Detailed curriculum vitæ

Laurent Battisti

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Laurent BATTISTI

Postdoctoral researcher in Ruhr-Universität Bochum

Mathematical interests: Complex geometry, Differential geometry, Toric varieties, Complex Lie groups, Cousin groups.

Coordinates of three experts for reference letters:
Prof. Laurent MEERSSEMAN, University of Angers (France), `meersseman@math.univ-angers.fr`
Prof. Karl OELJEKLAUS, University of Aix-Marseille (France), `karloelj@cmi.univ-mrs.fr`
Prof. Hassan YOUSSFI, University of Aix-Marseille, concerns teaching, `youssfi@cmi.univ-mrs.fr`

Positions

2013–present **Postdoctoral researcher (one year)**, *Ruhr-Universität Bochum*, Germany.
Invitation by Professors P. Heinzner and A. T. Huckleberry.

2012–2013 **ATER (Teaching assistant) in Mathematics**, *Université Aix-Marseille*, France.
The teaching load was 96hrs/year.

Oral interrogation, *Lycée Notre Dame de Sion*, Marseille, France, CPGE MPSI & MP.
The teaching load was 5hrs/week.

2009–2012 **PhD scholarship with teaching load**, *Université de Provence*, Marseille, France.
The teaching load was 64hrs/year. A part of my teaching was held in the Army Air Force School "**Ecole de l'Air**" in Salon de Provence between 2010 and 2012.
Funds: I was awarded both a government scholarship and a European one. I chose the European scholarship to allow my Doctoral School to obtain one more scholarship.

2008–2009 **Tutor in Mathematics**, *bachelor's-level, CTES (Université de Provence)*, Marseille, France.

Education

2009–2012 **PhD in Mathematics**, *Université Aix-Marseille (formerly Université de Provence)*.
Advisor: Professor Karl OELJEKLAUS.
Title: *Variétés toriques à éventail infini et construction de nouvelles variétés complexes compactes: quotients de groupes de Lie complexes et discrets.*
(translation: Toric manifolds with infinite fan and construction of new compact complex manifolds: quotient by complex Lie groups and discrete groups.)
Jury: Georges DLOUSSKY, Professor, Université d'Aix-Marseille - LATP, *Examiner*
Jean-Jacques LOEB, Professor, Université d'Angers - LAREMA, *Referee*
Laurent MEERSSEMAN, Professor, Université de Bourgogne - IMB, *Referee*
Karl OELJEKLAUS, Professor, Université d'Aix-Marseille - LATP, *Advisor*
Matei TOMA, Professor, Université de Lorraine - IEC, *President*
David TROTMAN, Professor, Université d'Aix-Marseille - LATP, *Examiner*
Mention : First Class Honours. **Date of defense**: December 10th, 2012.

2008–2009 **Master 2 (MSc) in Pure Mathematics**, *Université de Provence*, Marseille, France.
Honors: summa cum laude (rank 1).
Thesis written under direction of Professor Karl OELJEKLAUS.
Title: *Variétés toriques.* ("Toric varieties") **Date of defense**: June 15th, 2009.

2007–2008 **Agrégation de Mathématiques**, *Prepared in Université de Provence*, Marseille, France.
Rank: 172. This is a French national competitive exam for positions in the public education system. During the schoolyear 2007–2008 I was awarded a scholarship upon university results.

2006–2007 **Master 1 in Pure Mathematics**, *Université de Provence*, Marseille, France.
Honors: summa cum laude (rank 1).
Dissertation written under direction of Professor Karl OELJEKLAUS.
Title: *La courbure*. ("Curvature") **Date of defense**: May 22nd, 2007.

2005–2006 **Licence 3 (BSc) in Mathematics**, *Université de Provence*, Marseille, France.
Honors: summa cum laude (rank 1).

2003–2005 **Classes préparatoires scientifiques (MPSI, MP*)**, *Lycée Thiers*, Marseille, France.
Classes préparatoires are specific to the French education system, with competitive exam entrance.

2003 **Baccalauréat série Scientifique**, *Lycée de l'Empéri*, Salon de Provence, France.

Computer skills

Systems Mac OS X, Linux, Windows.

Softwares Maple, Mathematica, Matlab, Sage, bureautique (LaTeX, Office), vector graphics.

Languages Basic, C, Python, HTML, PHP, Java, Caml, AppleScript.

Spoken languages

French Mother tongue

English Fluent

Italian Fluent

German Notions - currently learning it.

Publications

Published in peer-reviewed journals:

Holomorphic line bundles over domains in Cousin groups and the algebraic dimension of OT-manifolds, *to appear in Proceedings of the Edinburgh Mathematical Society, preprint at arXiv:1306.3047v1*, with K. Oeljeklaus.

