

A NEW APPROACH FOR MODELING OF FUNDAMENTAL DIAGRAMS

With corrections in eqs. (10) and (11)

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ABSTRACT

According to the intra-vehicle interaction, a traffic flow can generally be divided into three homogenous states 1) that of free driving, 2) that of bunched driving, and 3) that of standing. The parameter describing the state of free driving is the desired speed, for the state of bunching it is the intra-vehicle gaps (time headway) within the convoy and the mean speed of the convoy, and for the state of standing its is the maximum jam density. These are the most essential parameters which do not depend on the actual traffic situation.

This paper introduces a new model which considers the Fundamental Diagram (equilibrium speed-flow-density relationship) as a function of the homogeneous states. All traffic situations in reality can be considered as combinations of the homogenous states and therefore can be described by the essential parameters mentioned above. The non-congested (fluid) traffic is a combination (superposition) of the states of free driving and bunched driving, the congested (jam, Stop and Go) traffic is a combination of the states of bunched driving (Go) and standing (Stop). The contribution of the traffic states within the differently

congested traffic situations can then be easily obtained from the queuing and probability theory. As a result, Fundamental Diagram in all equilibrium traffic situations is derived as simple functions of the essential parameters.

According to the new model the capacity of freeways and rural highways can be determined by measuring the essential parameters. This is much easier than measuring the capacity directly.

Furthermore, the probabilities of the various traffic states can be obtained from the new model. This leads to new possibilities in real-time controlling and telematics.

The new model is verified by comprehensive measurements carried out on freeways and rural highways in Germany.

Keywords:

Fundamental Diagram, speed-flow-density relationship, freeway, rural highway

1 INTRODUCTION

The traffic flow on freeways and rural highways is described traditionally in terms of three parameters: the mean speed v , the traffic flow rate q , and the traffic density k . The functional relationship between these three parameters is called Fundamental Diagram. The three parameters can in general be determined by on-site measurements. However, these parameters are defined as values under equilibrium conditions. These values cannot be measured exactly. All measured values on real-world roads should be considered as

approximations only. Normally, the measured mean values of the parameters over a long time interval can be used for describing the Fundamental Diagram. In the Fundamental Diagram, the traffic flow rate q is measured as a time-mean value, the traffic density k and the mean speed v as space-mean values. These three parameters are associated with each other by the equilibrium relationship $q = v \cdot k$. Accordingly, the Fundamental Diagram is defined clearly if a function between two of the three parameters is defined. The Fundamental Diagram is featured by the parameters: the desired speed v_0 , the maximum jam density k_{\max} , the maximum traffic flow rate q_{\max} , and the optimal density k_{opt} , where k_{opt} corresponds to q_{\max} . The relationship between k , q , and v - represented in the q - v Diagram - forms the elementary knowledge for dimensioning freeways and rural highways. The mean traffic flow rate attained at a given traffic density serves as a measure of the traffic quality. Normally, for analyzing traffic qualities, the k - v relationship is applied since this functional relationship is monotonous: v decreases continuously with increasing k .

The Fundamental Diagram is actually a 3-dimensional function (cf. Figure 1). The data for this figure is collected on the German freeway A43. The measure station is located between two major freeway interchanges, one 4 km downstream, one 3 km upstream. The locally measured time-mean speeds are converted to space-mean speeds according to the traffic flow theory. This 3D function has a maximum for q . This means that for v and k there exist optimal values at which q achieves its maximum value. For measurements in the reality, this maximum traffic flow of the road can hardly be determined because the measurements always indicate the bottleneck capacity in front or in back of the location of the measurement. Moreover, the capacity of a bottleneck (normally the capacity here is defined

as the maximum of the bottleneck) is different whether it is measured before or after the traffic breakdown. For a long term consideration, for instance, by determining the equilibrium capacity, the capacity must be averaged over a long time interval during which breakdown can occur by chance. The equilibrium capacity must be between the maximum output flow before breakdowns and the maximum queue discharge flow after breakdowns. The capacity obtained through a Fundamental Diagram is the capacity in sense of the equilibrium capacity. This equilibrium capacity is the subject of this paper.

Considering the three projections of this 3D figure, the individual relationships between q , k , and v are received. The measured data are described completely by each of the three (k - v , q - v , and k - q) relationships. Here, for further consideration, the k - v relationship is used predominantly. For example, a measured k - v relationship on the German freeway A43 (cf. also Figure 1) is shown in Figure 2, left.

Usually, measured data exhibit in the k - v relationship two concentrated data clusters. The two clusters have different characteristics. Here, the traffic density k can be distinguished into two regions. The region of the traffic density within which the traffic can operate with high speed is the region of fluid traffic. The region of the traffic density within which the traffic can only move with STOP and GO is the region of jam (congested) traffic.

It can be seen clearly that between the two regions there exist only very few data points. This indicates that the traffic state is unstable in this transition region and the traffic flow only persists in this state for very short periods. If the measured data is used as the basis of a regression -- a 1-part or a 2-part function can be used for the regression -- the

Fundamental Diagram in the transition region is always overridden by data within the regions of fluid traffic and jam traffic.

However, just the transition region is needed for determining the maximum traffic flow rate, q_{\max} . The distortion of the Fundamental Diagram in this region would decisively falsify the predicted capacity of roads (cf. Figure 2, right).

In order to represent uniformly the actual shape of the Fundamental Diagram over the entire data area the class means instead of the individual data points can be employed as a database. In this way only one value of v for one value of k remains. As a result, the real shape of the Fundamental Diagram is clearly emphasized (Figure 3, left).

Traditionally, the relationships between the traffic parameters are described by mathematical functions using regression techniques. The shape of these mathematical functions is normally determined by trial and error.

The classical model for describing the relationship between the equilibrium traffic parameters of the Fundamental Diagram (q , v , and k) is the one-part, linear model for the k - v relationship from Greenshields (1935). This model often fails because in general only very few measurement points are available in the transition region. From experience, the description of the traffic process at high traffic densities is unrealistic. The capacity of roads (especially for high-speed roads, e.g., freeways and rural highways) is overestimated by this model.

Because there are very few measured data points in the transition region, there exist different models and theories for describing the k - v relationship. Two-part models (e.g.

May and Keller, 1969) for fluid and jam traffic with separate approaches are an example of these models, another is the non-linear model for the k - v relationship from van Aerde (1995). Different models also result in different capacities C (= maximum flow rate q_{\max}) for the same database (Figure 3, right).

The traditional models are macroscopic, equilibrium models. Their disadvantage is the consolidation of traffic flows within the whole regression data region. The traffic flow in the considered region is always assumed as homogeneous. And no microscopic features of the traffic flow, e.g., gaps (time headways), desired speeds, are taken into account directly. The description of the capacity by these traditional models is dependent on the scope of the data and therefore not always reliable. The models are not extendable to traffic conditions with additional parameters (e.g. proportion of slow-traffic (e.g. trucks), gradient etc.).

Assuming that the traffic flow be always homogeneous does not correspond to the reality, neither for fluid traffic nor for jam traffic.

In the past decade, many efforts were devoted to developing the so-called high-order macroscopic models which generally not assume that traffic flow is homogenous. However, these high-order models still need the equilibrium v - k solutions as a parameter in their mathematical formulation. Although some of these macroscopic models (cf. Helbing 1997, Klar, 1999; and Hoogendoorn and Bovy 2000) are found on macroscopic behavior and include microscopic parameters (such as desired speeds, acceleration times, overtaking probabilities, densities, speed variances, length of vehicles, and reaction times), the relationship between the macroscopic behavior (such as the equilibrium flow-speed-density relationship) and these microscopic parameters are not explicitly defined.

The subject of this paper is to develop a reasonable macroscopic, equilibrium solution which is based on measurable microscopic parameters. Therefore, the high-order macroscopic models are not further taken into account.

2 A NEW APPROACH FOR MODELING THE FUNDAMENTAL DIAGRAM

2.1 Division of traffic flow into homogeneous states

According to the car-following behavior and the relative position of the vehicles, the traffic flow in both the fluid and jam regions can be sub-divided into states which are in fact homogeneous (cf. Table 1). These homogeneous states are traffic states in sense that they are describable by only one state parameter. For fluid traffic, one can distinguish between individual vehicles travelling freely (state FREE) and bunched vehicles travelling in succession (state CONVOY). For jam traffic, one can distinguish between bunched vehicles (state GO) and standing vehicles (state STOP).

One can compare the four homogeneous states of traffic flow with the three physical phases of water. The individual freely travelling vehicles are like molecules in the GAS (steam) state, the bunched vehicles are like molecules in the WATER state. The Standing vehicles are like molecules in the ICE state. Accordingly, the fluid traffic flow is a GAS-WATER mixture, the jam traffic flow is a WATER-ICE mixture. For our analogy, the WATER in fluid traffic can possess other features than the WATER in jam traffic. This difference results in different capacities of a bottleneck before and after a breakdown. Zhang (1999), argues that the difference in high flows prior breakdown and queue discharge rates is

(partly) due to retarded acceleration and deceleration in case of busy traffic, when drivers mainly react the vehicle direct in front of them. On the contrary, Dilker and Bovy (1997) argue that differences in car-following behavior stem from behavioral differences in free-flow and congested regimes. According to the approach proposed in this paper, these differences can be modeled by deferent reaction times for closing and opening phases in the STOP and GO traffic. In general, the reaction time following a deceleration (brake) action is shorter than that following an acceleration action. That leads to a larger time headway in congested (stop and go) traffic compared to the fluid traffic condition.

