Determination of Stochastic Bottlenecks Capacity on Freeways and Estimation of Congestion Probabilities

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Abstract

This paper presents an approach for estimating the stochastic distribution of bottlenecks capacities on freeways. Using the so-called Product-Limit Method, it is able to determine the distribution of capacities on freeways before a breakdown. The distribution of capacities after a breakdown can be observed directly by measuring the departure capacity behind the bottleneck. Combining the capacities before and after a breakdown, the presented paper introduces a mathematical method to obtain the distribution of the overall capacity which takes both the free traffic and sporadic breakdowns into account.

Using the results of the presented approach, the mean capacity and the risk of disruption (breakdowns, duration of congestion) of the traffic flow on freeways can be estimated and analyzed.

Keywords: capacity estimation, distribution of capacity, freeway, traffic flow control

Introduction

Capacity is one of the most important measures in traffic facilities analysis. As usually known, the bottleneck's capacity on freeways has a dual feature (duality): the capacity before a breakdown is higher than the capacity after a breakdown (cf. Figure 1). The difference between these two capacities is called "capacity drop". Furthermore, theses capacities are not constant but stochastically distributed values.

Normally, the mean value and the distribution of capacities after a breakdown can be directly measured at a bottleneck under congested traffic conditions. The distribution of capacities before a breakdown cannot be estimated so easily because it cannot be directly observed. However, using an mathematical approach from the life science, it is able to determine the so-called revival (residual) probability of bottleneck's capacities on freeways before a breakdown (Minderhoud, 1998 and Zurkinden, 2003). The approach is called "Product-Limit Method" (PLM). From the revival (residual) probability of

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capacities, the distribution of capacities can be calculated. Furthermore, combining the distributions of capacities before and after a breakdown, the distribution of overall capacities which takes both the free traffic flow and the sporadic breakdowns into account can be obtained.

Thus, according to the presented approach, it is able to estimate the overall capacity considering both the free (before a breakdown) and the congested (after a breakdown) traffic condition. Using the distributions of capacities before and after a breakdown and the overall capacity distribution, and combining them with a given distribution of traffic demands, the probability of breakdown (the demand is higher than the capacity before a breakdown), the probability of recovery (the demand is lower than the capacity after a breakdown), and the probability of congestion (the demand is higher than the overall capacity) can be calculated.

Using the results of the presented approach, the risk of disruption (breakdowns, duration of congestion) of the traffic flow on freeways can be estimated and analyzed. The approach can be used for planing and dimensioning of freeways and for controlling traffic flow on freeways in purposes of congestion warning or ramp access control.

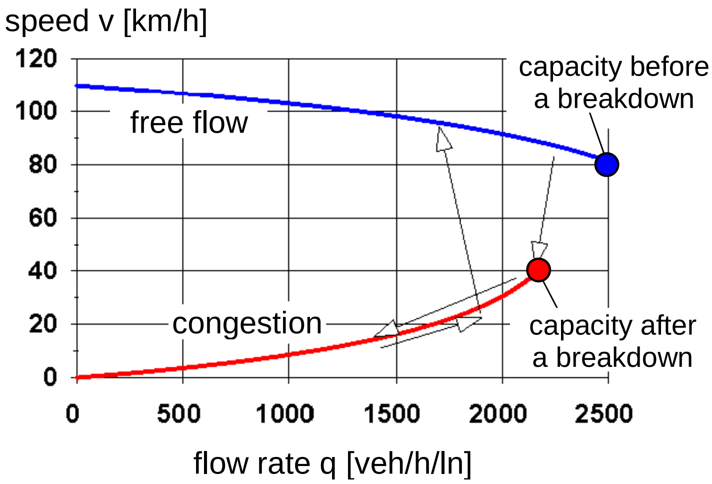


Figure 1: Duality of the capacity on a freeway

Product-Limit method for estimating the capacity before traffic breakdowns

The Product-Limit Method (PLM) for estimating bottlenecks capacity on a freeway is based on the general approach for statistical analysis of lifetime data (Lawless, 1981). The PLM can take both the uncensored data (observed lifetimes) that can only be observed in a portion of the observation period and the censored data (ages of people alive which are older than some observed lifetimes). In the work of Minderhoud et. al. (1998), a detailed formulation of the PLM for the capacity analysis of freeways is given. Zurlinden (2003) adopted the PLM for congestion analysis in Germany. The basic idea and some results of these works are demonstrated here for a better understanding of the PLM.

