

**Determining Capacity of Share Lanes  
with Permitted Left-turn Movements at Signalized Intersections  
- Modeling the Effect of Blockage**

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**Abstract**

At signalized intersections, turning vehicles often use the same share lane together with the through traffic. For example, at small and medium intersections, there is often only a single lane approach which is used both by left-turn and through movement. Obviously, in case of those single-share lanes, only a permitted control for the left-turn movement is applicable. Since a permitted left-turn movement has to give way to the opposing through movement, it has to wait if necessary and thus impedes the through movement in the same direction. In the reality, if the left-turn movement is permitted controlled, the through movement at the same approach can be totally blockaded by waiting turning vehicles during the green time. Thus, the green time for the share lane cannot be efficiently utilised and the lane capacity under consideration cannot be fully received.

In the existing capacity manuals for traffic quality assessment, there are not yet adequate procedures for taking into account that effect of blockage. In the current US Highway Capacity Manual there are only empirical regression functions for the effect of blockage based on measurements and simulation studies. Unfortunately, using this regression functions, some important margin conditions are not satisfied. As a result, the capacity of the share lane is calculated inaccurately under particular conditions.

In this paper, a mathematical model is presented for an exact calculation of the blockage probability caused by permitted turning vehicles and for the estimation of the capacity of single-share lanes at signalized intersections. According to the probability and combinatorial theory, a mathematically exact model is developed.

The proposed model can be applied to share lanes either with left-turn or with right-turn movement. Respectively, by extending the model, also the capacity for the

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Right-Turn-On-Right situation can be exactly calculated. In contrast to the regression functions in the US Highway Capacity Manual, the proposed new model satisfies all necessary boundary conditions.

The results of the model fill out a gap in the current procedures of capacity estimations at signalized intersections. The proposed model has the capability to be incorporated into the existing highway capacity manuals and other regulations. The model in this paper is developed for fixed time controlled and isolated intersections. In the future, an extension to actuated, adapted or coordinated intersections is expected.

**Key words:** Capacity, Signalized intersection, Share lane, Blockage Probability

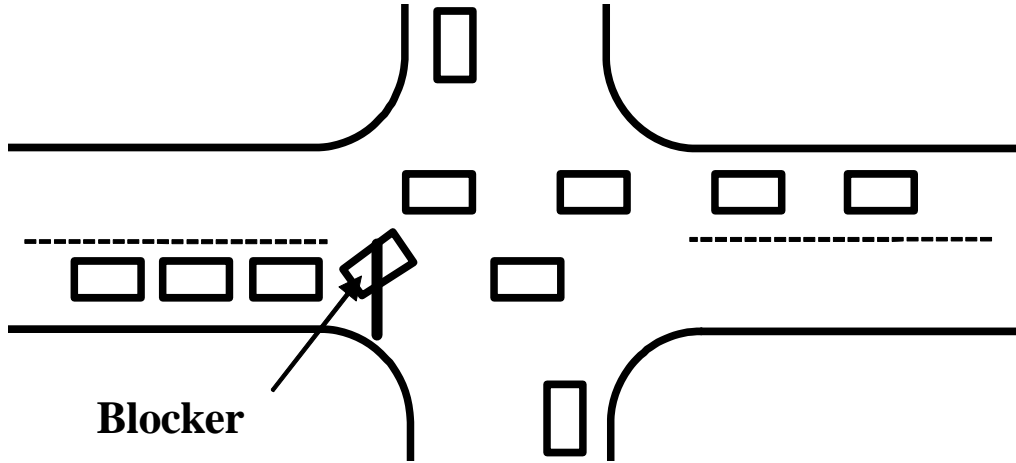
## **Introduction**

At signalised intersections, there are often traffic lanes which are used by different traffic movements. Those traffic lanes are called share lanes. Normally, the traffic movements in a share lane can obey different departure rules. The turning movements have often other departure rules than the through movements due to different traffic rules. For example, the permitted left-turn movements have to give way to the opposing through movements while the through movements can depart with a saturation flow rate at stop-lines without hesitation. Here, in general, we deal with a time interval (e.g. the green time) where in a certain probability the departure of through vehicles is blocked by a waiting permitted turning vehicle because of the give way regulations. The capacity of the share lane is reduced by the blockage. In the existing regulations for traffic quality assessment, the reduction of capacity for such a share lane is not sufficiently taken into account. For example, the US Highway Capacity Manual (HCM, TRB 2000) uses only a simple regression model for the effect of blockage caused by permitted turning movements. There is no accurate theoretical background in that model. That regression model does not satisfy all the necessary boundary conditions. In the Germany Highway Capacity Manual (HBS, FGSV 2001), the capacity reduction in the share lane is not considered at all. The capacity of share lanes with permitted turning movements cannot be calculated in HBS.