The four homogeneous states can be determined by a few simple parameters. For the state FREE, it is the desired (or free flow) speed v_0 , for the state CONVOY the mean net gap (or mean net time headway) τ_{ko} within the convoy (and the mean speed v_{ko} within the convoy), for the state GO the mean net gap τ_{go} , and for the state STOP the maximum jam density k_{max} . If the probabilities for the individual states are given, the Fundamental Diagram is clearly defined as superposition (averaged mean value over a large time-space terrain) of these homogeneous states (cf. Table 1).

In Figure 4, the shape of the k-v relationship of the individual homogeneous traffic states is represented: The state FREE is a level straight line; the state CONVOY is only a point; the state STOP is also a point. Under the assumption that the average net gaps τ between the bunched vehicles in a convoy is constant (i.e., it is independent of the speed), the state GO is a hyperbolic function ($v_{go}=f(k_{ko})$). This assumption is supported by numerous measurements. Figure 5 shows the measured mean gross gaps (time headway from the front of one vehicle to the front of the following vehicle) within convoys according to different

authors. Using a mean net gap (time headway from the rear of one vehicle to the front of the following vehicle) of a length of 1.2 s and an average vehicle length of 6 m, the mean gross gaps from the measured data can be obtained directly. The constant net gap within the convoys also corresponds to a car-following mechanism using the principle of the "relative distance".

It can be assumed that the mean net gap within the state CONVOY in fluid traffic flow is shorter than the mean net gap within the state GO in jam traffic flow. The difference is between zero and the difference between the reaction times for the opening phase and the reaction time for closing phase in a convoy, $d_R = T_{R,open} - T_{R,close}$. In such a way, a small shift here between the state CONVOY and the state GO is established (cf. Figure 4).

The parameters of the individual states can be determined from experience or from measurements. For instance, $v_0 = 130$ km/h, $v_{ko} = 80$ km/h, $\tau_{ko} = 1.5$ s, $\tau_{go} = 2.0$ s, and $k_{max} = \text{veh}/7.5 \text{ m} = 133$ veh/km can be used as default values for German freeways. These values are average values over the total cross-section of the carriageway.

When the probabilities for the individual states are given, the shape of the k-v relationship can be constructed. The k-v relationship for fluid traffic is then the mean value of the state FREE and the state CONVOY. The k-v relationship for jam traffic is the mean value of the state STOP and the state GO. The mean value of the parameters k and v is understood as space-mean value according to their space-related definitions.

2.2 Determination of probabilities of the individual homogeneous states

How can the probabilities for the individual states be determined?

First, the jam traffic is considered. The traffic density k within the jam traffic can be constructed from k_{go} and $k_{stop}=k_{max}$. The following relationship between the traffic density in the state GO, the state STOP, and the traffic density of the entire region of jam traffic can be stated:

$$\frac{1}{k} = \frac{p_{go}}{k_{go}} + \frac{p_{stop}}{k_{stop}} = \frac{p_{go}}{k_{go}} + \frac{1-p_{go}}{k_{max}} \quad (1)$$

This function yields the probability p_{go} of the state GO as a function of k , k_{go} , and k_{max} :

$$p_{go} = \left(\frac{1}{k} - \frac{1}{k_{max}} \right) / \left(\frac{1}{k_{go}} - \frac{1}{k_{max}} \right) \quad (2)$$

Note that the value p_{go} is a space-mean value. p_{go} means that, for a given time instance, from totally M vehicles on a road (which is long enough to fulfil the equilibrium condition) $p_{go} * M$ vehicles are in state GO and $(1-p_{go}) * M$ vehicles are in state STOP. Under the assumption that the mean net gap τ_{go} within the convoy is a constant value, the space-mean speed of the state GO is then a function of τ_{go} , k_{go} , and k_{max} :

$$v_{go} = \frac{1}{\tau_{go}} \left(\frac{1}{k_{go}} - \frac{1}{k_{max}} \right) \quad (3)$$

The space-mean speed of in the jam traffic region, v_{jam} , is then a function of τ_{go} , k and k_{max} :

$$v_{\text{jam}} = p_{\text{go}} \cdot v_{\text{go}} = \frac{1}{\tau_{\text{go}}} \left(\frac{1}{k} - \frac{1}{k_{\text{max}}} \right) \quad (4)$$

Since τ_{go} and k_{max} are constant values, the equation (4) is a hyperbolic function of the traffic density k . This applies to all street types. Figure 6a shows the good agreement between the measured data on a 2-lane carriageway and results from the model computation.

Accordingly, in the region of jam traffic, the shape of the Fundamental Diagram is always a hyperbolic function in the k - v relationship and therefore a straight line in the k - q relationship. The shape of the Fundamental Diagram in the region of jam traffic is always the same for all street types (freeways, rural highways etc.). Unfortunately, this feature cannot be obtained for the fluid traffic.

However, it is possible to derive the functional k - v relationship of the Fundamental Diagram in the region of fluid traffic for all street types according to queuing and probability theory.

Every vehicle which must be overtaken by other vehicles can be considered to be a flying counter in the sense of queuing theory. According Tanner (1962) the time-mean probability of bunching on a single traffic lane is given by (assuming an $M/G/1$ queue system)

$$p_{\text{bunch, single lane}} = \text{saturation degree} = \frac{q}{q_{\text{max}}}$$

Analogously, the space-mean probability of bunching (=vehicles travelling in convoy = p_{ko}) can be obtain by

$$p_{k_o, \text{singel lane}} = \text{relative occupancy quotient} = \frac{k}{k_{k_o}}$$

The capacity of the queue system is achieved if all of the vehicles are travelling in a convoy, say, in the case of $k=k_{k_o}$.

For a 2-lane carriageway, a vehicle cannot overtake and must put itself into a convoy if on the overtaking lane already a convoy exists. The probability that a vehicle must join a convoy is then equal to the probability that there is a convoy on the overtaking lane. Thus, $p_{k_o} = p_{k_o, \text{overtaking lane}} = k/k_{k_o}$ (assuming homogeneous distribution of traffic on both traffic lanes for simplicity) and $p_{\text{free}} = 1 - k/k_{k_o}$. Again, p_{k_o} and p_{free} are space-mean values. The resulting function for the space-mean speed in the fluid traffic region, v_{fluid} , is a linear equation of k :

$$v_{\text{fluid}} = p_{\text{free}} \cdot v_0 + (1 - p_{\text{free}}) \cdot v_{k_o} = v_0 - (v_0 - v_{k_o}) \cdot \frac{k}{k_{k_o}} \quad (5)$$

with

$$k_{k_o} = (v_{k_o} \cdot \tau_{k_o, \text{gross}})^{-1} = \left[v_{k_o} \cdot \left(\tau_{k_o} + \frac{l_{\text{veh}}}{v_{k_o}} \right) \right]^{-1} = \left[v_{k_o} \cdot \left(\tau_{k_o} + \frac{1}{v_{k_o} \cdot k_{\text{max}}} \right) \right]^{-1}$$

Therefore, the k - v relationship for fluid traffic on a 2-lane carriageway is always a linear function. The measured data confirm this linear k - v relationship (Figure 6b).

For a 3-lane carriageway on a 6-lane freeway, the derivation can be carried out correspondingly. Since 2 traffic lanes are available for overtaking, the probability that a

vehicle must join a convoy is equal to the probability that these are convoys on both of the overtaking lanes. Thus, $p_{ko}=(k/k_{ko})^2$ and $p_{free}=1-p_{ko}$.

Therefore, the k-v-relationship for the fluid traffic on a 3-lane carriageway is always a square function of traffic density k:

$$v_{fluid} = p_{free} \cdot v_0 + (1 - p_{free}) \cdot v_{ko} = v_0 - (v_0 - v_{ko}) \cdot \left(\frac{k}{k_{ko}} \right)^2 \quad (6)$$

Here, the measured data confirm also the result of the model, i.e., a square function in the k-v relationship (Figure 6c).

In the same way, the k-v relationship for carriageways with more than three traffic lanes can be determined. It only needs to raise the quotient k/k_{ko} to the power of the number of the traffic lanes minus one. That is:

$$v_{fluid} = v_0 - (v_0 - v_{ko}) \cdot \left(\frac{k}{k_{ko}} \right)^{N-1} \quad (7)$$

with N = number traffic lanes of the carriageway

In the model, the overtaking maneuver is not limited to the left-hand overtaking which is strictly stipulated in Germany. Thus, the results of the new model can be used also in countries (such as U.S.A., Canada etc..) where overtaking on the right-hand is allowed.