Considering a bottleneck on a freeway (e.g.: lane reduction, downstream area of a on-ramp etc.) for traffic flow analysis (cf. Figure 2), the uncensored data are related to the capacities (here, the traffic flow values immediate before a breakdown are considered as capacities), and the censored data are related to the uncongested (free) traffic flow values which are higher than some observed capacity values.

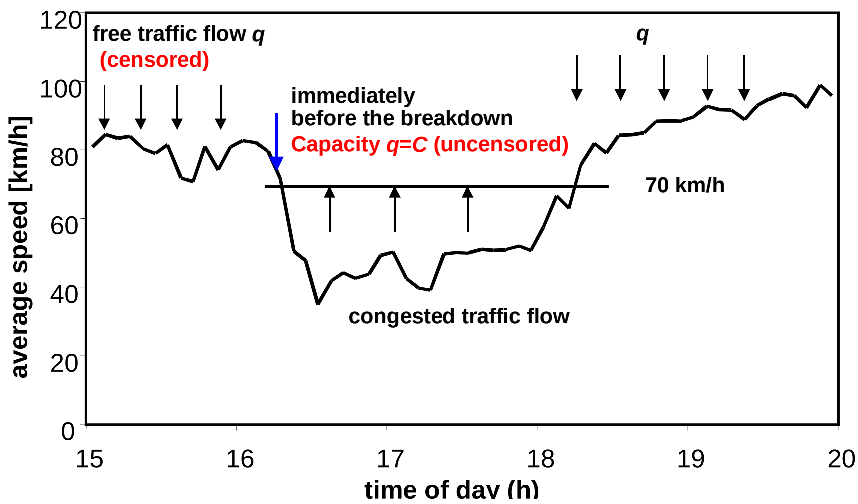


Figure 2: Traffic flow values before and after a breakdown

The PLM is constituted on the idea that each uncongested flow observation having a higher flow rate than the lowest observed capacity rate contributes to the capacity estimate since this observation gives additional information about the capacity value. Such uncongested flow observations indicate that the corresponding capacity value has not been reached, although its flow value exceeds other observed capacity values. These observations are called "censored" observations, since the actual capacity value is not directly measured but has an unknown, higher value. The censored values can be taken into account by considering the survival (residual) function of the capacity.

Formally, the observation is called "right censored" at flow rate q if the unknown capacity value of the observation is only known to be a greater than or equal to q. The capacity estimation is apparently a problem of "right censoring" (Minderhoud et. al., 1998).

In order to estimate the distribution of the capacity, the capacity observations are assumed to be identically and independently with probability density function $f_C(q)$, probability distribution function $F_C(q)$, and probability survival (residual) function $S_C(q)$ = $1-F_C(q)$. Then, the likelihood of a sample is (Minderhoud et. al., 1998):

$$L = \prod_{i=1}^{n} f_C(q_i)^{1-\delta_i} \cdot S_C(q_i)^{\delta_i} \qquad (1)$$

where n = number of the observation periods

$\delta_i = 0$ for uncensored q-value ($q = C$ = observed capacity)

$\delta_i = 1$ for censored q-value (all other q-values)

For the capacity estimation, Zurlinden (2003) used a Weibull-distribution as a capacity distribution function. That is,

$$F_C(q) = 1 - e^{-(\frac{q}{b})^a} \tag{2}$$

The likelihood is then given by

$$L = \prod_{i=1}^{n} \left(a \cdot b^{-a} \cdot q_i^{a-1} \cdot e^{-(\frac{q_i}{b})^a} \right)^{1-\delta_i} \cdot \left(e^{-(\frac{q_i}{b})^a} \right)^{\delta_i} \tag{3}$$

Here, a is the shape parameter and b the scale parameter (=mean value) of the Weibull-distribution. Note, that the Weibull-distribution is a asymmetric, right-shifted distribution. The value of the median is lower than the mean value. For the special case of $q=b$ (q at mean value), the cumulative probability of the Weibull-distribution (eq.(2)) is equal to 0.63, which is much higher than the probability of the median (=0.5).