In this paper, the capacity reduction within a share lane caused by waiting permitted turning vehicles is qualified through a general mathematical model.

### Probability for Un-blockaded Through Movements within a Share lane

Firstly,  $m$  vehicles within a share lane consisting of two movements  $L$  (left-turn) and  $T$  (Through) are investigated. These  $m$  vehicles can depart in a time interval  $I$ . The proportions of left-turn and through vehicles are  $a_L$  and  $a_T$  respectively. We are looking for the average number  $m_T^*$  for the through vehicles which arrive consecutively before a waiting left-turn vehicle (Blocker) blockades the share lane (see Figure 1).



**Figure 1** – Effect of blockage caused by a permitted left-turn vehicle at signalised intersections with single lane approaches

The probability that exact  $n$  vehicles ( $n < m$ ) in the through movement arrive consecutively before a left-turn vehicle arrives, is according to the probability and combinatory theory:

$$p_n = a_T^n \cdot a_L = a_T^n \cdot (1 - a_T) \quad \text{für } 0 \leq n < m \quad (1)$$

The probability that all  $m$  vehicles are from the through movement is

$$p_m = a_T^m \quad (2)$$

Combining eq. (1) with eq. (2), one obtains the probability that exact  $n \leq m$  vehicles in the through movement arrive consecutively before a left-turn vehicle arrives:

$$p_n = \begin{cases} a_T^n \cdot (1 - a_T) & \text{für } 0 \leq n < m \\ a_T^m & \text{für } n = m \end{cases} \quad (3)$$

The following necessary boundary condition for probability holds:

$$\begin{aligned}
\sum_{n=0}^m p_n &= a_T^m + \sum_{n=0}^{m-1} a_T^n \cdot (1-a_T) = a_T^m + (1-a_T) \sum_{n=0}^{m-1} a_T^n \\
&= a_T^m + (1-a_T) \frac{(1-a_T^m)}{(1-a_T)} = 1
\end{aligned} \tag{4}$$

Therefore, the average number of through vehicles in consecutive order before a left-turn vehicle arrives or the interval  $I$  terminates is

$$m_T^* = \sum_{n=0}^m np_n = (1-a_T) \left( \sum_{n=0}^{m-1} na_T^n \right) + ma_T^m = \left( \frac{1-a_T^{m-1}}{1-a_T} a_T - (m-1)a_T^m \right) + ma_T^m \quad [\text{veh}] \tag{5}$$

Eliminating this equation yields the expectation (mean value) of the number of through vehicles in consecutive order under the given conditions:

$$m_T^* = \frac{a_T(1-a_T^m)}{1-a_T} \quad [\text{veh}] \tag{6}$$

For  $a_T = 0$  is  $m_T^* = 0$  and for  $a_T \Rightarrow 1$  we have  $m_T^* \Rightarrow m$ . For  $m \Rightarrow \infty$  one obtains with  $a_T < 1$  the upper limit:

$$m_{T,m=\infty}^* = \frac{a_T}{1-a_T} = \frac{a_T}{a_L} \quad [\text{veh}] \tag{7}$$

The here derived function for  $m_T^*$  can be very important for different traffic facilities. One of the direct applications is the estimation of capacity for share lanes at signalised intersections with permitted left-turn movements (also for share lanes at signalised intersections with permitted right-turn movements). Assuming in case of protected left-turn movement  $m$  vehicles can pass through the stop-line, the value  $m_T^*$  states for the exact number of through vehicles which can pass the stop-line during the green time (interval under consideration) without being blocked by permitted left-turn vehicles.