Even the k-v relationship for 2-lane 2-way rural highways can be obtained from the queuing and gap-acceptance theory. Assuming that a vehicle needs a gap $t_{0,2}$ within the opposing traffic flow for overtaking and a gap $t_{0,1}$ within the direction traffic flow for

remerging, the probability that a vehicle can carry out the overtaking maneuver, i.e., the vehicle does not need to join a convoy, is equal to the probability that the gap in the opposing traffic flow is larger than $t_{0,2}$, and the gap in the direction traffic flow is larger than $t_{0,1}$. This yields

$$p_{\text{free}} = P(t_1 > t_{0,1} \cap t_2 > t_{0,2}) \quad (8)$$

It can be yet assumed that either in the opposing traffic flow or in the direction traffic flow the gaps are exponentially distributed (this assumption is suitable because the traffic flow rate is seldom larger than 1200 veh/h on 2-lane 2-way rural highways). Then

$$P(t > t_0) = e^{-k \cdot v_0 \cdot t_0} = e^{-k/k_0} \quad \text{with } k_0 = 1/(v_0 \cdot t_0) \quad , \quad (9)$$

$$p_{\text{free}} = e^{-(k_1 \cdot v_{0,1} \cdot t_{0,1} + k_2 \cdot v_{0,2} \cdot t_{0,2})} \quad , \quad (10)$$

and therefore for a 2-lane rural highway

$$v_{\text{fluid}} = p_{\text{free}} \cdot v_{\text{free}} + (1 - p_{\text{free}}) \cdot v_{\text{ko}} \quad . \quad (10)^*$$

That is an exponential k-v relationship.

Up to now, the k-v relationships separately for fluid and jam traffic are derived. Since the mean net gap τ_{ko} within the fluid convoy (state CONVOY) is smaller than the mean net gap τ_{go} within the jam convoy (state GO) there exists a region within which the two k-v relationships are overlapping themselves. In this region, the state of the traffic cannot be determined clearly. The traffic may be observed either in the fluid convoy or in the jam convoy. The state of the traffic can jump up and down. The probability that the traffic goes

from the fluid convoy into the jam convoy increases with increasing traffic density. In reverse, the probability that the traffic goes from jam convoy into fluid convoy increases with decreasing traffic density.

It can be expected that the fluid traffic immediately breaks down into the jam traffic at $k=k_{ko}$ (i.e., at the state that the average length of the net gaps between the vehicles is equal to τ_{ko} in the fluid convoy), and that the jam traffic immediately turns back upwards into the fluid traffic at $k=k_{go,min}$ (i.e., at the state that the average length of the net gaps between the vehicles is equal to τ_{go} in the jam convoy). Defining the return probability that the traffic turns back from jam traffic into fluid traffic as p_u , then $p_u=1$ at $k=k_{go,min}$ and $p_u=0$ at $k=k_{ko}$. Here, the shape of p_u is assumed to be a linear function for simplicity. Accordingly, the shape of the k - v relationship in the overlapping region can be determined as a superposition (space-mean value) of the fluid convoy and of the jam convoy.

Over the whole area of the traffic density, the Fundamental Diagram can now be represented as a combination (superposition) of the four homogeneous states (the speed v is a space-mean value of the speeds in the corresponding homogeneous states). The k - v relationship is represented by

$$v(k) = \begin{cases} v_{fluid}, \text{ eq.(7) or (10)} & \text{for } k \leq k_{ko} & \text{fluid traffic} \\ v_{trans}, v_{fluid} \cdot p_u + v_{jam} \cdot (1 - p_u) & \text{for } k_{go,min} \leq k \leq k_{ko} & \text{transition} \\ v_{jam}, \text{ eq.(4)} & \text{for } k \geq k_{go,min} & \text{jam traffic} \end{cases} \quad (11)$$

with $p_u = 1 - (k - k_{go,min}) / (k_{ko} - k_{go,min})$

$$k_{ko} = \left[v_{ko} \cdot \left(\tau_{ko} + \frac{1}{v_{ko} \cdot k_{max}} \right) \right]^{-1}$$

$$k_{go,min} = \left[v_{ko} \cdot \left(\tau_{go} + \frac{1}{v_{ko} \cdot k_{max}} \right) \right]^{-1}$$

Thus, the Fundamental Diagram is completely described by the five parameters: v_0 , τ_{ko} , v_{ko} , τ_{go} , and k_{max} . All of these parameters can be empirically measured on real-world roads.

In the region of fluid traffic, the shape of the Fundamental Diagram in the k - v relationship is either an exponential, or a linear, or a square, or a cubic function and so forth, depending on how many traffic lanes are available. In the region of the jam traffic, the shape of the Fundamental Diagram in the k - v relationship is always a hyperbolic function for all street types. In the transition region, it is the overlapping of the two functions in the fluid and jam traffic. The transition region is limited by k_{ko} and $k_{go,min}$. The optimal traffic density k_{opt} is located between k_{ko} and $k_{go,min}$. The phenomenon of "Capacity Drops" can be modeled by the difference between τ_{ko} and τ_{go} .

According to the new model, the Fundamental Diagram can be described completely in the k - v relationship with a few parameters since the shape of the k - v -relationship are prescribed by the model. These parameters are the desired speed v_0 , the mean speed v_{ko} of the fluid convoy, the mean net gap τ_{ko} in the fluid convoy, the mean net gap τ_{go} in the jam convoy, and the maximum traffic density k_{max} in jam traffic.

In order to take the proportions of slow-traffic (e.g., trucks) and other variables into account, further parameters that affect the mean net gap τ within the convoy must be

included (cf. Table 2). All parameters mentioned here are to be determined microscopically. Some parameters depend even on the Road Traffic Act (e.g., the speed-limits). If no measured values are given, the recommended values (Table 2) can be used.

It can be recognized that the k-v relationship is mainly determined by the mean net gap τ within the convoy. The mean net gap τ is again dependent on other variables. These variables include a) the minimum gap in the fluid convoy, b) the discharging gap of a queue, c) the intra-vehicle distance in the jam traffic, d) the proportion of slow-traffic, e) the environment conditions, e.g., wetness and darkness, f) the lane distribution of traffic flow, g) the gradient and slope, h) the curvature, i) and the regulation of speed limits. In addition, the length of the measurement intervals has a role to play for the determination of the mean net gap with the convoy. The net gap within the convoy, averaged over all lanes of a carriageway, can be calculated as following:

$$\begin{aligned}\tau_x^* &= \tau_x \cdot f_{\text{truck},\tau} \cdot f_{\text{flow-split},\tau,x} \\ l_{\text{veh}}^* &= l_{\text{veh,pc}} \cdot f_{\text{truck,lveh}}\end{aligned}\tag{12}$$

Where the index can be substituted either by "ko" or by "go". Using the given values τ_x^* and l_{veh}^* , Fundamental Diagrams - that take into account the proportion of slow-traffic and the flow-split on the traffic lanes - can be constructed.

Using the recommended values from Table 2 the minimum (for queue discharge) and maximum (prior breakdown) capacities can be obtained for a single traffic lane:

$$C_{\text{max,lane}} = \frac{3600}{\tau_{\text{ko}} + 3600/(v_{\text{ko}} \cdot k_{\text{max}})} = \frac{3600}{1.2 + 3600/(80 \cdot 155)} = 2415 \text{ pc/h}\tag{13}$$

$$C_{\min, \text{lane}} = \frac{3600}{\tau_{\text{go}} + 3600/(v_{\text{ko}} \cdot k_{\text{max}})} = \frac{3600}{1.6 + 3600/(80 \cdot 155)} = 1904 \text{ pc/h} \quad (14)$$

For a 2-lane carriageway the minimum and maximum capacities are

$$C_{\max, 2\text{-lane}} = \frac{3600 \cdot N}{\tau_{\text{ko}} \cdot f_{\text{flow-split,ko}} + 3600/(v_{\text{ko}} \cdot k_{\text{max}})} = \frac{3600 \cdot 2}{1.2 \cdot 1.2 + 3600/(80 \cdot 155)} = 4161 \text{ pc/h} \quad (15)$$

$$C_{\min, 2\text{-lane}} = \frac{3600 \cdot N}{\tau_{\text{ko}} \cdot f_{\text{flow-split,go}} + 3600/(v_{\text{ko}} \cdot k_{\text{max}})} = \frac{3600 \cdot 2}{1.6 \cdot 1.1 + 3600/(80 \cdot 155)} = 3512 \text{ pc/h} \quad (16)$$

The equilibrium mean capacity is between C_{\max} and C_{\min} . The factor $f_{\text{flow-split},\tau}=1,1$ or 1.2 represents the fact that at capacity 55% or 60% of all vehicles are on the left traffic lane.

By varying the parameters different Fundamental Diagrams can be constructed, thereby different environment conditions can be taken into account. In Figure 9 the variations considering the proportions of trucks and the gradients for a 4-lane freeway are depicted. Here, only the mean speed within the convoy v_{ko} and the net gap τ are varied. It can be recognized that realistic capacities can be obtained through these simple calculations.