For freeway capacity estimation a discrete non-parametric form of the capacity survival (residual) function $S_C(q)=1-F_C(q)$ can be used. In order to deal with both censored and uncensored data, the Product-Limit estimate of the survival function is given by (cf. Lawless, 1981 and Minderhoud et. al., 1998)

$$\hat{S}_C(q = C_k) = \Pr(m_k \geq C_k \mid m_{k-1} \geq C_{k-1} \mid m_{k-2} \geq C_{k-2} \mid \ldots \mid m_1 \geq C_1) \tag{4}$$

That is

$$\hat{S}_C(q = C_k) = \prod_{j \in k} \frac{m_j - 1}{m_j}, \quad j = 1..k, 1 \leq k \leq n \tag{5}$$

1	2	3	4	5	6	7	8
intervall	q (veh/h)	capacity (C) free flow (F)	order j	k	mj	(mj-1)/mj	SC (q)
1	3000	F	2	-	-		
2	2500	F	1	-	-		
3	3500	C	3	1	6	5/6	5/6=0.83
4	4000	F	4	-	-		
5	4300	C	6	3	3	2/3	5/6*3/4*2/3=0.41
6	4500	F	7	-	-		
7	4600	C	8	4	1	0/1	5/6*3/4*2/3*0/1=0
8	4100	C	5	2	4	3/4	5/6*3/4=0.62

Table 1: Example of PLM calculation (source Minderhoud et. al., 1998)

In eqs.(4) and (5), k is the number of observed capacities having value lower than or equal to the value of C_k; index j indicates the all observed capacities C_j having value lower than or equal to the value of C_k; m_j is the number of all observations (all q values, also by $q=C$) having value higher than or equal to the value of C_j. In Table 1, an example of the PLM calculation is presented to explain the PLM calculation procedure. Note, the values of observed capacities depend on the length of the observation period T. In this paper, the T = 5 min is used for estimating the capacity.

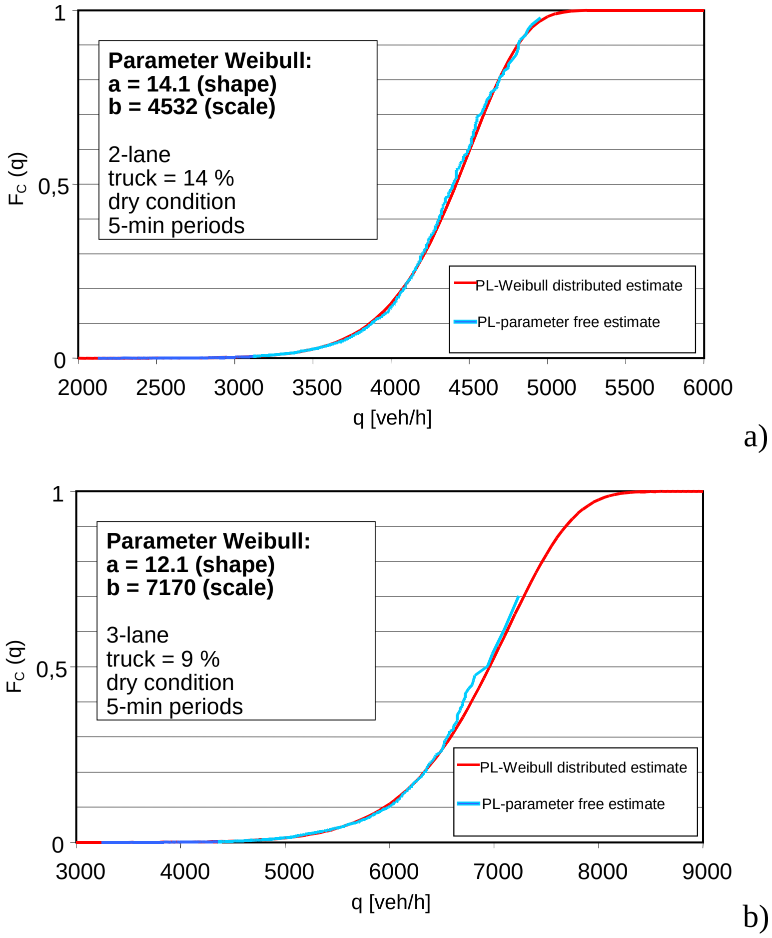


Figure 3: Results of the Product-Limit estimation for two freeways in Germany (5-min intervals, Source Zurlinden, 2003)

