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**DERIVATION OF THE VAN AERDE TRAFFIC STREAM MODEL FROM
TANDEM-QUEUEING THEORY**

Ning Wu, Ph.D.

Institute for Traffic Engineering, Ruhr-University Bochum, 44780 Bochum, Germany
phone: +49 234 3226557, fax: +49 234 3214151
e-mail: ning.wu@rub.de, <http://homepage.rub.de/ning.wu>

Hesham Rakha, Ph.D., P.Eng.

Charles Via Jr. Department of Civil and Environmental Engineering
3500 Transportation Research Plaza, Blacksburg, VA 24061
Phone: (540) 231-1505 - Fax: (540) 231-1555
e-mail: hrakha@vt.edu, <http://filebox.vt.edu/users/hrakha>

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Ning Wu, Ph.D.

Institute for Traffic Engineering, Ruhr-University Bochum, 44780 Bochum, Germany

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e-mail: ning.wu@rub.de, <http://homepage.rub.de/ning.wu>

Hesham Rakha, Ph.D., P.Eng.

Charles Via, Jr. Department of Civil and Environmental Engineering

3500 Transportation Research Plaza, Blacksburg, VA 24061

Phone: (540) 231-1505 - Fax: (540) 231-1555

e-mail: hrakha@vt.edu, <http://filebox.vt.edu/users/hrakha>

ABSTRACT

Modeling of traffic stream behavior requires establishing some relationship between the traffic stream flow, speed, and density (also known as the Fundamental Diagram). A reasonable model can estimate the maximum flow rate of motorways and the corresponding traffic quality more accurately. Using different theories such as car-following theory, queuing theory, or theory of fluid dynamics, different models for the Fundamental Diagram can be established. Based on queuing theory, several models can be found in the literature, for example, the model from Brilon and Ponzlet [1] and the model from Heidemann [2]. The existing queuing models consider a roadway cross section as a stand-alone queuing counter and thus the interaction between consecutive vehicles cannot be considered sufficiently. This paper presents a new model treating a road stretch as a series of many queuing counters. The new model yields a single-regime Fundamental Diagram that fully corresponds to the well-known Van Aerde model [3, 4]. In the Van Aerde model, the model parameters can be easily measured and calibrated using loop detector data. The application and calibration of the model is demonstrated using sample of datasets and compared to other models. The results demonstrate the superiority of this model in capturing traffic stream behavior.

Keyword: Fundamental Diagram, Capacity, Queuing theory, Tandem queue

INTRODUCTION

The rapid development of personal computers over the last few decades has provided the necessary computing power for advanced traffic micro-simulators. Today, microscopic traffic simulation software are widely accepted and applied in all branches of transportation engineering as an efficient and cost effective analysis tool. One of the main reasons for this popularity is the ability of microscopic traffic simulation software to reflect the dynamic nature of the transportation system in a stochastic fashion.

The core of microscopic traffic simulation software is a car-following model that characterizes the longitudinal motion of vehicles. The process of car-following consists of two levels, namely modeling steady-state and non-steady-state behavior [5]. Ozaki defined steady state as conditions in which the vehicle acceleration and deceleration rate is within a range of $\pm 0.05g$ [6]. Another definition of steady-state or stationary conditions is provided by Rakha [7] as the conditions when traffic states remain practically constant over a short time and distance. Steady-state car-following is extremely critical to traffic stream modeling given that it influences the overall behavior of the traffic stream. Specifically, it determines the desirable speed of vehicles at different levels of congestion, the roadway capacity, and the spatial extent of queues. Alternatively, non-steady-state conditions govern the behavior of vehicles while moving from one steady state to another through the use of acceleration and deceleration models. The acceleration model is typically a function of the vehicle dynamics while the deceleration model ensures that vehicles maintain a safe relative distance to the preceding vehicle thus ensuring that the traffic stream is asymptotically stable. Both acceleration and deceleration models can affect steady-state conditions by reducing queue discharge saturation flow rates.

Traffic stream models describe the motion of a traffic stream by approximating for the flow of a continuous compressible fluid. The traffic stream models relate three traffic stream measures, namely: flow rate (q), density (k), and space-mean-speed (u). Gazis et al. [8] were the first to derive the bridge between microscopic car-following and macroscopic traffic stream models. Specifically, the flow rate can be expressed as the inverse of the average vehicle time headway. Similarly, the traffic stream density can be approximated for the inverse of the average vehicle spacing for all vehicles within a section of roadway. Therefore every car-following model can be represented by its resulting steady-state traffic stream model. Different graphs relating each pair of the above parameters can be used to show the steady-state properties of a particular model; including the speed-spacing ($u-s$) and speed-flow-density ($u-q-k$) relationships. The latter curve is of more interest, since it is more sensitive to the calibration process and the shape and nose position of the curve determines the behavior of the resulting traffic stream.

A reliable use of micro-simulation software requires a rigorous calibration effort. Because traffic simulation software are commonly used to estimate macroscopic traffic stream measures, such as average travel time, roadway capacity, and average speed, the state-of-the-practice is to systematically alter the model input parameters to achieve a reasonable match between desired macroscopic model output and field data [9].

This paper first derives the Van Aerde traffic stream model using tandem queuing theory. Subsequently, the paper presents methods for calibrating the model and compares the model to other state-of-the-art traffic stream models. Finally, the conclusions of the paper are presented.

MODEL DESCRIPTION

First, a single cross section (Figure 1, cross section A) of a road stretch is considered. When no vehicles are within the distance $L_0 = \tau V_0/3.6$ (length of the interaction zone A to B) downstream of the subject vehicle, a vehicle just behind the cross section A can travel the interaction zone without being impeded by any vehicles ahead of it. The minimum intra-vehicle headway τ corresponds to the reciprocal of the maximum potential capacity C_0 of a stand-alone cross section under free-flow conditions. The potential capacity C_0 for free-flow conditions cannot be obtained in the real-world because the traffic flow will break into congestion before this capacity is reached. However, the potential capacity C_0 can be estimated by the so-called Product-Limit-Method (PLM) [10] which is based on the theory of probability and statistics. Using the PLM, the distribution function of the potential capacity at a cross section under consideration can be estimated. C_0 is then the mean value of the distributed potential capacity.