In Figure 10 variations taking into account the gradients for a 2-lane 2-way rural highway are depicted. The calculated capacities also agree very well with the measured values.

The new model was applied on different measured data in Germany. In Figure 11, the measured data in the k-v and q-v relationships for two 4-lane freeways are represented. The agreement between the model and the measured data is very good. The applied parameters

are indicated in the illustrations. All parameters lie in a plausible range. In Figure 12, the measured data in the k-v and q-v relationship for two 6-lane freeways are represented. The agreement also appears good. The measured data and the model calculation for a 2-lane 2-way rural highway are depicted in Figure 13.

It can be recognized that the Fundamental Diagrams of all types of Highways can be described very well by the new model using suitable parameters. This shows that the new approach can model the regularity between the macroscopic and the microscopic parameters realistically. The new model is able to generate Fundamental Diagrams from microscopic parameters for all street types.

Fundamental Diagrams can be generated appropriately by varying the parameters. Different environment conditions can thereby be considered. By fine-tuning the parameters, it can be expected that the new model can describe the Fundamental Diagrams of all street types very realistically.

3 STANDARD DEVIATIONS OF MEAN SPEEDS

It is even possible to compose the standard deviations of the mean speeds s_v using the standard deviations of the four homogenous traffic states. It is to be pointed out, that the standard deviation of the mean speeds is not identical with the deviation of the local speed distribution. For the computation example, the standard deviations of the mean speeds in 1-minute-intervals are calculated for a 20-minutes period. Then, one can consider the standard deviation s_{free} of the state "Free" and the standard deviation s_{k_0} of the state

"Convoy" as constant values. Then the standard deviation s_{fluid} of the fluid traffic is a linear function of the traffic density k . The standard deviation s_{jam} of the jam traffic is a also linear function of k . The function $s_{\text{jam}}=f(k)$ go through the point $(k_{\text{max}}, 0)$. As a result, the standard deviation s_{trans} in the transition region is also a linear function of k (cf. Figure 14). Likewise, the measurements agree with model calculations with respect to the standard deviations. In Figure 14, $s_{\text{free}}=25$ km/h, $s_{k_0}=15$ km/h, and $s_{\text{jam.max}}=36$ km/h are applied. These values confirm the measured values in the real world.

4 CONCLUSION

According to measurements cited in the literature, it can be determined that the mean net gap within a convoy can be considered as a constant value. Base on this fact a new model for constructing Fundamental Diagrams is established.

According to this model, the Fundamental Diagram can be represented as a superposition of four homogeneous traffic states. The shape of the k - v relationship of the Fundamental Diagram can be defined by this model:

- In the region of fluid traffic it is exponential for 2-lane 2-way rural highways, linear for 2-lane carriageways, square for 3-lane carriageways, cubic for 4-lane carriageways.
- In the region of jam traffic it is hyperbolic for all street types.
- In the transition region there is an overlapping of the two functions of fluid and jam traffics.

According to the model, the Fundamental Diagram can be described completely by five essential parameters:

- the desired speed v_0 ,
- the net gap τ_{ko} within a fluid convoy,
- the mean speed v_{ko} of the fluid convoy,
- the net mean gap τ_{go} of the jam convoy,
- and the maximum jam density k_{max} .

The relationship between the microscopic and macroscopic parameters can be constructed. Using the microscopic parameters, Fundamental Diagrams of all type of roads can be easily generated.

The new model is verified by comprehensive measurements carried out on freeways and rural highways in Germany.

According to the new model the capacity of freeways and rural highways can be determined by measuring the essential parameters. This is much easier than measuring the capacity directly.

Furthermore, the probabilities of the various traffic states can be obtained from the new model. This leads to new possibilities in real-time controlling and telematics.

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LEGENDS TO TABLES AND FIGURES

Table 1 - Homogeneous traffic states and their corresponding parameters

Table 2 - Recommendation for applied parameters

Figure 1 - Fundamental Diagram as a 3D function (data: German freeway A43, measured between two major freeway interchanges, one 4 km downstream, one 3 km upstream)

Figure 2 - Traffic statuses in k-v and q-v relationships (data: cf. Figure 1)

Figure 3 - k-v relationship with the class means and some traditional approaches as regressions with the corresponding capacities (data: cf. Figure 1)

Figure 4 - Homogeneous states in the k-v relationship

Figure 5 - Measured gaps within convoys from different authors

Figure 6 - Shape of the Fundamental Diagram in the k-v relationship, a) in the region of jam traffic (data: German freeway A43), b) in the region of fluid traffic, 2-lane carriageway (data: German freeway A43), c) in the region of fluid traffic, 3-lane carriageway (data: German freeway A8), d) in the region of fluid traffic, 2-lane rural highway (data: 2-lane rural highways in the state Baden-Württemberg)

Figure 7 - Overtaking possibilities on different roads

- a) 2-lane carriageway
- b) 3-lane carriageway
- c) 2-lane 2-way rural highway

Figure 8 - Overlapping of the fluid and jam traffic

Figure 9 - Variations taking into account the proportions of trucks and the gradients for a 4-lane freeway

Figure 10 - Variations taking into account the gradients for a 2-lane rural highway

Figure 11 – Comparison of k-v and q-v relationships on 4-lane freeways

- top: German freeway A43, speed limit
- bottom: German freeway A1, no speed limit

Figure 12 - Comparison of k-v- and q-v-relationships on 6-lane freeways

- top: German freeway A1, speed limit
- bottom: German freeway A8, no speed limit

Figure 13 - Comparison of k-v- and q-v-relationships on 2-lane rural highways (rural highways in state Baden-Württemberg)

Figure 14 - Standard deviation of mean speeds from measurements and model calculations for a 2-lane carriageway (data: German freeway A43, speed limit) along with the mean speed v

region	homogeneous state	Short cut	Analogy to the physical states of water	descriptive parameters	corresponding probability
fluid traffic	free vehicles	FREE	Gas (steam)	v_0	p_{free}
	bunched convoy	CONVOY	Water I	$\tau_{k_0} (+ v_{k_0})$	$p_{k_0=1} - p_{\text{free}}$
jam traffic	bunched convoy	GO	Water II	τ_{g_0}	p_{g_0}
	standing vehicles	STOP	Ice	k_{max}	$p_{\text{stop}=1} - p_{g_0}$

Table 1 - Homogeneous traffic states and their corresponding parameters

Parameter	Measured value	recommendation	Note
v_0 [km/h]	fast (pc):130-140 slow (truck): 80-90	130 80	desired speed in state FREE
v_{ko} [km/h]	75-80	80	speed in state CONVOY $\approx v_{truck}$
τ_{ko} [s]	1.1-1.4	1.2	minimum mean net gap in state CONVOY in the far-left lane
τ_{go} [s]	1.5-2.0	1.6	mean net gap in state GO in the far-left lane
k_{max} [pcu/km)	150-160	155	for a length of a vehicle $l_{veh}=6.5m$
$f_{truck,lveh}$ [truck/pc]	1.61	1.6	$f_{truck,lveh} = l_{truck}/l_{pc}$ $l =$ length of a vehicle
$f_{truck,\tau}$ [truck/pc]	1.7-1.9	1.8	$f_{truck,\tau} = \tau_{truck}/\tau_{pc}$
$f_{flow-split,\tau}$ [-]	2 lane: 1.2-1.4 3 lane: 1.2-1.5	1.2 1.3	$f_{flow-split,\tau} = \tau_{carriageway}/\tau$