Using the PLM, Zurlinden (2003) investigated the capacity on German freeway. He estimated the distribution of capacities for two freeways (one 2-lane and one 3-lane freeway) in Germany (Figure 3). In Figure 3, the results of both Product-Limit-Weibull distributed estimation and Product-Limit-parameter free estimation are depicted together. It can be seen, that the Weibull-distribution is a very good approximation of the parameter free estimation. From this example (cf. Figure 3), the capacity of a two-lane freeway is 4532 veh/h and that of a three-lane freeway is 7170 veh/h under the actual traffic conditions. Because the traffic flow will break down if the actual flow rate q is higher than the capacity C, the probability of breakdown $P_{br}(q)$ is simply equal to the value of the distribution function $F_C(q)$. For example, on the two-lane freeway, a traffic flow rate of q=4000 veh/h would causes a breakdown with a probability of 0.15 (cf. Figure 3, a)).

It is to be point out, that the capacity estimated from the PLM is not the "conventional" capacity that can be utilized by a real freeway. They are theoretical limiting value in the sense of queuing theory. This theoretical capacity can never be reached under real traffic conditions because of the disruption of the traffic flow. If the actual flow rate reaches the value of this capacity, the probability of a breakdown is already higher than 0.5 (0.63 for Weibull-distributed capacities). This capacity from PLM is higher than the "conventional" capacity by a factor 1.25 (Zurlinden, 2003; cf. Figure 4). The "conventional" capacity is a overall capacity taking into account all the possible conditions under consideration, also the congested traffic condition.

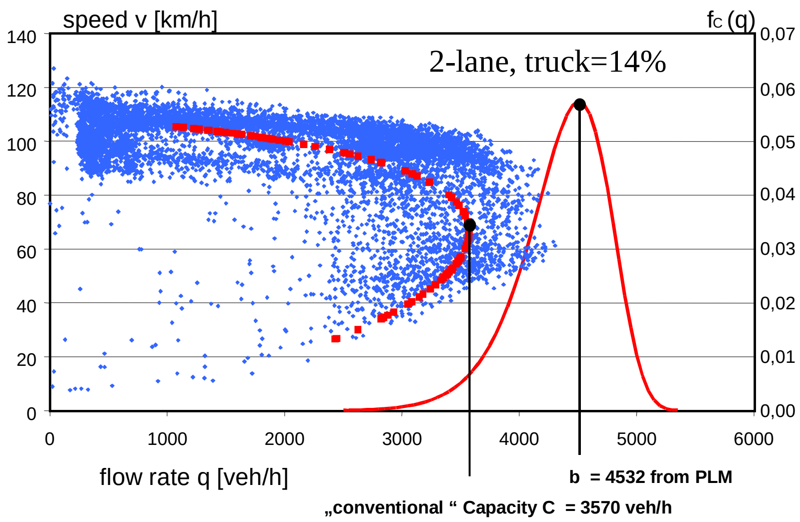


Figure 4: Difference between the "conventional" capacity and the capacity from PLM (5-min intervals, Source Zurlinden, 2003)

The capacity of a bottleneck and its distribution after a breakdown can be directly observed. It can be defined as the departure capacity of the bottleneck during the congestion. For the same freeways mentioned in Figure 3 the capacity distribution functions after a breakdown are illustrated in Figure 5 together with the capacity distribution functions before a breakdown. Denote the capacity distribution function after a breakdown with $F_C(q)^*$ and the probability that the traffic flow can recover from the congestion with $P_{re}(q)$, then is $P_{re}(q)=1-F_C(q)^*$.

Transition of probability between capacities before and after a breakdown

According to the previous section, the capacity distribution functions both for "free" (before a breakdown) and "congested" (after a breakdown) traffic flow are known and therefore also the probabilities that the traffic flow changes from "free" to "congested" (P_{br}) and that the traffic flow changes from "congested" to "free" (P_{re}). Denote the probability that the traffic flow is in the state "free" with P_{free} and that the traffic flow is in the state "congested" with P_{cong}, then the following equation system (Markov Chain) is true:

$$\begin{pmatrix} P_{free}^{k+1}(q) \\ P_{cong}^{k+1}(q) \end{pmatrix} = \begin{pmatrix} 1-P_{br}(q) & P_{re}(q) \\ P_{br}(q) & 1-P_{re}(q) \end{pmatrix} \begin{pmatrix} P_{free}^{k}(q) \\ P_{cong}^{k}(q) \end{pmatrix} \tag{6}$$