Approximating the queuing system under consideration as a generalized $G/G/1$ queuing system, the travel time through the interaction can be computed as

$$\Delta t = t_0 + t_w = 3.6 \frac{L_0}{V_0} + t_w = \tau + t_w. \quad (1)$$

Where $t_w = \frac{x}{C_0(1-x)} k_{st}$ (queuing delay) (s);

$x = \frac{q}{C_0}$ = the degree of saturation of the queuing system,

q = flow rate in veh/h,

C_0 = capacity of the queuing system (estimated by the Product-Limit-Method or as the reciprocal of minimum intra-vehicle headway τ within bunched vehicles) in veh/h,

k_{st} = factor that accounts for geometric roadway restrictions and the stochastic nature of traffic,

τ = minimum intra-vehicle headway (s) = time threshold of interaction between two vehicles = $3600/C_0$

V_0 = free-flow speed (km/h), and

L_0 = length of interaction zone (m).

Under real-world traffic conditions, the parameter k_{st} is calibrated using measured field data. Normally, the value of k_{st} is smaller than 0.5 and can be assumed to be constant.

Thus, the actual speed V can be expressed as a function of the actual flow rate (q) as

$$V(q) = \frac{L_0}{\Delta t} = \frac{\tau V_0}{\tau + t_w} = \frac{V_0}{1 + \frac{t_w}{\tau}} = \frac{V_0}{1 + \frac{\frac{x}{C_0(1-x)} k_{st}}{\tau}} = \frac{V_0}{1 + \frac{q}{C_0 - q} k_{st}} \quad (2)$$

If the traffic stream becomes denser, one or more vehicles may be present in the interaction zone L_0 (A to B). In this case, a simple $G/G/1$ queuing system is no longer suitable for the modeling of the traffic stream motion.

Now, a more realistic configuration is considered (Figure 2). In this case, each vehicle is considered to pass its own counter. We have a Tandem-queue system that consists of at least two counters.

Obviously, the Tandem-queue system may have a lower capacity in comparison to a single-queue system because impedance occurs in the area L_0 . The capacity C_0^* of the Tandem-queue system can be estimated by applying a factor k_C to the capacity C_0 of the single cross section.

Substituting C_0 by C_0^* in equation (2) yields the flow-speed relationship for the Tandem-queue system as

$$V(q) = \frac{V_0}{1 + \frac{q}{C_0^* - q} k_{st}} = \frac{V_0}{1 + \frac{q}{C_0 k_C - q} k_{st}}, \quad (3)$$

where $k_C = f(\text{density } d)$. When $d = d_{\max}$ then $k_C = 0$. In this case, all vehicles are standing and the number of queuing counters under consideration is infinite. For $d = 0$ then $k_C = 1$. In this case, the single counter situation applies. The function $k_C = f(d)$ may be obtained using complicated mathematic formulations. In general, it is a monotonically descending function. Under real-world conditions, this function can be assumed using an approximately linear function (cf. also [11]). That is,

$$k_C = 1 - \frac{d}{d_{\max}} = \frac{d_{\max} - d}{d_{\max}} = \frac{d_{\max} - \frac{q}{V}}{d_{\max}}$$

Thus, equation (3) can be written as

$$V(q) = \frac{V_0}{1 + \frac{q}{C_0 \frac{d_{\max} - \frac{q}{V}}{d_{\max}} - q} k_{st}} = \frac{V_0}{1 + \frac{1}{C_0 \frac{d_{\max} - \frac{1}{V}}{d_{\max}} - 1} k_{st}} \quad (4)$$

By re-arranging the variables in Equation (4) and solving for the flow rate q the following speed-flow relationship is derived

$$\begin{aligned} q(V) &= \frac{d_{\max}}{\left(\frac{k_{st}}{\frac{V_0}{V} - 1} + 1 \right) \frac{d_{\max}}{C_0} + \frac{1}{V}} \\ &= \frac{1}{\frac{k_{st} V}{V_0 - V} \frac{1}{C_0} + \frac{1}{C_0} + \frac{1}{V} \frac{1}{d_{\max}}} \end{aligned} \quad (5)$$

The speed-density relationship can be obtained by substituting d for q/V as

$$d(V) = \frac{q}{V} = \frac{1}{\frac{k_{st} V^2}{V_0 - V} \frac{1}{C_0} + \frac{V}{C_0} + \frac{1}{d_{\max}}} . \quad (6)$$

This function can also be used as a car-following model using the distance-speed relationship as

$$\begin{aligned} \Delta X &= \frac{1}{d} = \frac{k_{st} V^2}{V_0 - V} \frac{1}{C_0} + \frac{V}{C_0} + \frac{1}{d_{\max}} \\ &= \frac{\frac{k_{st}}{C_0} V^2 + \frac{1}{C_0} V(V_0 - V) + \frac{1}{d_{\max}} (V_0 - V)}{V_0 - V} \\ &= \frac{-\frac{1 - k_{st}}{C_0} V^2 + (\frac{V_0}{C_0} - \frac{1}{d_{\max}}) V + \frac{V_0}{d_{\max}}}{V_0 - V} \end{aligned} \quad (7)$$

This function is identical to the Van Aerde model [3, 4] which was established earlier heuristically. The model by Van Aerde is expressed as

$$\begin{aligned} \Delta X &= \frac{c_2}{V_0 - V} + c_3 V + c_1 \\ &= \frac{c_2}{V_0 - V} + \frac{c_3 V(V_0 - V)}{V_0 - V} + \frac{c_1(V_0 - V)}{V_0 - V} . \\ &= \frac{c_2 + c_1 V_0 + (c_3 V_0 - c_1) V - c_3 V^2}{V_0 - V} \end{aligned} \quad (8)$$

Solving Equations (7) and (8) simultaneously yields

$$\begin{aligned} c_3 &= \frac{1 - k_{st}}{C_0} \\ c_3 V_0 - c_1 &= \frac{V_0}{C_0} - \frac{1}{d_{\max}} \\ c_2 + c_1 V_0 &= \frac{V_0}{d_{\max}} \end{aligned}$$

Solving for the constants c_1 , c_2 , and c_3 we derive

$$\begin{aligned} c_1 &= \frac{1}{d_{\max}} - \frac{k_{st} V_0}{C_0} \\ c_2 &= \frac{k_{st} V_0^2}{C_0} \\ c_3 &= \frac{1 - k_{st}}{C_0} \end{aligned} \quad (9)$$

Equations (7) and (8) can be rewritten as

$$\Delta X = \frac{1}{V_0 - V} \frac{k_{st} V_0^2}{C_0} + \frac{(1 - k_{st})}{C_0} V + \frac{1}{d_{\max}} - \frac{k_{st} V_0}{C_0} \quad (10)$$

Thus, the Van Aerde [3, 4, 7] heuristic model is hereby derived theoretically considering a tandem queue and thus demonstrating the theoretical basis for the model. The parameters c_1 , c_2 , and c_3 can be related to macroscopic traffic stream parameters that can be directly estimated and calibrated in the field, as demonstrated in Equation (9). Other publications have also demonstrated how these parameters can be calibrated using field traffic stream data [4, 12]. In Figure 3, the properties of the various model parameters V_0 , C_0 , d_{\max} , and k_{st} are illustrated. It can be seen, that the shape of the density-flow relationship is defined by these four parameters.