Table 2 - Recommendation for applied parameters

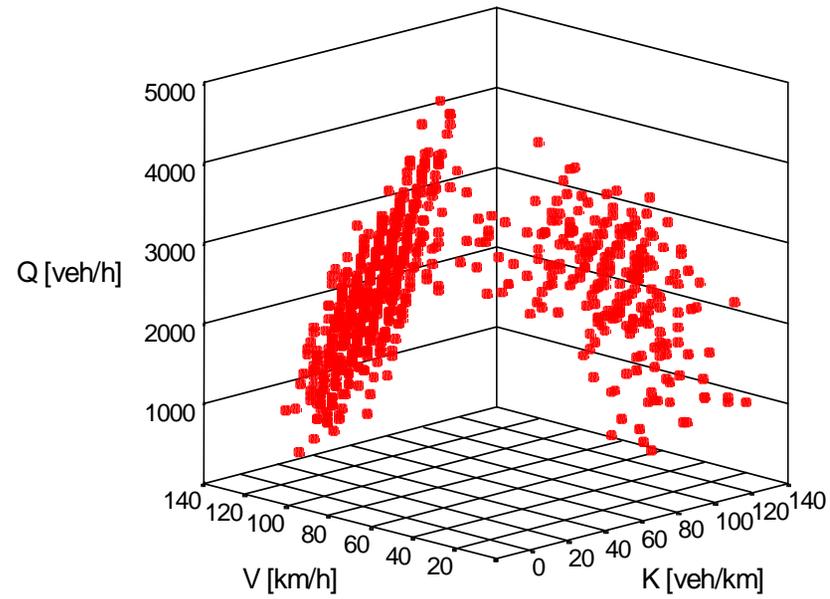


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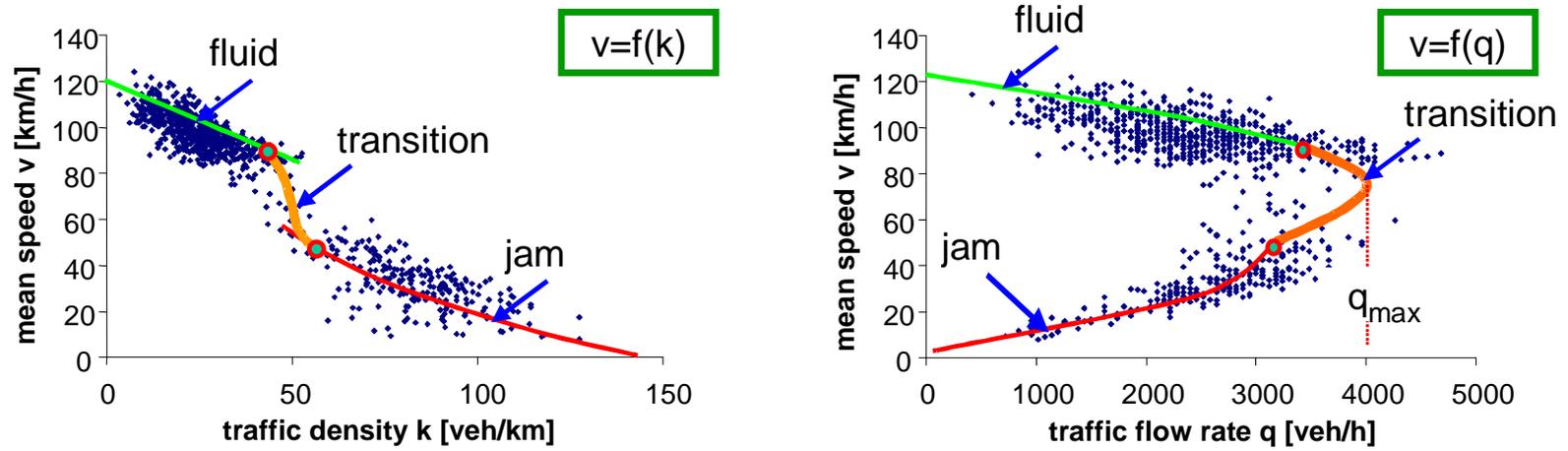


Figure 2 - Traffic statuses in k - v and q - v relationships (data: cf. Figure 1)

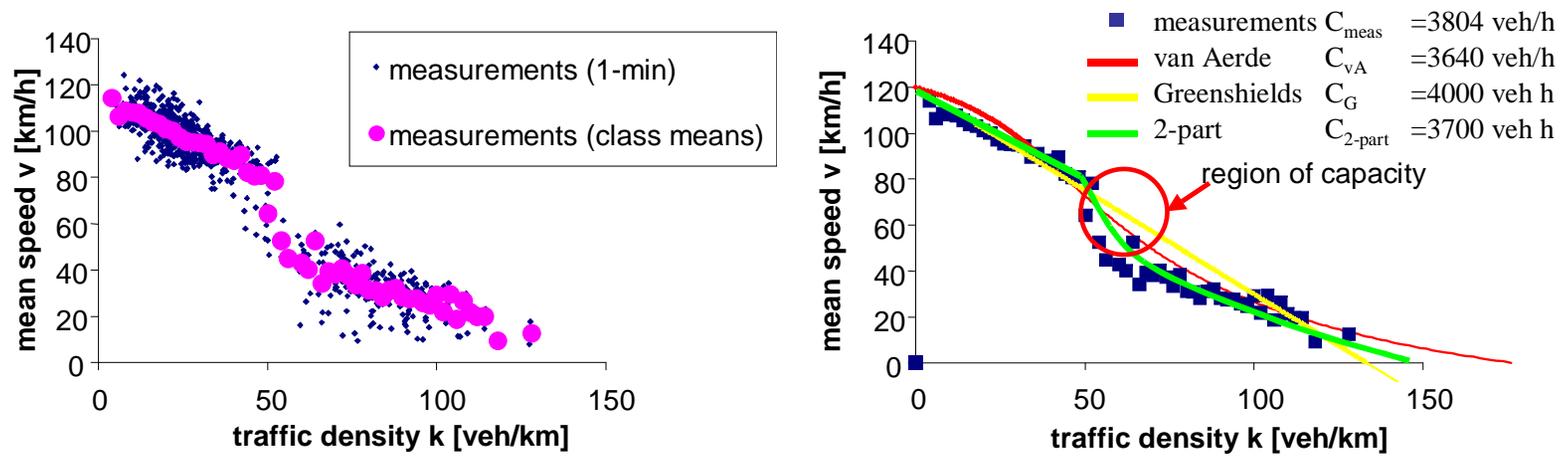


Figure 3 - k-v relationship with the class means and some traditional approaches as regressions with the corresponding capacities

(data: cf. Figure 1)