Since in the total capacity of the share lane the proportions of left-turn and through vehicles do not change, it is always true:

$$m_T^* = m_{sh}^* \cdot a_T \quad [\text{veh}]$$

Thus, the total capacity of the share lane during the green time (corresponding to the capacity of a cycle) with the green time  $g$  (i.e.,  $I = g$ ) before the share lane is blocked by a left-turn vehicle is

$$m_{sh}^* = \frac{m_T^*}{a_T} = \frac{1 - a_T^m}{1 - a_T} \quad [\text{veh}] \quad (8)$$

Respectively, the capacity of the left-turn movement is

$$m_L^* = m_{sh}^* \cdot a_L = m_T^* \frac{a_L}{a_T} = \frac{1 - a_T^m}{1 - a_T} a_L = 1 - a_T^m \quad [\text{veh}] \quad (9)$$

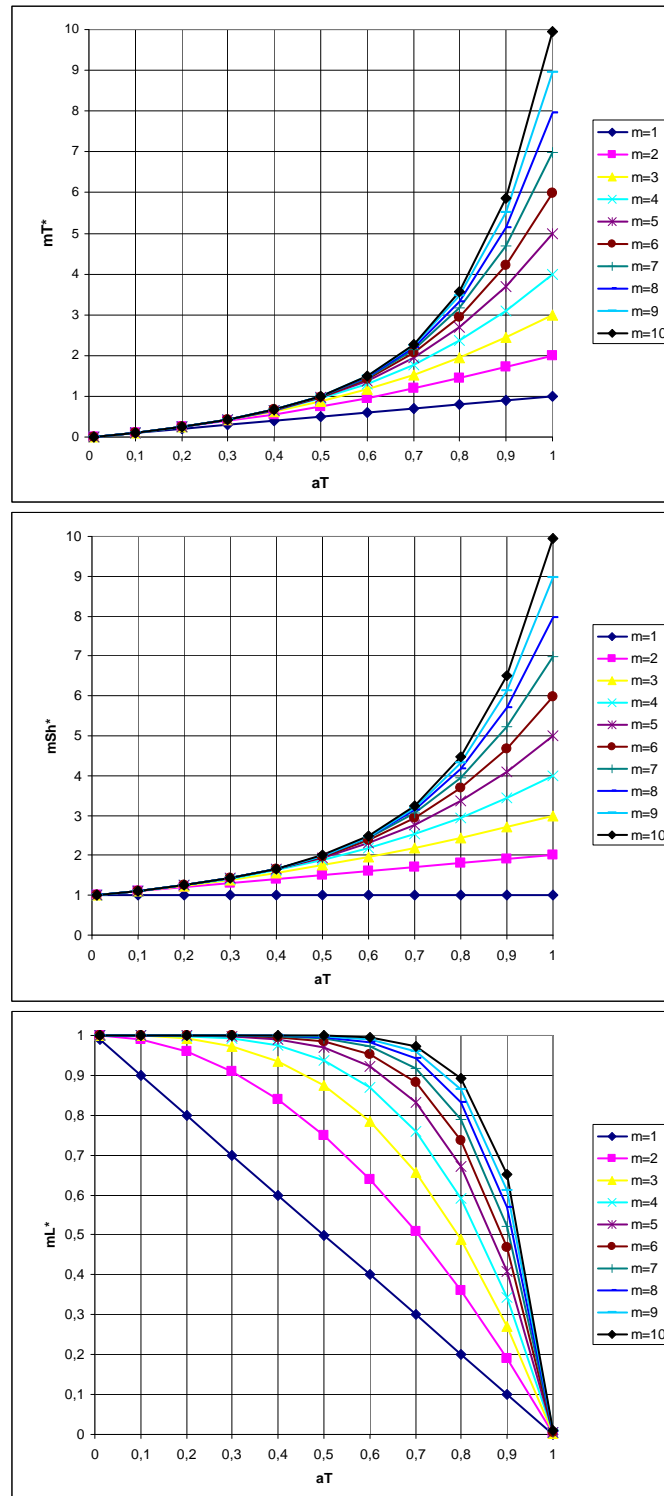
According to eq. (2),  $m$  is the maximum number (capacity) of through vehicles which can pass through the stop-line during the green time  $g$  ( $I = g$ ). This maximum number of through vehicles can be achieved if all vehicles are from the through movement. That is,