Where k and k+1 are indices for the k-th and k+1-th interval. For the equilibrium condition, this equation system becomes

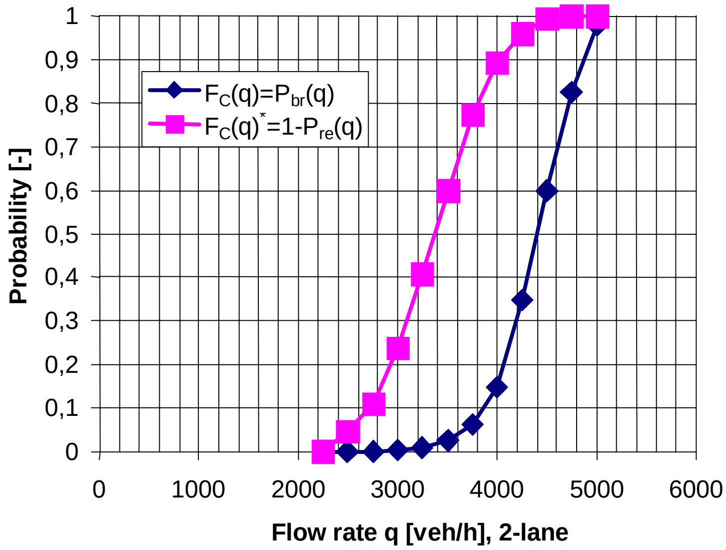
$$\begin{pmatrix} P_{free}(q) \\ P_{cong}(q) \end{pmatrix} = \begin{pmatrix} 1-P_{br}(q) & P_{re}(q) \\ P_{br}(q) & 1-P_{re}(q) \end{pmatrix} \begin{pmatrix} P_{free}(q) \\ P_{cong}(q) \end{pmatrix} \tag{7}$$

with $P_{free}(q) + P_{cong}(q)= 1$. The equation (7) yields the solution:

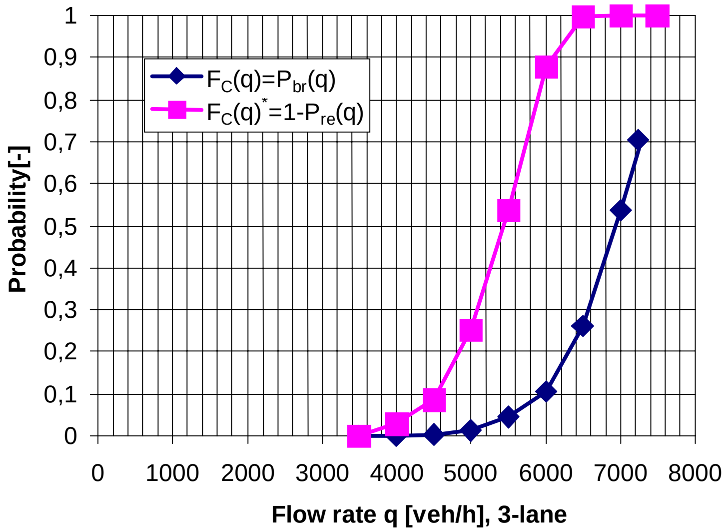
$$P_{cong}(q) = 1 - P_{free}(q) = \frac{P_{br}(q)}{P_{br}(q) + P_{re}(q)} = \frac{P_C(q)}{P_C(q) + (1 - P_C^*(q))} \tag{8}$$

and

$$P_{free}(q) = 1 - P_{cong}(q) = \frac{(1 - P_C^*(q))}{P_C(q) + (1 - P_C^*(q))} \tag{9}$$



a)



b)

Figure 5: Capacity distribution functions before a breakdown $F_C(q)$ and after a breakdown $F_C(q)^*$ (5-min intervals)

For the example illustrated in Figure 6, the meaning of the three probabilities $P_{br}(q)$, $P_{re}(q)$, and $P_{cong}(q)$ can be understood as following for a two-lane freeway with an actual flow rate of 4000 veh/h: if the traffic is in the state "free", the probability that the traffic flow within a 5-min period could break down from the free flow state to the congested flow state is ca. 0.13 (P_{br}); if the traffic is in the state "congested", the probability that the traffic flow within a 5-min period could recover from the congested flow state to the free flow state is 0.11 (P_{br}); thus, the probability that the traffic within a 5-min period is in the state "congested" is 0.14/(0.14+0.11)=0.56. That is, the traffic flow under this predefined condition is ca 56% congested and 44% free.