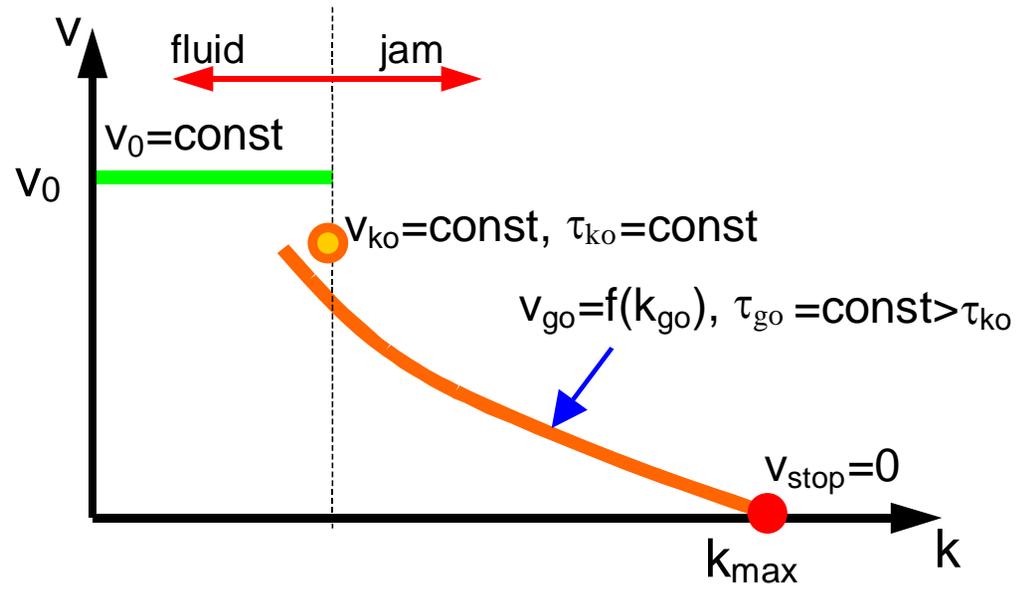


Figure 4 - Homogeneous states in the k - v relationship

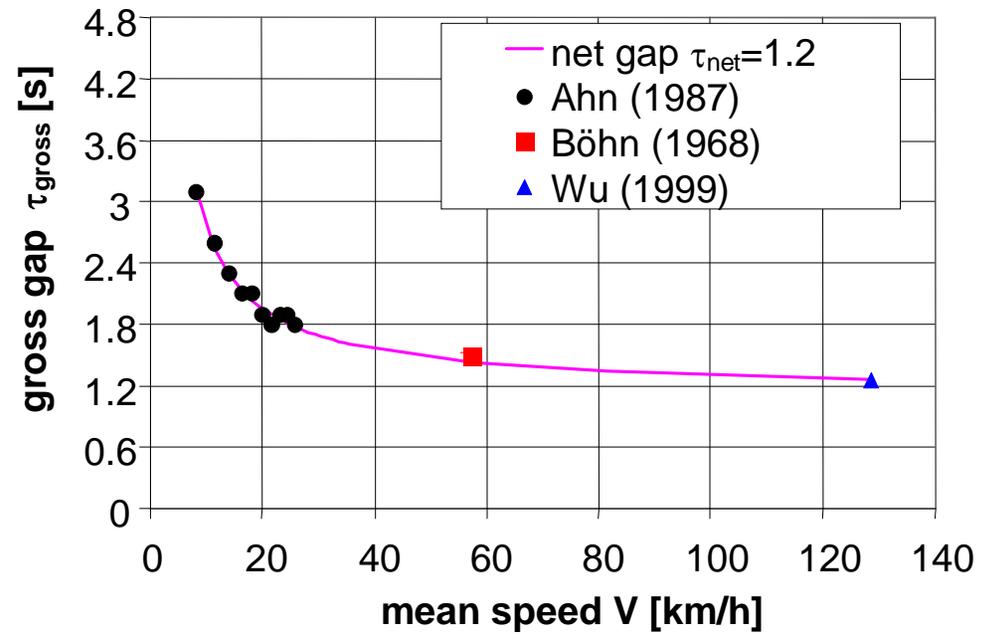


Figure 5 - Measured gaps within convoys from different authors

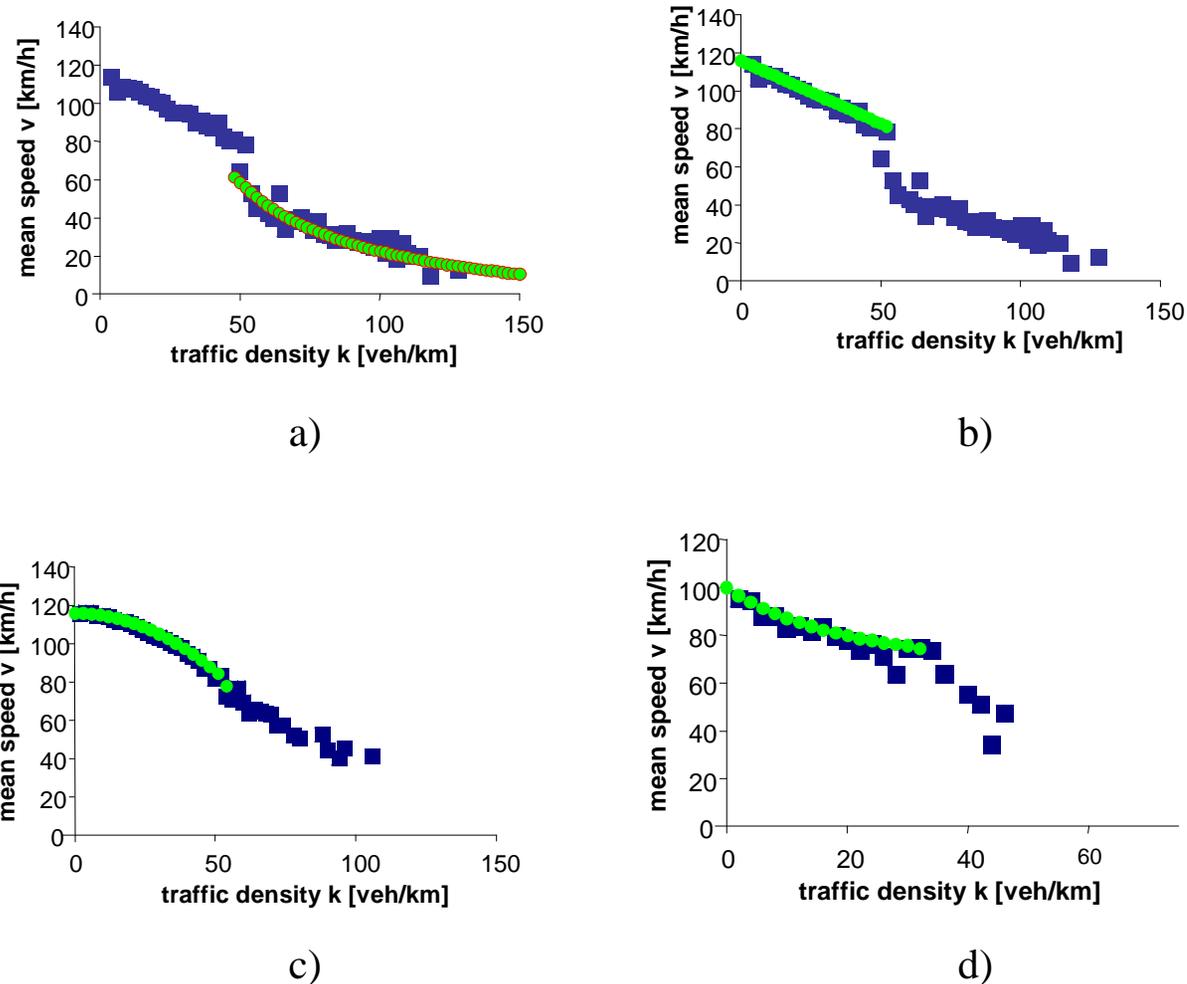


Figure 6 - Shape of the Fundamental Diagram in the k - v relationship, a) in the region of jam traffic (data: German freeway A43), b) in the region of fluid traffic, 2-lane carriageway (data: German freeway A43), c) in the region of fluid traffic, 3-lane carriageway (data: German freeway A8), d) in the region of fluid traffic, 2-lane rural highway (data: 2-lane rural highways in the state Baden-Württemberg)