DISCUSSION

By computing the derivative of Equation (5) and setting it equal to zero, the speed-at-capacity can be computed as

$$q'(V) = \frac{-d_{\max} \left[\left(\frac{k_{st} \frac{V_0}{V^2}}{\left(\frac{V_0}{V} - 1 \right)^2} \right) \frac{d_{\max}}{C_0} - \frac{1}{V^2} \right]}{\left[\left(\frac{k_{st}}{\frac{V_0}{V} - 1} + 1 \right) \frac{d_{\max}}{C_0} + \frac{1}{V} \right]^2} = 0.$$

This equation yields

$$\left(\frac{k_{st} \frac{V_0}{V^2}}{\left(\frac{V_0}{V} - 1 \right)^2} \right) \frac{d_{\max}}{C_0} - \frac{1}{V^2} = 0, \text{ or } \left(\frac{k_{st} V_0}{(V_0 - V)^2} \right) \frac{d_{\max}}{C_0} - \frac{1}{V^2} = 0. \quad (11)$$

This is the condition under which the maximum flow rate q_{\max} is reached. If the optimum speed V_{opt} corresponding to the maximum flow rate q_{\max} is pre-defined, the k_{st} parameter can be computed as

$$k_{st} = \frac{C_0}{d_{\max} V_{\text{opt}}^2} \frac{(V_0 - V_{\text{opt}})^2}{V_0} \quad (12)$$

Otherwise, Equation (12) yields

$$V_{\text{opt}} = \frac{-2V_0 + \sqrt{4V_0^2 + 4\left(\frac{d_{\max} V_0 k_{st}}{C_0} - 1\right)V_0^2}}{2\left(\frac{d_{\max} V_0 k_{st}}{C_0} - 1\right)} = \frac{V_0 \left(\sqrt{1 + \left(\frac{d_{\max} V_0 k_{st}}{C_0} - 1\right)} - 1 \right)}{\left(\frac{d_{\max} V_0 k_{st}}{C_0} - 1\right)} \quad (13)$$

Correspondingly, the maximum flow q_{\max} can be computed as

$$\begin{aligned} q_{\max} &= \frac{d_{\max}}{\left[\frac{C_0}{d_{\max} V_{\text{opt}}} \frac{(V_0 - V_{\text{opt}})}{V_0} + 1 \right] \frac{d_{\max}}{C_0} + \frac{1}{V_{\text{opt}}}} \\ &= \frac{d_{\max}}{\frac{(V_0 - V_{\text{opt}})}{V_{\text{opt}} V_0} + \frac{d_{\max}}{C_0} + \frac{1}{V_{\text{opt}}}} = \frac{1}{\frac{2}{V_{\text{opt}} d_{\max}} - \frac{1}{V_0 d_{\max}} + \frac{1}{C_0}} \end{aligned} \quad (14)$$

The corresponding optimum density d_{opt} can then be computed as

$$d_{\text{opt}} = \frac{q_{\max}}{V_{\text{opt}}} = \frac{1}{\frac{1}{d_{\max}} \left(2 - \frac{V_{\text{opt}}}{V_0} \right) + \frac{1}{C_0} V_{\text{opt}}} \quad (15)$$

The value of q_{\max} is significantly smaller than the capacity (C_0) considering a stand-alone queuing counter.

Rakha [7] demonstrated that two conditions need to be satisfied in order to ensure that the Van Aerde model does not produce densities that exceed the jam density at speeds greater than zero. These conditions are cast as

$$V_{\text{opt}} \geq \frac{V_0}{2} \quad \text{and} \quad q_{\max} \leq \frac{d_{\max} V_0 V_{\text{opt}}}{2V_0 - V_{\text{opt}}} \quad (16)$$

Rakha [7] also derived the wave speed as

$$w = - \left[\left(\frac{d_{\max}}{q_{\max}} - \frac{V_0}{V_{\text{opt}}^2} \right) + \frac{(V_0 - V_{\text{opt}})^2}{V_0 V_{\text{opt}}^2} \right]^{-1} \quad (17)$$

Consequently, the capacity of the single tandem queuing system (C_0) can then be computed as

$$C_0 = d_{\max} \left[\left(\frac{d_{\max}}{q_{\max}} - \frac{V_0}{V_{opt}^2} \right) + \frac{(V_0 - V_{opt})^2}{V_0 V_{opt}^2} \right]^{-1}. \quad (18)$$

$$q_{\max} = \frac{1}{\frac{1}{V_0 d_{\max}} + \frac{1}{C_0}} = \frac{C_0 V_0 d_{\max}}{C_0 + V_0 d_{\max}}. \quad (19)$$

The density at which the two waves intersect (d^*) can be computed by equating the flow at the intersect as

$$V_0 d^* = C_0 \left(1 - \frac{d^*}{d_{\max}} \right). \quad (20)$$

Solving Equation (20) for the density and subsequently the flow rate we obtain the following

$$d^* = \frac{C_0 d_{\max}}{V_0 d_{\max} + C_0}, \text{ and} \quad (21)$$

$$q_{\max}^* = \frac{C_0 V_0 d_{\max}}{V_0 d_{\max} + C_0}. \quad (22)$$

Special Cases

Earlier publications [7] demonstrated that by setting the speed-at-capacity (V_{opt}) equal to half the free-flow speed (V_0) and the optimum density (d_{opt}) equals to half the jam density (d_{\max}), the c_1 and c_3 parameters revert to zero generating the Greenshields model [13]. Furthermore, Rakha [7] demonstrated that the wave speed of Equation (17) equals the free-flow speed in the case that the speed-at-capacity is half the free-flow speed (Greenshields model).