$$m = g \cdot s_T \quad [\text{veh}] \quad (10)$$

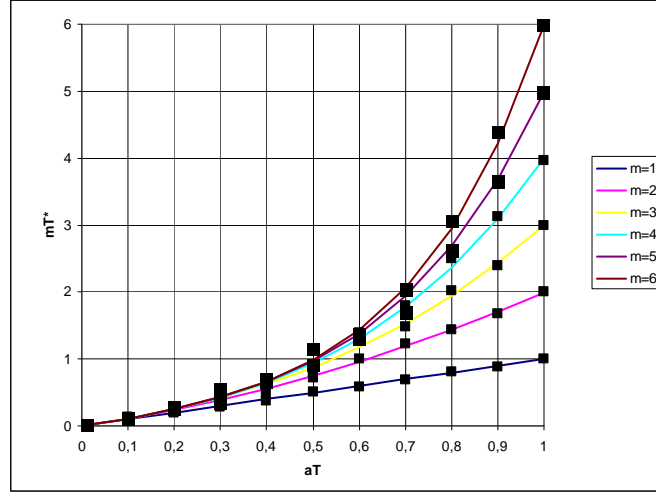
The parameter  $s_T$  is the saturation flow rate in veh/s for through vehicles at the stop-line.

In Figure 2, the capacities  $m_T^*$ ,  $m_{sh}^*$ , and  $m_L^*$  in veh/cycle as functions of the proportion of through vehicles  $a_T$  and the maximum number  $m$  of through vehicles are depicted. It can be seen that the capacities  $m_T^*$ ,  $m_{sh}^*$ , and  $m_L^*$  do not have a linear shape. The eqs. (6), (8), and (9) are derived under the assumption that the blocker in the left-turn movement can only depart at the end of green time. Therefore, the number of departures for the left-turn vehicles is between 0 and 1 per cycle.

In order to check the equation for the capacity  $m_T^*$ , a simplified simulation study is conducted. In Figure 3, the results of the simulation are depicted together with the values of the theoretical calculations. It can be seen that the derived model is confirmed by the simulation study.



**Figure 2** – Capacities  $m_T^*$ ,  $m_{sh}^*$ , and  $m_L^*$  as functions of the proportion of the through vehicles  $a_T$  and the maximum number  $m$  of through vehicles



**Figure 3** – Number of un-blockaded through vehicles  $m_T^*$  as a function of  $a_T$  and  $m$ .  
Lines = Model (eq. (6)); Points = Simulation

The proposed derivation can also be applied for the calculation of Right-Turn-On-Red (RTOR) regulation. The possible capacity for the right-turn vehicles during the red time  $r$  is then:

$$m_{RTOR}^* = \frac{a_R(1 - a_R^{m_r})}{1 - a_R} \quad [\text{veh}] \quad (11)$$

with

$$m_r = r \cdot s_R, \text{ veh}$$

$r$  = red time for the share lane ( $I = r$ ), s

$s_R$  = saturation flow rate headway for the right-turn vehicles at the stop-line, veh/s

In this case, any arriving through vehicle blockades the departure of right-turn vehicles during the red time.

### Total Capacity of Share Lanes with Filtered Left-turn Vehicles

In the real world, once the opposing queue clears, subject left-turning vehicles can filter through an unsaturated opposing flow at a rate by magnitude of the opposing flow. This capacity resulted from filtering for the left-turn movement is defined here as  $n_{L,filter}$ . The values of  $n_{L,filter}$  can be calculated by common procedures (e.g. according to HBS, FGSV 2001; HCM, TRB 2000).