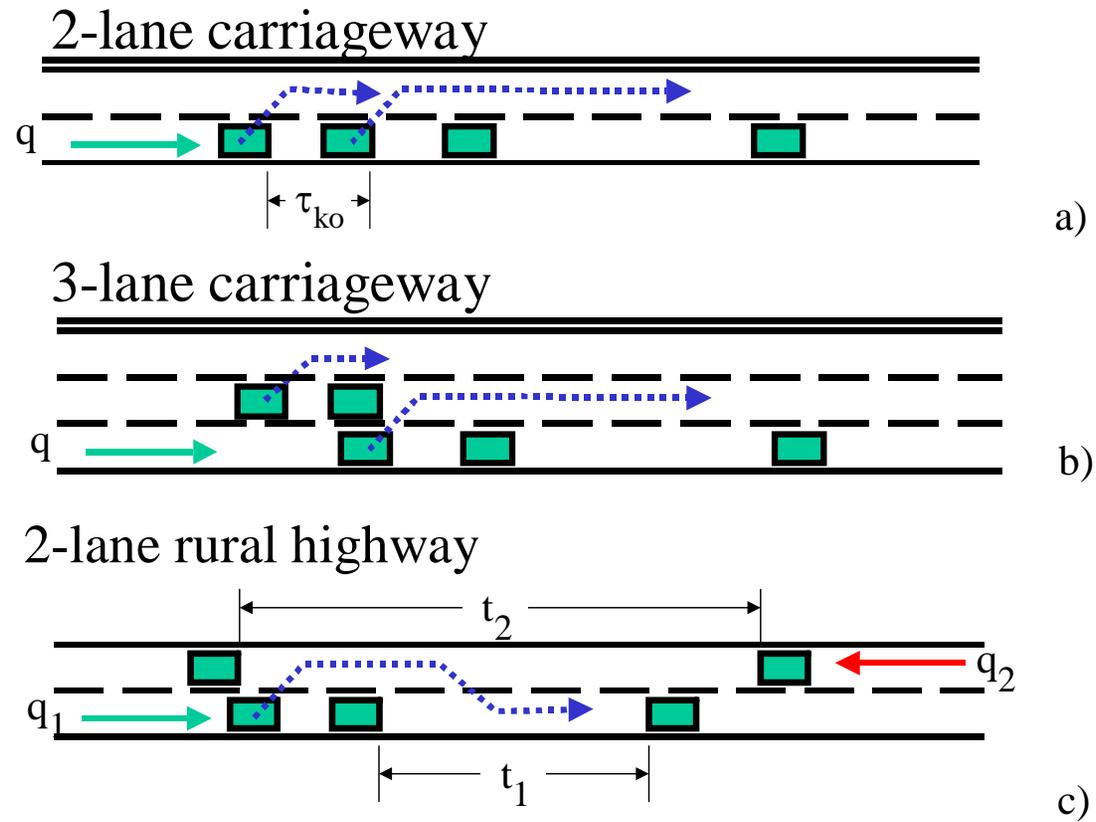


Figure 7 - Overtaking possibilities on different roads

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b) 3-lane carriageway

c) 2-lane 2-way rural highway

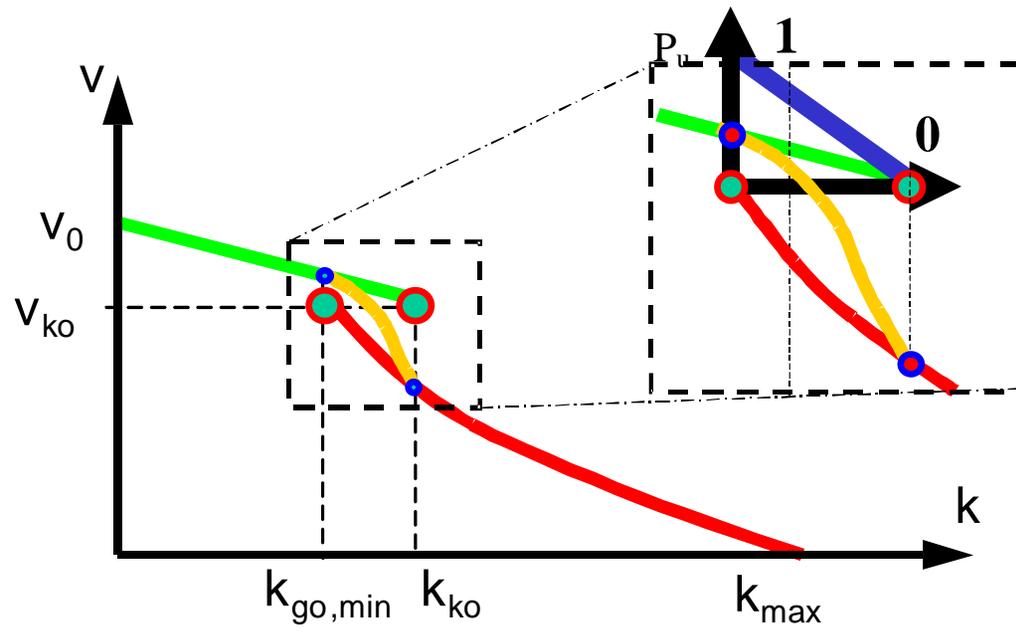


Figure 8 - Overlapping of the fluid and jam traffic

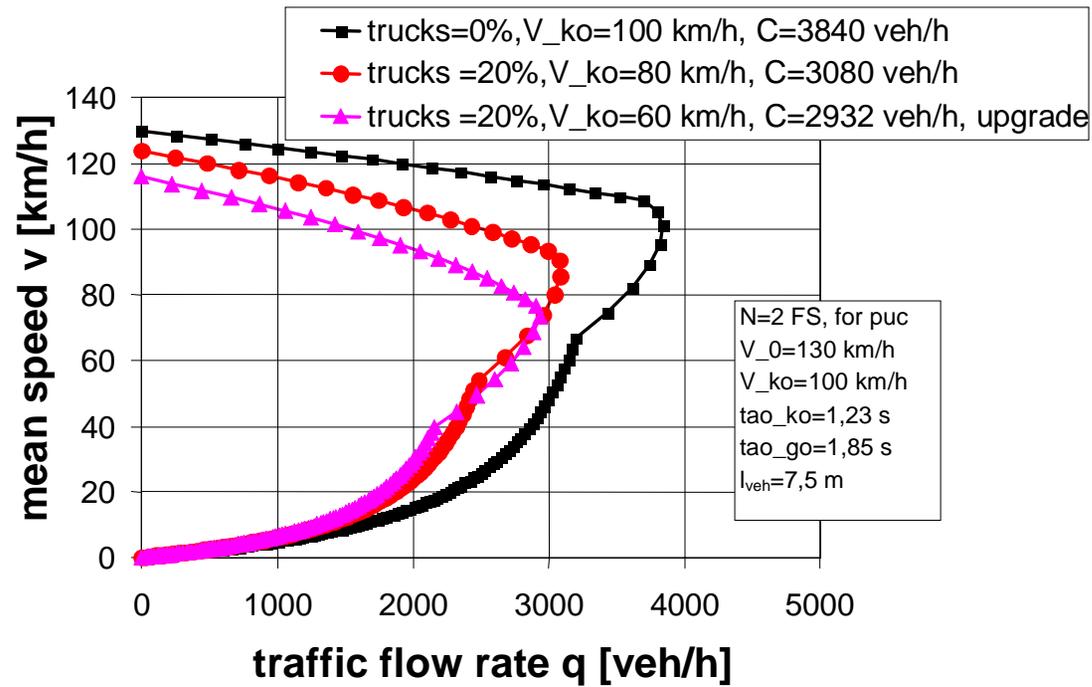


Figure 9 - Variations taking into account the proportions of trucks and the gradients for a 4-lane freeway

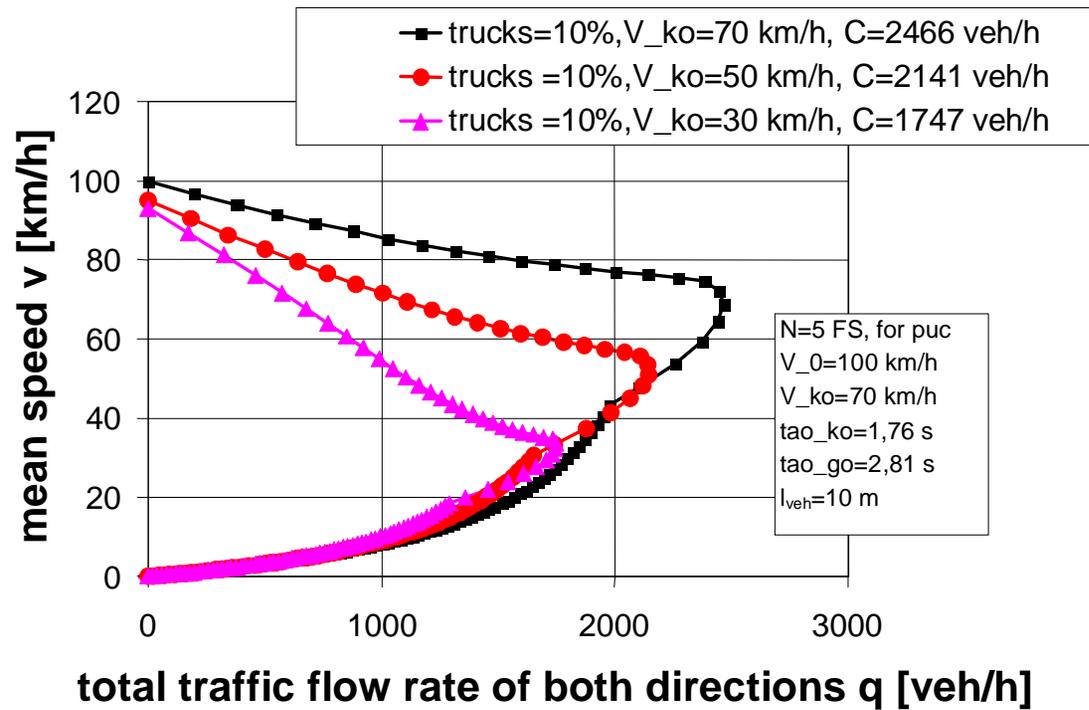


Figure 10 - Variations taking into account the gradients for a 2-lane rural highway