Consequently, by setting c_1 and c_3 to zero as

$$c_1 = \frac{1}{d_{\max}} - \frac{k_{st} V_0}{C_0} = 0 \quad \text{and} \quad c_3 = \frac{1 - k_{st}}{C_0} = 0 \quad (23)$$

The parameters can be computed as $k_{st} = 1$ and $C_0 = V_0 d_{\max}$ which when substituted in Equation (18) considering that the wave speed equals the free-flow speed (V_0) and in Equation (14) yields $V_{opt} = \frac{V_0}{2}$ (Equation (18)) and $q_{\max} = \frac{V_0 d_{\max}}{4}$ (Equation (14)). This is consistent with the Greenshields' model parameter values.

Substituting the values of V_{opt} and q_{max} in Equation (10) the car-following model can be cast as

$$\Delta X = \frac{1}{V_0 - V} \frac{V_0^2}{C_0}. \quad (24)$$

The speed-density relationship can be written as

$$d(V) = (V_0 - V) \frac{C_0}{V_0^2}. \quad (25)$$

The Greenshields model was calibrated to 5-min. aggregated field data gathered from the Split Cycle and Offset Optimization Tool (SCOOT) system along an arterial, as illustrated in the sample calibration of Figure 4. The figure demonstrates that because the Greenshields model only offers two degrees of freedom, it fails to provide a good fit for all regimes across all data planes. Specifically, in order to provide a reasonable estimate of roadway capacity, the jam density is underestimated.

Rakha [7] also demonstrated that by setting the speed-at-capacity (V_{opt}) equal to the free-flow speed (V_0), the c_2 parameter reverts to zero. This model is commonly known as the Pipes model [14] and is used as the steady-state car-following model in a number of commercial traffic simulation software including CORSIM, VISSIM (Weidemann 99 model), and Paramics. The Pipes model is also known as the triangular fundamental diagram in some literature given that the flow-density relationship is triangular, as illustrated in Figure 5. Substituting c_3 for zero in Equation (9) results in a k_{st} value of zero. Furthermore, the maximum flow can be estimated by replacing V_0 for V_{opt} in Equation (14). In this case q_{max} equals q_{max}^* . In other words the capacity is computed using Equation (22).

The Pipes model was calibrated to the same arterial data that were presented earlier, as illustrated in Figure 5. The Pipes model provides a better fit to the data in comparison to the Greenshields model fit; however the model tends to over-estimate the traffic stream speed at higher flow levels in the uncongested regime. The Pipes model offers three degrees of freedom by calibrating three model parameters (V_0 , d_{max} , and q_{max}).

Finally, the full Van Aerde model offers four degrees of freedom by calibrating four model parameters (V_0 , V_{opt} , d_{max} , and q_{max}). The addition of the fourth parameter demonstrates the model superiority in capturing the full range of data, as illustrated in Figure 6. The model offers a good fit to the data for the full range of data across the three data planes.

In addition, further analysis was conducted using data from an 88 km/h (55 mi/h) speed limit freeway. Table 1 summarizes the estimates of the four traffic stream parameters considering different traffic stream models, as calibrated and tabulated in the literature [15]. In addition, the Van Aerde functional form was calibrated to the data as part of this research effort and the four traffic stream parameters estimated by the model are summarized. The literature also provides independent estimates of the valid ranges of observed values for each of the four parameters of interest. These observed validity ranges serve as an independent measure of the quality of fit of the various models to the subject data. The results of Table 1 demonstrate that the Van Aerde functional form is the only functional form that ensures that all key traffic stream parameters are within the valid ranges. These results demonstrate that the Van Aerde functional form provides the required level of flexibility to capture all four parameters and match the field data, as illustrated in Figure 7.

Finally a sensitivity analysis of the model parameters was conducted considering a free-flow speed of 100 km/h, a desired capacity of 2200 veh/h/lane, and a jam density of 150 veh/km/lane. The speed-at-capacity (V_{opt}) was varied from 50% to 100% the free-flow speed. The value of the k_{st} parameter varied from 0.25 to 0.00, as illustrated in Figure 8. Furthermore, Figure 8 demonstrates that the C_0 and q_{max}^* parameters approach q_{max} as the speed-at-capacity approaches the free-flow speed. Specifically, q_{max}^* equals q_{max} when the speed-at-capacity equals the free-flow speed.

CALIBRATION AND EVALUATION

The calibration of the Van Aerde model can be achieved using two approaches. The first approach involves computing the C_0 parameter and then estimating q_{max} from C_0 . Alternatively, the q_{max} parameter can be calibrated using a heuristic approach that was presented earlier [12]. The first approach is illustrated in this paper.

For a typical 4-lane motorway (two lanes each direction) in Germany, the new model is validated using the parameters $C_0 = 4532$ veh/h (obtained by using the Product-Limit method, cf. [16]), $V_{opt} = 80$ km/h, $d_{max} = 285.7$ veh/km ($= 1$ km / length of a car * number of lanes $= 1000\text{m} / 7\text{m} * 2$), and $V_0 = 130$ km/h. As a result $k_{st} = 0.048$, $q_{max} = 3556$ veh/h, and $d_{opt} = 44.45$ veh/km are obtained. These values correspond very well to the real-world traffic conditions. For this example, the $q - V$, $d - V$, and $d - q$ relationships are illustrated in Figure 9.

Using the proposed new model the number of lanes and the proportion heavy vehicles (HV) can be modeled by simply setting different values for the 4 parameters d_{max} , V_0 , C_0 , and k_{st} . The mean values of C_0 , V_0 , and d_{max} from passenger cars and trucks can be used in order to account for the proportion of heavy vehicles (here, the value of k_{st} is considered as a constant). In Figure 10, an example of the flow-speed relationships for different proportions of heavy vehicles is illustrated. In this example, the parameter $V_{opt} = 80$ km/h for both passenger cars and trucks, $d_{max} = 1000\text{m} / 7\text{m} * 2 = 285.7$ veh/km for passenger cars, $d_{max} = 1000\text{m} / 14\text{m}$ (length of a truck) $* 2 = 142.6$ veh/km for trucks, $V_0 = 130$ km/h for passenger cars, $V_0 = 80$ km/h for trucks, and a passenger car equivalency of 1.5 for trucks are used as input parameters.

In Figure 11, the proposed new model is illustrated together with the Greenshields model and a measured data set. It can be seen that the new model describes traffic flow under real-world traffic conditions quite well. Again, it can be seen that the value of q_{max} is always smaller than the value of C_0 .