The additional capacity  $m_{filter}$  per cycle for the share lane resulting from the filtering can approximately be calculated according to the so-called share-lane formula from Harders (1968):

$$m_{\text{filter}} = \frac{1}{\frac{a_T}{m_T} + \frac{a_L}{n_{L,\text{filter}}}} = \frac{1}{g \cdot s_T + \frac{a_L}{n_{L,\text{filter}}}} = \frac{1}{g \cdot s_T + \frac{a_L}{n_{L,\text{filter}}}} \quad [\text{veh}] \quad (12)$$

The total capacity of the share lane per cycle is therefore (cf. eq. (8)):

$$m_{sh,\text{total}}^* = m_{sh}^* + m_{\text{filter}} = \frac{1 - a_T^m}{1 - a_T} + \frac{1}{\frac{a_T}{g \cdot s_T} + \frac{a_L}{n_{L,\text{filter}}}} \quad [\text{veh}] \quad (13)$$

In addition, the capacity at the stop-line is limited to the boundary condition:

$$m_{sh,\text{total}}^* \leq \frac{1}{\frac{a_T}{g \cdot s_T} + \frac{a_L}{g \cdot s_L}} \quad [\text{veh}] \quad (14)$$

That is,

$$m_{sh,\text{total}}^* = \min \left( \frac{1}{\frac{a_T}{g \cdot s_T} + \frac{a_L}{g \cdot s_L}}, \frac{1 - a_T^m}{1 - a_T} + \frac{1}{\frac{a_T}{g \cdot s_T} + \frac{a_L}{n_{L,\text{filter}}}} \right) \quad [\text{veh}] \quad (15)$$

$$m_T^* = m_{sh,\text{total}}^* \cdot a_T \quad [\text{veh}] \quad (16)$$

$$m_L^* = m_{sh,\text{total}}^* \cdot a_L \quad [\text{veh}] \quad (17)$$

with

$$m = \max(0, g \cdot s_T - m_{\text{filter}}) = \max \left( 0, g \cdot s_T - \frac{1}{\frac{a_T}{g \cdot s_T} + \frac{a_L}{n_{L,\text{filter}}}} \right), \text{ veh}$$

$n_{L,\text{filter}}$  = capacity of the permitted left-turn movement during the green time by filtering, veh

For  $g \cdot s_L \leq n_{L,\text{filter}}$  the through vehicles can not be blockaded by left-turn vehicles. In this case is  $m = 0$ ,  $m_{sh}^* = 0$  and  $m_{sh,\text{total}}^* = m_{\text{filter}}$ .

Respectively, the approximation formula for share lanes with permitted right-turn movements is



$$m_{sh,total}^* = \min \left( \frac{1}{\frac{a_T}{g \cdot s_T} + \frac{a_R}{g \cdot s_R}}, \frac{1 - a_T^m}{1 - a_T} + \frac{1}{\frac{a_T}{g \cdot s_T} + \frac{a_R}{n_{R,filter}}} \right) \quad [\text{veh}] \quad (18)$$

$$m_T^* = m_{sh,total}^* \cdot a_T \quad [\text{veh}] \quad (19)$$

$$m_R^* = m_{sh,total}^* \cdot a_R \quad [\text{veh}] \quad (20)$$

$$m = \max \left( 0, g \cdot s_T + m_{RTOR}^* - \frac{1}{\frac{a_T}{g \cdot s_T} + \frac{a_R}{n_{R,filter}}} \right) \quad [\text{veh}] \quad (21)$$

$n_{R,filter}$  = capacity of the permitted right-turn movement during the green time by filtering, veh

$m_{RTOR}^*$  = capacity of the permitted right-turn movement with RTOR regulation during the red time (cf. eq. (11)), veh

### Comparison with the Regression Formula in HCM

In appendix C of HCM, the Left-Turn Adjustment Factor for permitted phasing is considered explicitly. There are two formulas for accounting the un-blocked green time for approaches with shared permitted left-turn lanes, one for multilane approaches with opposing multilane approaches (eq. C16-5 in HCM) and one for single lane approaches opposed by single lane approaches (eq. C16-10 in HCM). Both equations are derived by regression analysis. In HCM, the portion of effective green time until the arrival of the first left-turn vehicle is designated  $g_f$  by the following formula:

$$g_f = g \cdot e^{-a \cdot LTC^b} - t_L \quad [\text{s}] \quad (22)$$

where

$g$  = actual green time for the permitted phase, s

$LTC$  = left turns per cycle

$t_L$  = lost time for subject left-turn lane group, s

$a, b$  = model parameters, for multilane approaches  $a = 0.822$  and  $b = 0.717$  (eq. C16-5 in HCM), for single lane approaches  $a = 0.860$  and  $b = 0.629$  (eq. C16-10 in HCM).