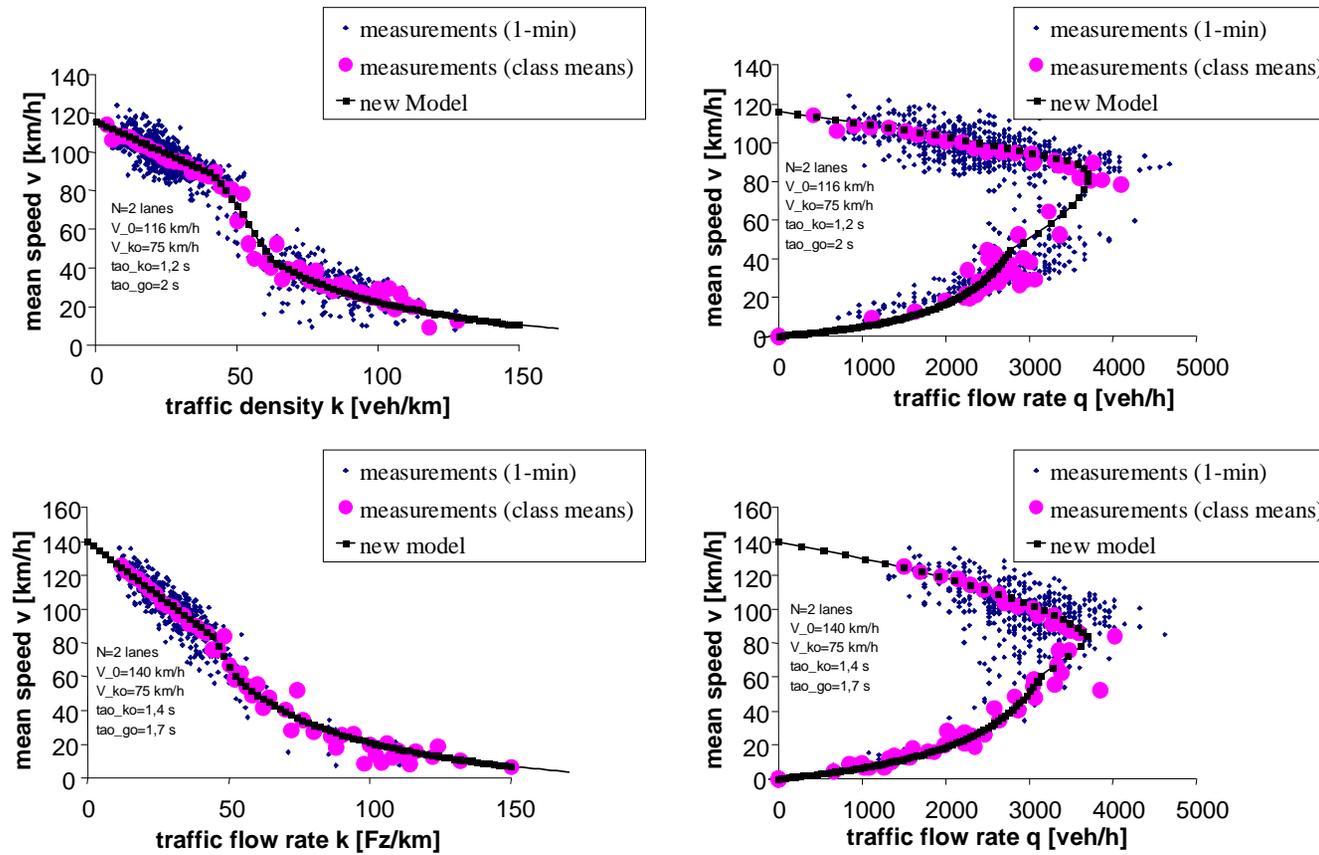


Figure 11 – Comparison of k-v and q-v relationships on 4-lane freeways

top: German freeway A43, speed limit

bottom: German freeway A1, no speed limit

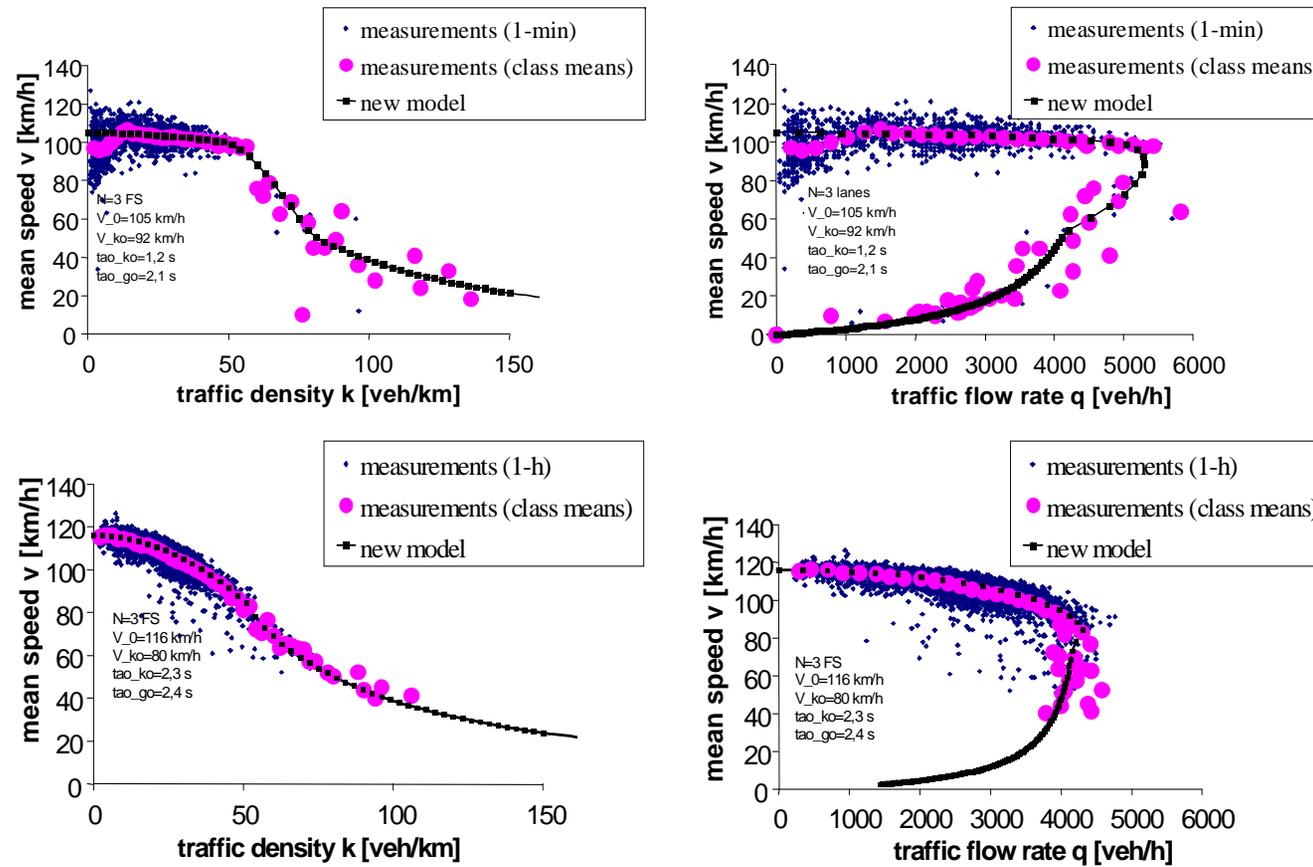


Figure 12 - Comparison of k-v- and q-v-relationships on 6-lane freeways

top: German freeway A1, speed limit

bottom: German freeway A8, no speed limit

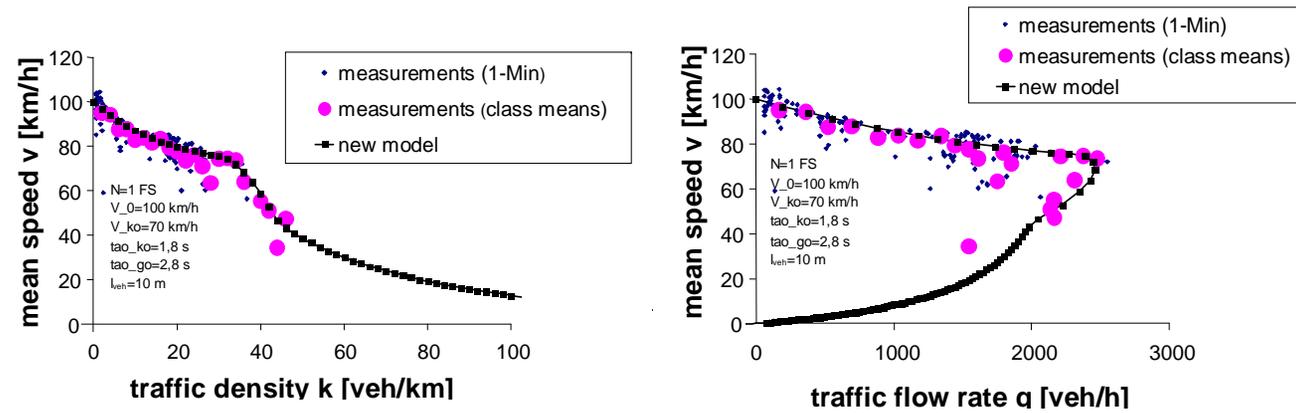


Figure 13 - Comparison of k-v- and q-v-relationships on 2-lane rural highways (rural highways in state Baden-Württemberg)

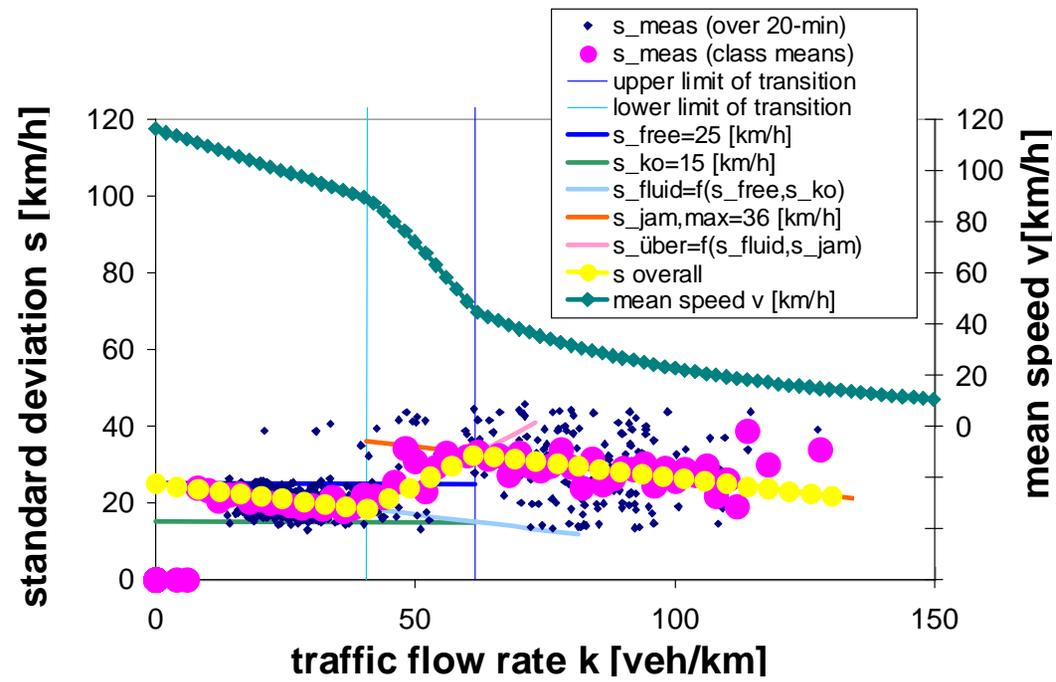


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