The Van Aerde model was calibrated to a number of datasets from Europe and North America using the SPD_CAL calibration heuristic [12], as summarized in Table 2. The results demonstrate that on freeways the speed-at-capacity is approximately 80% of the free-flow speed. Alternatively, the speed-at-capacity tends to half the free-flow speed for lower geometric design facilities (e.g. arterial and tunnel facility). The k_{st} parameter ranges from 0.0058 to 0.2700 and is typically higher on lower design facilities. The wave speed on North American freeways ranges from -21 to -25 km/h, which is consistent with what is reported in the literature (~ 20 km/h). The study, however demonstrates that the wave speed may be very different on arterials or on freeways with a very high free-flow speed, like for example an Autobahn.

CONCLUSION

The paper demonstrated the theoretical background model for the Van Aerde model using the tandem-queuing theory. This theoretical background demonstrates that, unlike single-tandem queuing approaches, the tandem-queue approach captures the interaction between consecutive vehicles within the traffic stream. This theoretical background also allows for a generalization of the Van Aerde model.

According to this Van Aerde model, a Fundamental Diagram can be defined completely by four parameters: 1) the free flow speed V_0 , 2) the jam density d_{\max} , 3) the potential capacity of the cross section C_0 which can be estimated by the so-called Product-Limit Method (PLM), and 4) a system parameter k_{st} that describes the stochastic properties of the queuing system. The first three parameters can be measured directly. Only the system parameter k_{st} has to be calibrated.

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Table 1: Comparison of Flow Parameters for Single-Regime, Multiple-Regime Models, and Proposed Model

Type of Model	Model	Free-speed (km/h)	Speed-at-Cap. (km/h)	Capacity (veh/h/lane)	Jam density (veh/km/lane)
Valid Data Range [15]		80-88	45-61	1800-2000	116-156
Single-Regime	Greenshields	91	46	1800	78
	Greenberg	∞	37	1565	116
	Underwood	120	45	1590	∞
	Northwestern	77	48	1810	∞
Multi-Regime	Edie	88	64	2025	101
	2-Regime	98	48	1800	94
	Modified Greenberg	77	53	1760	91
	3-Regime	80	66	1815	94
Van Aerde		80	60	1827	116

Source: May, [15] pp. 300 and 303.

Highlighted cells: Outside the valid data range for specified parameter.

Table 2: Summary Results on Different Facilities

Data	n	V_0 (km/h)	V_{opt} (km/h)	q_{max} (veh/h)	d_{max} (veh/km)	C_0 (veh/h)	q^*_{max} (veh/h)	k_{st}	w (km/h)
Freeway	24	80	61	1827	116	2685	2082	0.0281	-23.15
Tunnel	24	68	34	1263	125	2287	1799	0.2695	-18.30
Arterial	33	45	23	582	102	939	779	0.2047	-9.21
I-4 Fwy	288	87	76	1906	116	2530	2023	0.0058	-21.81
401 Fwy	282	106	90	1888	100	2487	2013	0.0071	-24.87
Amsterdam Fwy	1199	99	86	2481	115	3460	2652	0.0066	-30.16
Autobahn	3215	150	108	2100	90	2911	2395	0.0337	-32.34

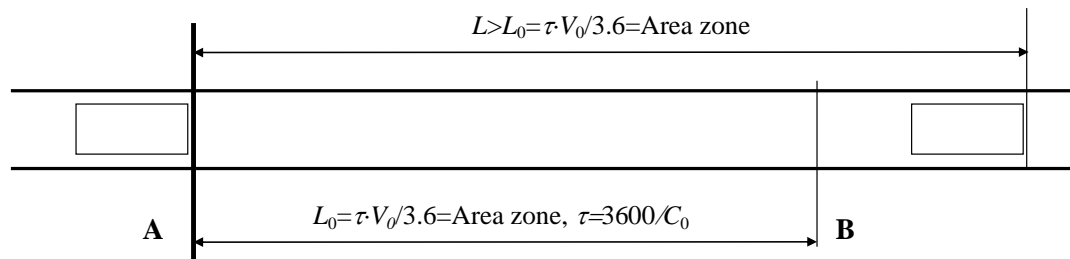


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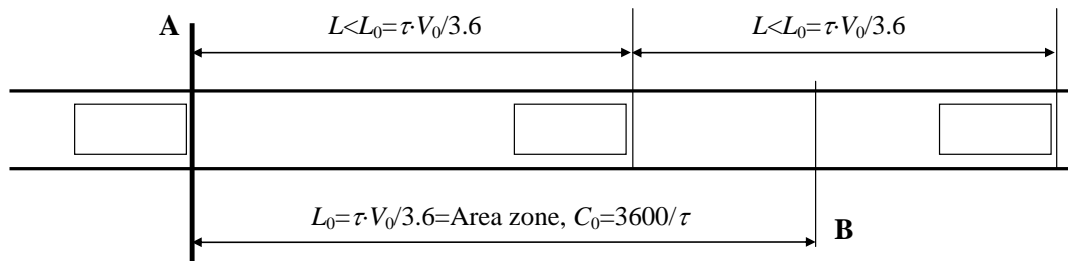


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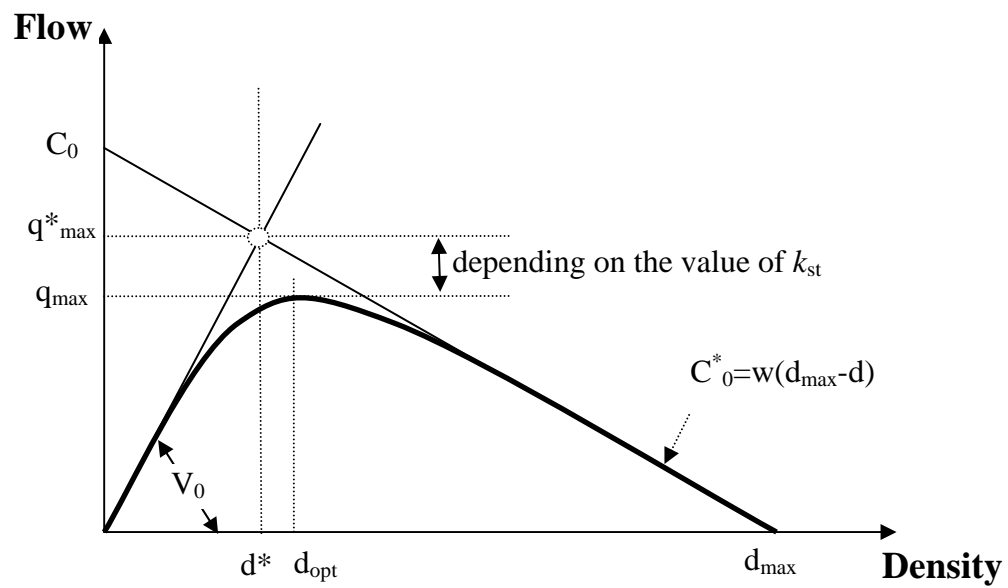