Because the eq. C16-10 in HCM (for single lane approaches opposed by single lane approaches) considers similar preconditions as the proposed model in this paper, the formula with  $a = 0.860$  and  $b = 0.629$  is used for further comparison.

Compared to the new model, the parameter  $LTC$  can be calculated from maximum number of through vehicles  $m$  in combination with the proportion of left-turn vehicles  $a_L$ . Setting  $LTC = a_L \cdot m = (1 - a_T) \cdot m$  in eq. (22) yields

$$g_f = g \cdot e^{-a \cdot [(1-a_T) \cdot m]^b} - t_L = g_f = g \cdot a_T^* - t_L \quad [\text{s}] \quad (23)$$

with

$$a_T^* = e^{-a \cdot [(1-a_T) \cdot m]^b} \quad (24)$$

The term  $a_T^* = e^{-a \cdot [(1-a_T) \cdot m]^b}$  represents the proportion of green time in which the through vehicles are not blocked by the first left-turn vehicle. According to the definition in the new model this proportion can be expressed by

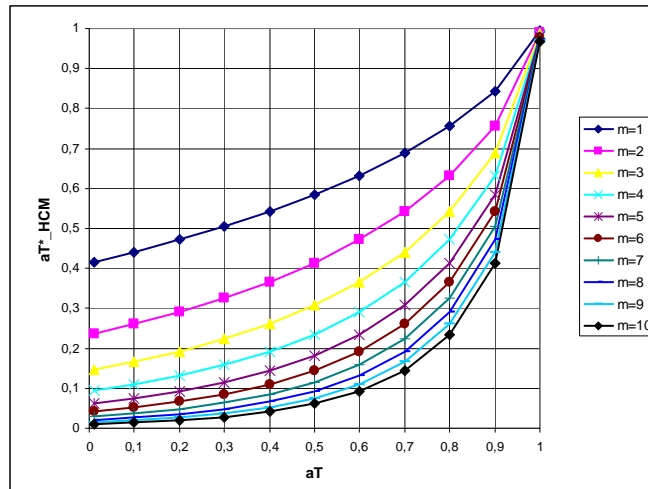
$$a_T^* = \frac{m_T^*}{m} = \frac{a_T(1 - a_T^m)}{m(1 - a_T)} \quad (25)$$

The proportion  $a_T^*$  of green time, in which the through vehicles are not blocked by the first left-turn vehicle, must satisfies the boundary conditions:

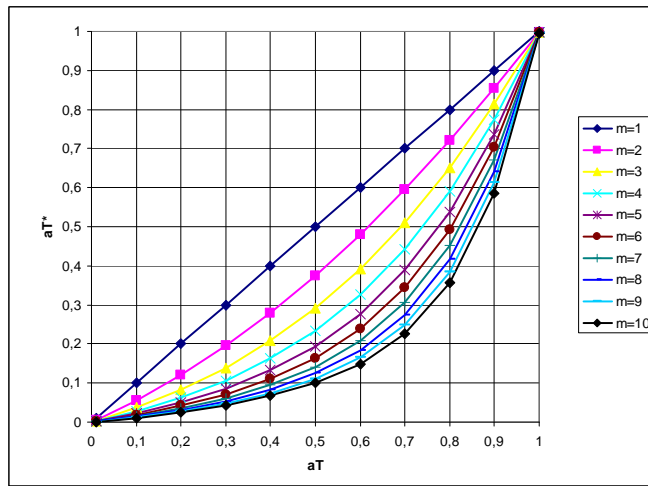
$$\begin{aligned} a_T^* &= 1 \quad \text{for } a_T = 1 \\ a_T^* &= 0 \quad \text{for } a_T = 0 \end{aligned} \quad (26)$$

Eq. (24) does not satisfy all the necessary boundary conditions. For  $a_T = 1$  we have  $a_T^* = e^{-a \cdot 0^b} = 1$  and this is correct. For  $a_T = 0$  is  $a_T^* = e^{-a \cdot m^b} \neq 0$ . The boundary condition by  $a_T = 0$  is not fulfilled.