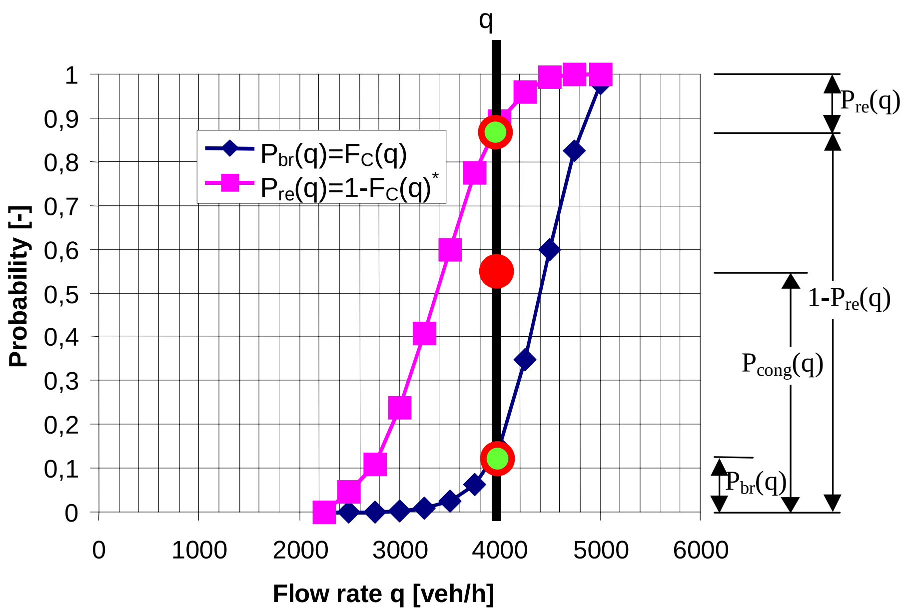


Figure 6: Probability of breakdown $P_{br}(q)$, recovery $P_{re}(q)$, and congestion $P_{cong}(q)$ (5-min intervals)

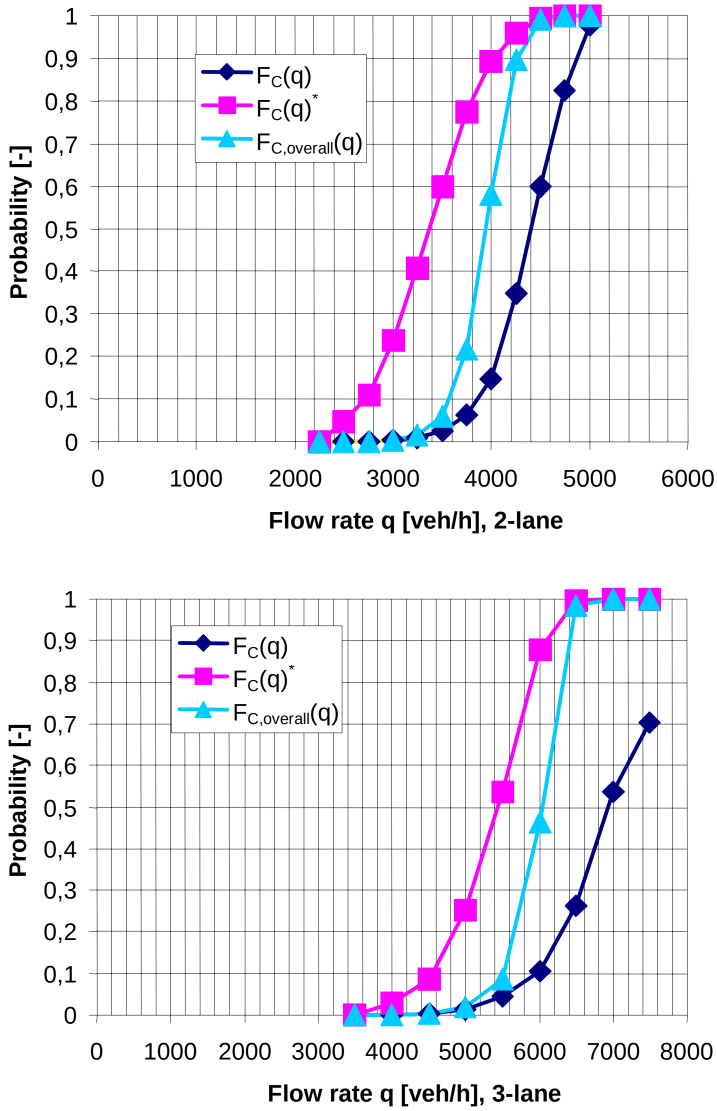
The probability that the traffic flow is congested is equal to the probability that the flow rate is higher than the overall capacity, that is