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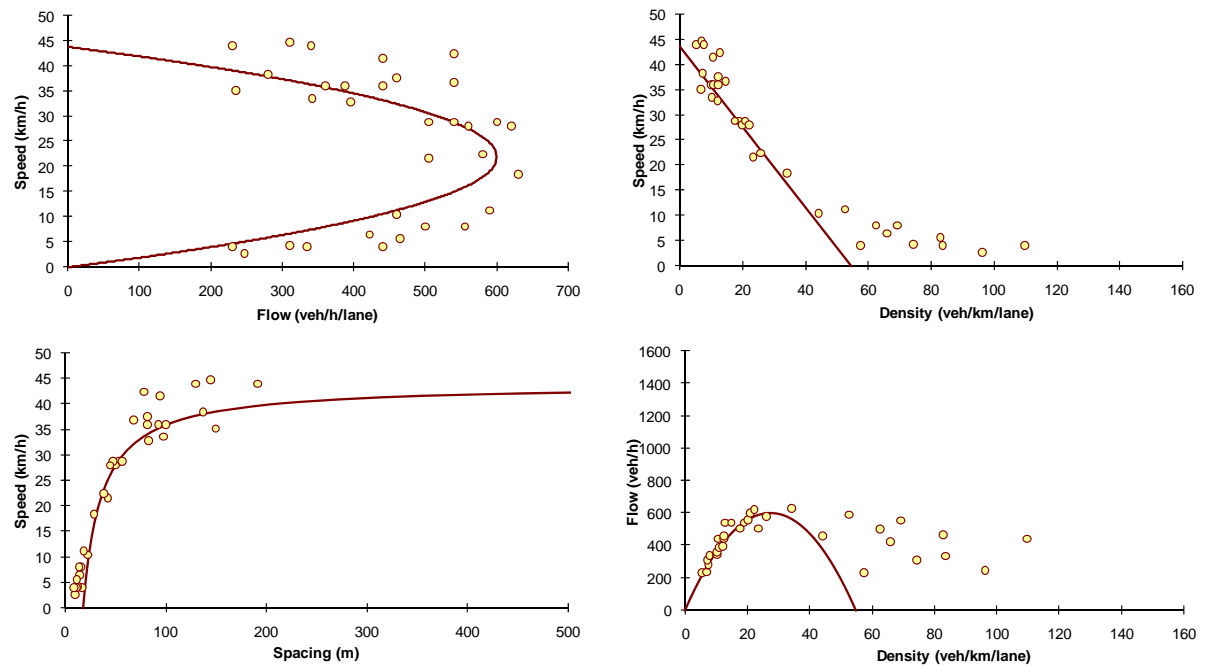


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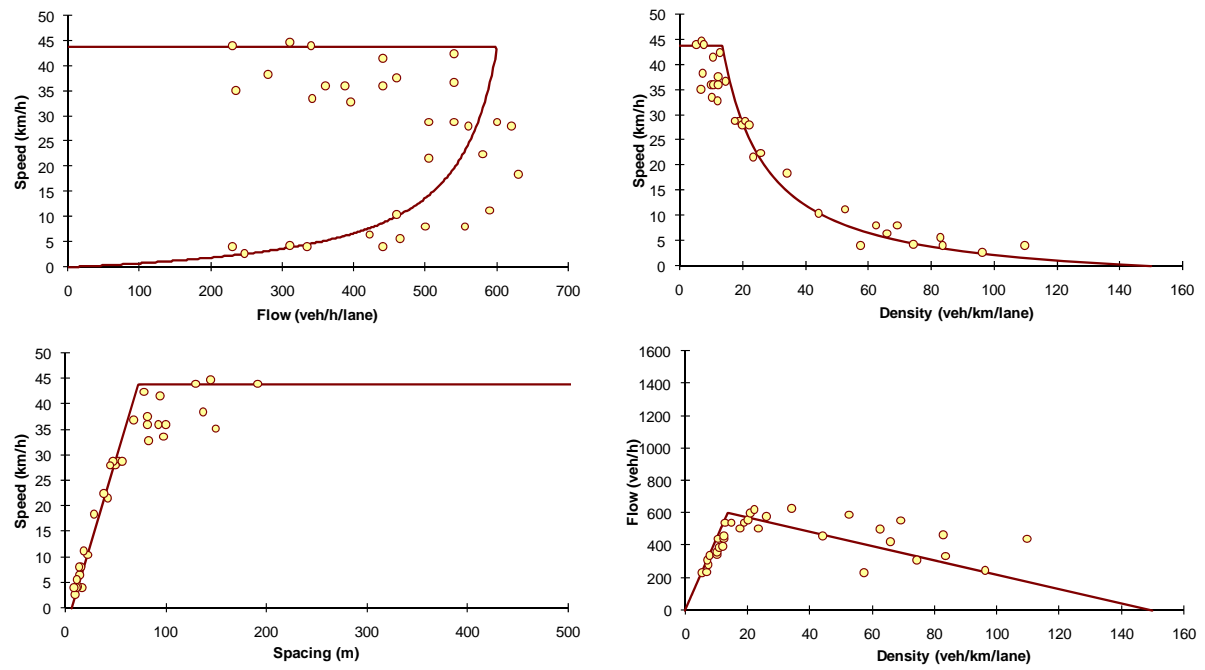