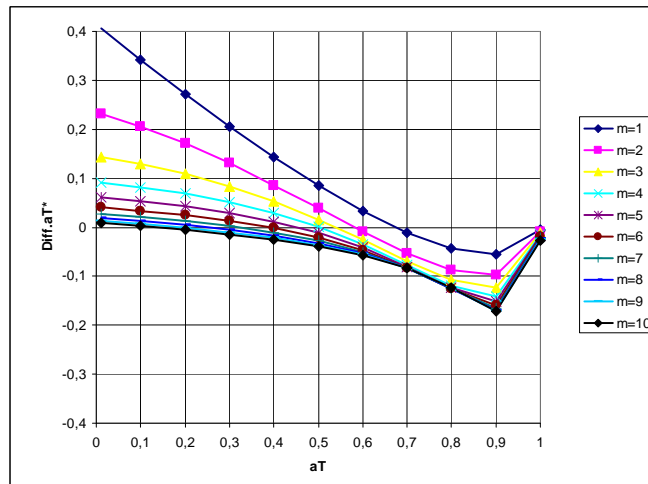
In Figure 4 the proportions  $a_T^*$  of green time, in which the through vehicles are not blocked by the first left-turn vehicle, both from the HCM formula (eq. (24)) and from the new model (eq. (25)) as well the differences, are presented. The proportions  $a_T^*$  are depicted as functions of the proportion of through vehicles  $a_T$ , and the maximum number of through vehicles  $m$ . It can be seen that the difference are not negligible. For  $a_T < 0.5$  the HCM formula gives totally wrong (too high) values. For  $a_T > 0.5$ , which corresponds to realistic traffic conditions, the HCM formula has similar but not the same values as the new model. It can also be seen that the deviations increase with increasing maximum number of through vehicles  $m$ . The differences are positive (overestimation of capacities) for  $a_T < 0.5$  and negative (underestimation of capacities) for  $a_T > 0.5$ .



a)



b)



c)

**Figure 4** – The proportions  $a_T^*$  of green time, in which the through vehicles are not blocked by the first left-turn vehicle: a) from the HCM formula ((24)), b) from the new model ((25)), and c) differences between a) and b)

Using the new theoretical model, the portion of effective green time until the arrival of the first left-turn vehicle can be rewritten as:

$$\begin{aligned}
 g_f &= g \cdot \frac{m_T^*}{m} - t_L \\
 &= g \cdot \frac{a_T(1 - a_T^m)}{m(1 - a_T)} - t_L
 \end{aligned}
 \tag{27} \quad [\text{s}]$$

where

- $g$  = actual green time for the permitted phase, s
- $m$  = maximum number of through vehicles per cycle, veh  
=  $g \cdot s$
- $s$  = saturation flow rate, veh/s
- $a_T$  = proportion of through vehicles
- $t_L$  = lost time for subject left-turn lane group, s

This equation can be easily incorporated in to HCM in place of eq. (22).

## Summary and Conclusions

In this paper, the influence of permitted turning vehicles on the total capacity of share lanes at signalised intersections is quantified through a mathematical model. With this model, the probability that the share lane is blockaded through a permitted turning vehicle can be exact calculated. Based on this probability, the average capacity of the share lane can be estimated. The proposed model can be used for share lanes with either permitted left-turn or permitted right-turn movements. Also the Right-Turn-On-Red regulation can be calculated by the proposed model.

The derivation of the model is based of the assumption that the permitted turning vehicles can clear the intersection after the green time. This assumption is necessary because the model is only valid for the case that at end of green time the blockage is cleared and the arrivals of the through and turning vehicles in the new interval under consideration are random. This assumption is not critical since in the reality traffic regulations allow the permitted waiting vehicles to clear the intersection immediately after the green time.

The major findings of the paper are the derivations of eqs. (3) and (6). According to those equations, the number of un-blockaded through vehicles in the share lane and therefore the total capacity of the shared lane before a blockage can be calculated exactly. Those equations can be easily incorporated into the existing highway capacity manuals. In contrast to the regression functions in HCM, the proposed functions satisfied all necessary margin conditions.

The model in this paper is developed for fixed time controlled and isolated intersections. In the future, an extension to actuated, adapted or coordinated intersections is expected.

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