$$P_{cong}(q) = \Pr(q > C_{overall})$$

Thus, the overall capacity distribution function is (Figure 7)

$$F_{C,overall}(q) = \Pr(q > C_{overall}) = P_{cong}(q)$$
$$= \frac{P_{br}(q)}{P_{br}(q) + P_{re}(q)} = \frac{F_C(q)}{F_C(q) + (1 - F_C(q)^*)} \qquad (10)$$

Assuming the resulting capacities are Weibull-distributed, the mean capacity corresponds to $F_C(q)$=0.63. Thus, from Figure 7, the mean overall capacity is about 2000 veh/h/lane both for the 2-lane and 3-lane freeway. This corresponds very well to the experience values in Europe.



a)

b)

Figure 7: Distribution function of capacities before a breakdown $F_C(q)$, after a breakdown $F_C(q)^*$, and of the overall capacities $F_{C,overall}(q)$ (5-min intervals)

The probability $P_{cong}(q)$ can be used for estimating the average duration of free and congested period and its distribution. For example, given the length of the measurement intervals T (e.g. 5 min), the probability that the traffic flow is in N_{cong} consecutive intervals, that is in the time period $L_{cong}=T{\cdot}N_{cong}$, congested is

$$\Pr(L_{cong} = T \cdot N_{cong}) = P_{cong}(q)^{N_{cong}} \tag{11}$$

The average length of the congested period is

$$\overline{L}_{cong}(q) = T \cdot \overline{N}_{cong}(q) = \frac{P_{cong}(q)}{\left(1 - P_{cong}(q)\right)^2} \cdot T \tag{12}$$

The average length of the free flow period is respectively

$$\overline{L}_{free}(q) = T \cdot \overline{N}_{free}(q) = \frac{1 - P_{cong}(q)}{P_{cong}(q)^2} \cdot T \qquad (13)$$

Certainly, the calculation of the average congested or free period can only be conducted if the average flow rate $\overline{q}$ is lower than the overall capacity $C_{overall}$. Otherwise the traffic flow is non-stationary and the congestion would increase with time. For the example of a two-lane freeway, the probability of congestion $P_{cong}(q)$ is ca. 0.56 with $\overline{q}$ =4000 veh/h (cf. Figure 6 a)). Thus, the average congested time period $\overline{L}_{cong}(q)$ is ca. $0.56/(1\text{-}0.56)^2 \cdot 5$ = 14.42 min and the average free flow time period $\overline{L}_{free}(q)$ is ca. $(1\text{-}0.56)/0.56^2 \cdot 5$ = 7.02 min. The average circulation time of stop-and-go is then ca. 17.42+7.02=21.44 min.

Conclusion

Using the Product-Limit method, distributions of bottlenecks capacities (eq.(5)) before a breakdown can be estimated for freeways (Minderhoud et. al., 1998; Zurlinden, 2003). According to this distribution functions, the probability of breakdown (from free to congested traffic condition) by a given traffic flow rate can be calculated. Using the measured bottlenecks capacities after a breakdown, the distribution of capacities after a breakdown and the probability of recovery (from congested to free traffic condition) by a given traffic flow rate can be obtained as well. Combining the capacity distribution functions both before and after a breakdown, the probability of congestion (portion of time that the traffic is in a congested state) and the distribution function of the overall capacities (average value of capacities over all free and congested traffic conditions) can be calculated by the presented approach (eqs.(8) and (10)).

The results of the presented approach can be useful for estimating the risk of disruption (breakdowns, duration of congestion) of the traffic flow on freeways (cf. also Wu, 2002) and for planing and dimensioning of freeways as well as for controlling traffic flow on freeways in purposes of congestion warning or ramp access control.

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