Figure 5: Sample Pipes Model Calibration

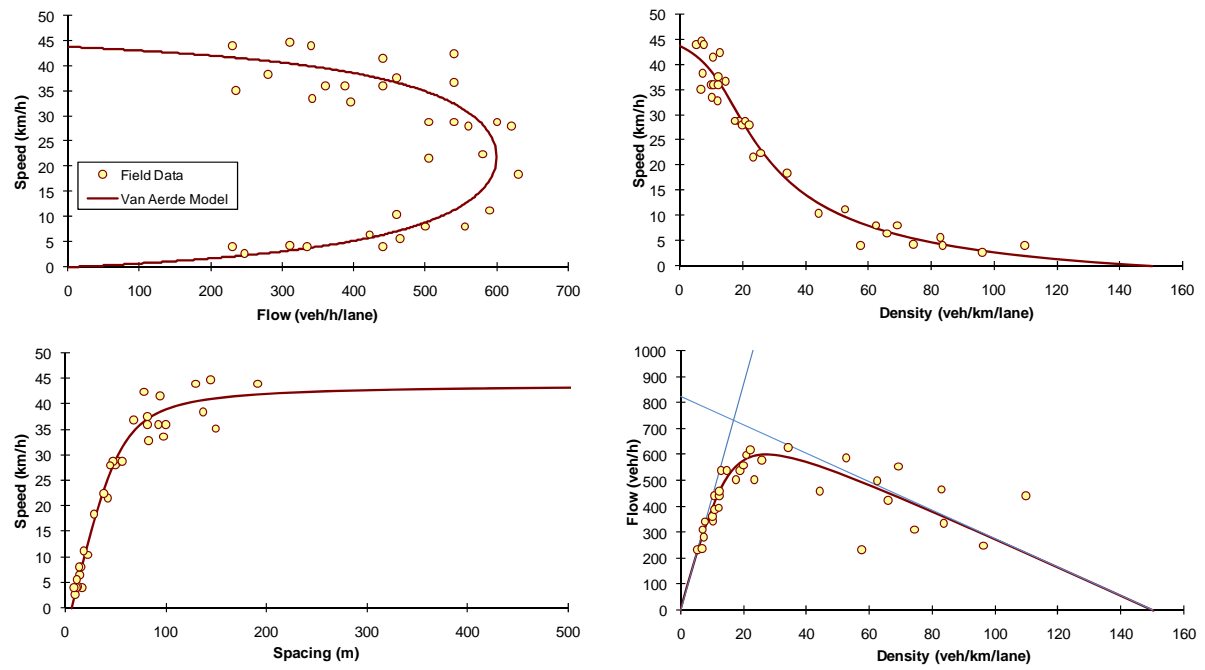


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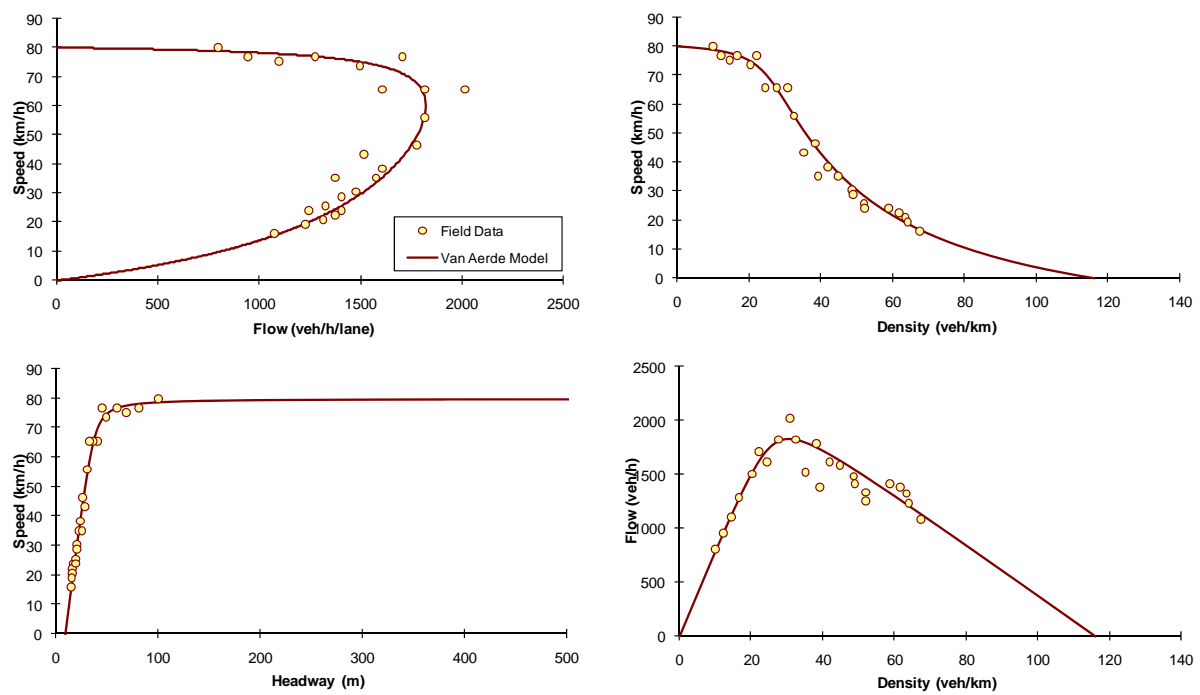


