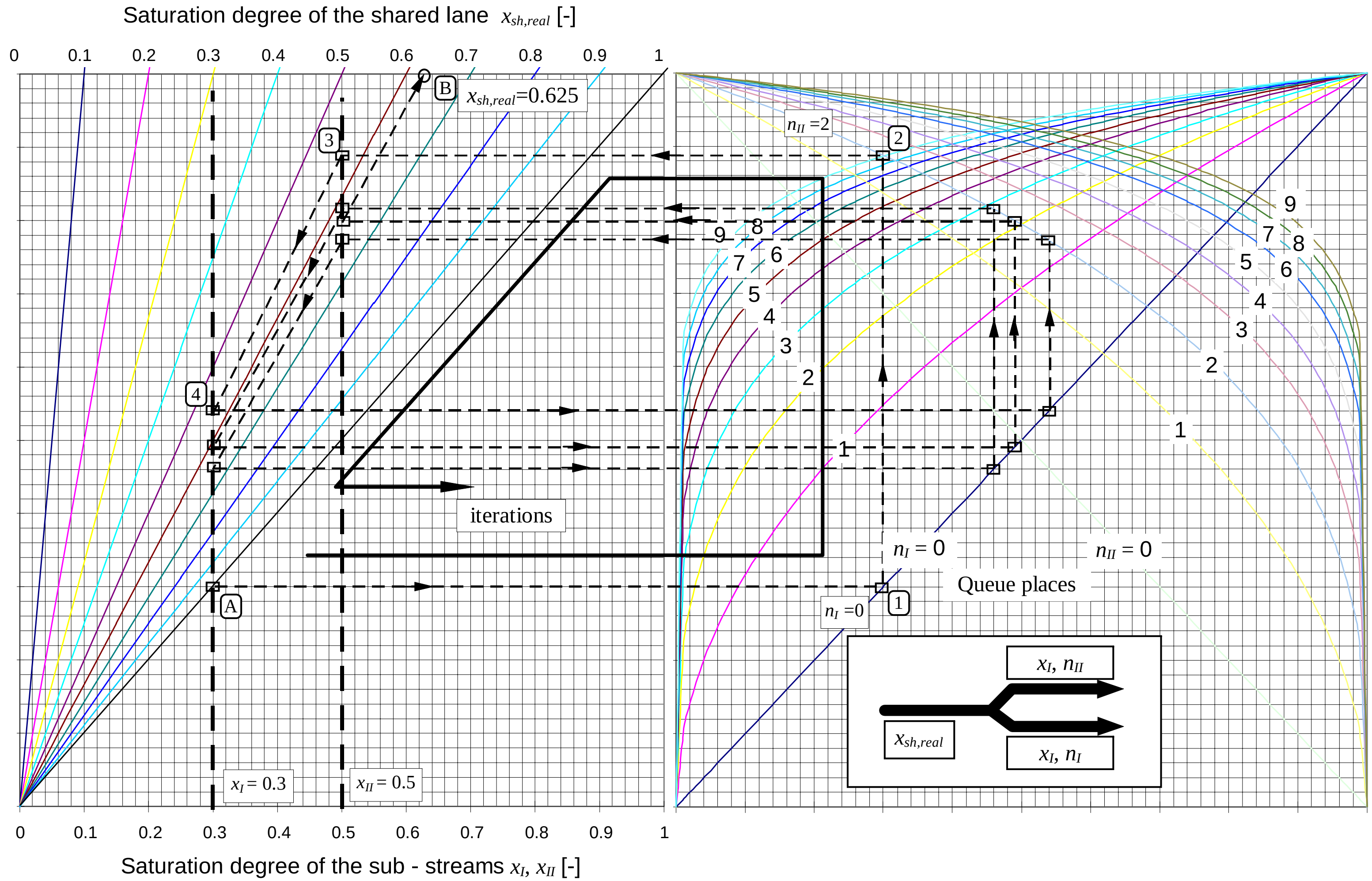
4) **J. Harders.** *Die Leistungsfähigkeit nicht signalgeregelter städtischer Verkehrsknoten.* Schriftenreihe "Straßenbau und Straßenverkehrstechnik", Heft 76, 1968.

5) **W. Siegloch.** ***D**ie Leistungsermittlung an Knotenpunkten ohne Lichtsignalsteuerung.* Schriftenreihe "Straßenbau und Straßenverkehrstechnik", Heft 154, 1973.

6) **N. Wu.** *An Approximation for the Distribution of Queue Lengths at Unsignalised Intersections.* Proceeding of the second International Symposium on Highway Capacity, Sydney, 1994. In Akcelik, R. (ed.), volume 2, pp. 717-736.

7) **M. Grossmann.** *Methoden zur Berechnung und Beurteilung von Leistungsfähigkeit und Verkehrsqualität an Knotenpunkten ohne Lichtsignalanlagen.* Ruhr-Universität Bochum, Schriftenreihe Lehrstuhl für Verkehrswesen, Heft 9, 1991.

8) **Kyte, M. et al.** *Capacity and level of service at unsignalised intersections. Final report, volume 2 - Two-way stop-controlled intersections.* National cooperative highway research program, Project 3-46. Dec. 1995.



Appendix - Capacity of shared - short lanes with two sub - streams : Iterations for $\left(\frac{x_I}{x_{sh,real}}\right)^{n_I+1} + \left(\frac{x_{II}}{x_{sh,real}}\right)^{n_{II}+1} = 1$

Example : $x_I = 0.3$, $x_{II} = 0.5$, $n_I = 0$, $n_{II} = 2$ $\Rightarrow$ Iterations from point A through 1,2,3,4, ... to B $\Rightarrow$ $x_{sh,real} = 0.625$ (cf. page 18)