Figure 7: Example Illustration of Model Calibration to Freeway Data

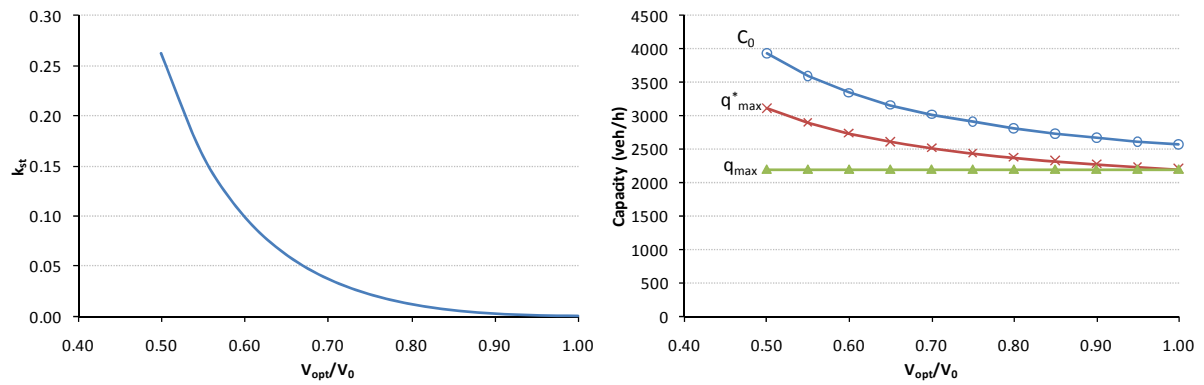


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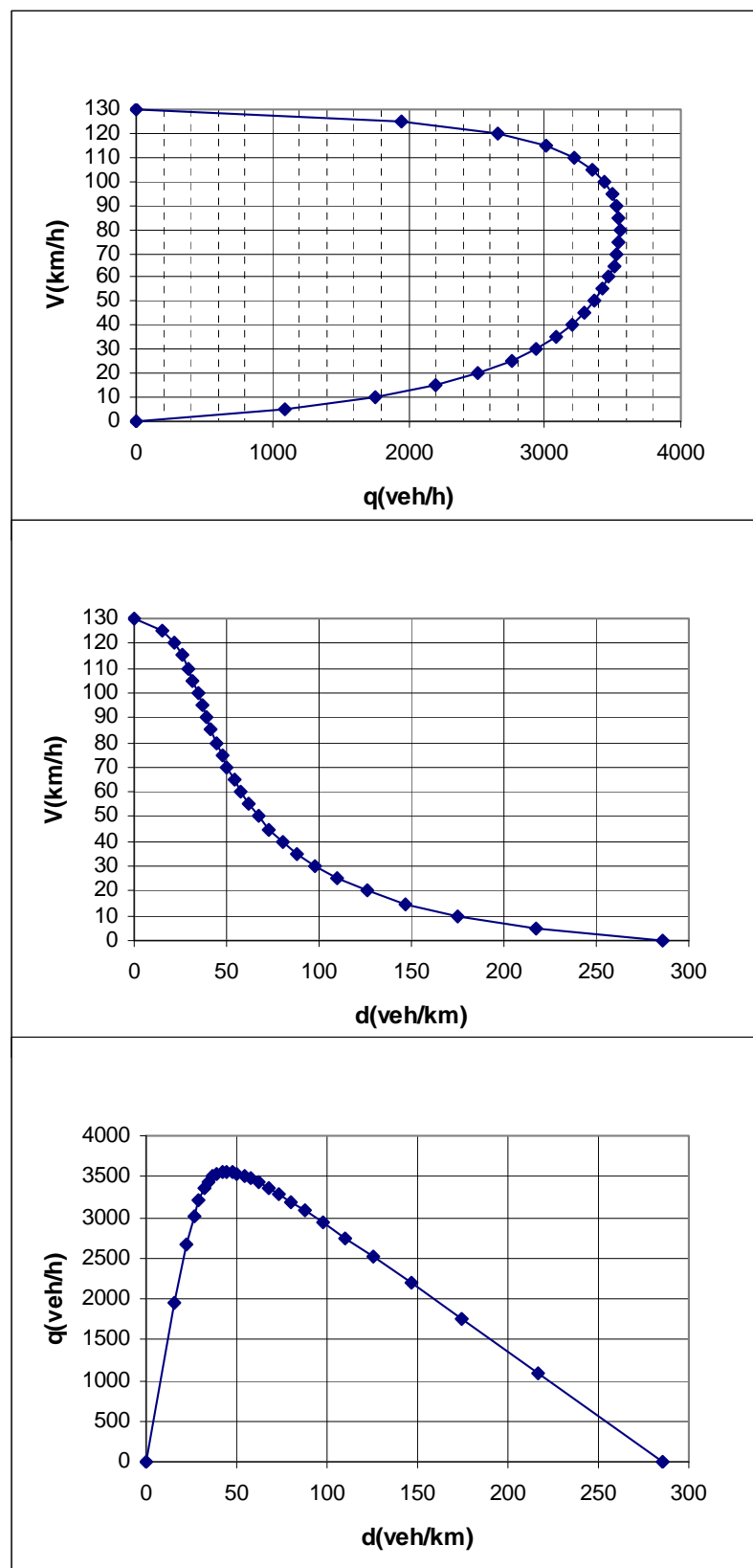


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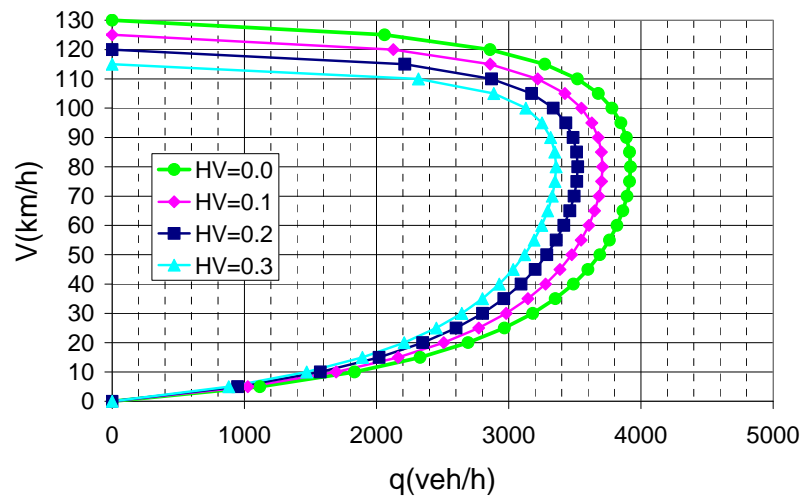


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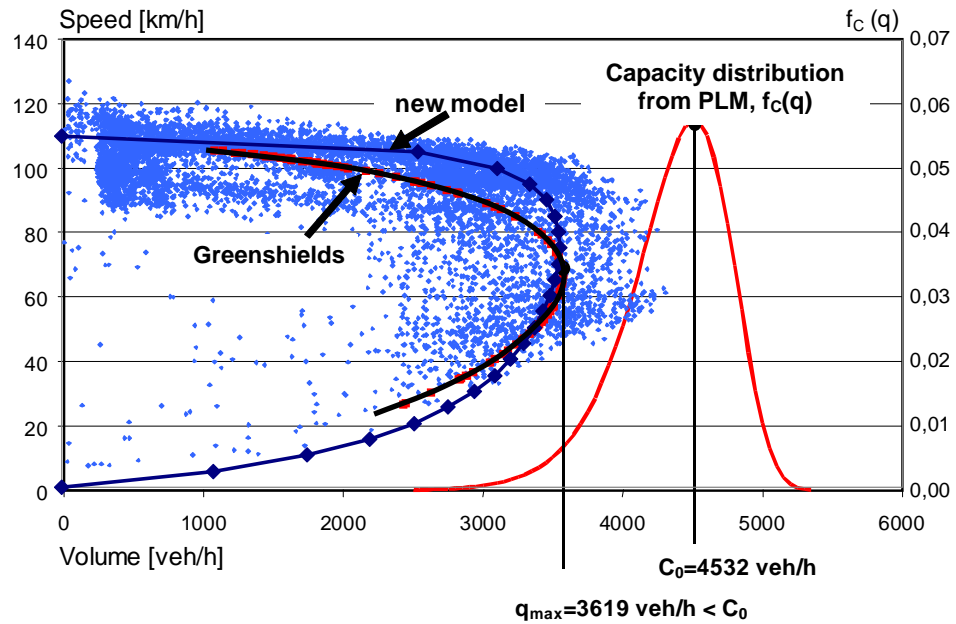


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