In this paper we extend results due to Vogt on line bundles over Cousin groups to the case of domains stable by the maximal compact subgroup. This is used in the sequel to show that the algebraic dimension of OT-manifolds is zero. In the last part we establish that certain Cousin groups, in particular those arising from the construction of OT-manifolds, have finite-dimensional irregularity.

LVMB manifolds and quotients of toric varieties, *Mathematische Zeitschrift: Volume 275, Issue 1 (2013), pp. 549-568*.

In this article, we study a class of manifolds introduced by Bosio, called LVMB manifolds. We provide an interpretation of his construction in terms of quotient of toric manifolds by complex Lie groups. Furthermore, LVMB manifolds extend a class of manifolds obtained by Meersseman, called LVM manifolds, and we give a characterization of these manifolds using our toric description. Finally, we give an answer to a question asked by Cupit-Foutou and Zaffran.

Surfaces de Stein associées aux surfaces de Kato intermédiaires, *(French) Documenta Mathematica, Volume 16 (2011), pp. 355–371*.

Let S be an intermediate Kato surface, D the divisor consisting of all rational curves of S, $\widetilde{S}$ the universal covering of S and $\widetilde{D}$ the preimage of D in $\widetilde{S}$. We prove two results about the surface $\widetilde{S} \setminus \widetilde{D}$: it is Stein (which was already known when S is either a Enoki or a Inoue-Hirzebruch surface) and we give a necessary and sufficient condition so that its holomorphic tangent bundle is holomorphically trivialisable.

Submitted:

Appendix, *to the article "Nonreciprocal units in a number field with an application to Oeljeklaus-Toma manifolds" by A. Dubickas*.

In his paper, Artūras Dubickas studies a number-theoretic problem that arose from a criterion on the existence of LCK on OT manifolds that I proved. This criterion is linked to the existence of a number field K of degree $s + 2t$ whose group of units $\mathcal{O}_K^*$ contains a subgroup of rank s such that all its elements have their $2t$ complex (non real) embeddings of same modulus.
The first part of the appendix is the proof of this criterion. In the second part, I give an alternative proof of a main result of the article, by using a geometric property of OT manifolds (namely, their non-Kählerianity) in place of a number-theoric argument.
The main article along with the appendix have been written simultaneously and we adopted this "format" of publication.

In preparation:

A generalization of Sankaran and LVMB manifolds, with K. Oeljeklaus.

In this paper we describe the construction of a new class of complex compact manifolds. They can be seen as a generalization of both Sankaran, OT and LVMB manifolds. Moreover, we give first properties about these new spaces.

Some remarks on LVMB manifolds.

In this paper we extend a characterization of LVM manifolds among LVMB ones and also give another possible generalization of this construction.

Administrative duties and responsibilities

2013–2014 Coorganizer of the seminar “Complex Geometry” in Ruhr-Universität Bochum

With V. Tsanov. Program, speakers’ welcoming and management of the seminar’s web page.

2012–2013 Participation to the presentation of Masters in Pure Mathematics to Bachelor students

Students’ welcoming, presentation of courses of study.

2012–2013 Member of the laboratory’s informatics committee (LATP, Marseille, France)

2011–2012 Participation in the making of timetable for BSc in Mathematics, spring semester.

2010–2013 Representative of PhD students to the laboratory’s council (LATP, Marseille, France)

2009–2013 Computer consultancy for laboratory’s members.

Configuration of new hardware, advising before acquiring.

2009 Creation and maintenance of a utility to open VNC connections with the laboratory’s server

Teaching statement

Since september 2009, I have been teaching as Lecturer or Tutor to a wide range of students, in various topics (algebra, analysis, geometry, probability theory and computer science).

In the University I taught to students in Mathematics, Biology, Physics and Engineering Science, from 1st to 3rd year of Bachelor. I also participated to the "Centre de Télé-Enseignement" of Université d'Aix-Marseille, which is a recognized distance education system. In this context, we contact the students using electronic messaging and a Moodle platform. Between september 2012 and june 2013 I also participated to oral interrogations in 1st and 2nd-year of "Classes Préparatoires" (they are part of the French education system and consist of two intensive years, training undergraduate students for enrolment in one of the "grandes écoles", including Polytechnique or Ecoles Normales Supérieures).

The Ecole de l'Air in Salon de Provence is a military school created in 1933, training officers for the French Air Force. This is a "grande école". There, I taught to students coming from "Classes Préparatoires" (EA 1, the future "Patrouille de France" pilots), foreign students (CSEA 1) and also military personnel to whom the success to a 2-year diploma lead to a higher military rank (EMA 1).

The ampleness of profiles of students gave me the possibility to constantly face new situations and renew myself, for the teaching point of view as well as the personal one.

During these first years of teaching I realized how important this is in a researcher's work. First of all, it is his duty to constantly meet new students to share scientific knowledge and give them the will to follow the same path. Moreover, teaching is of utmost importance if one wants to keep a clear way of thinking and explaining things. The best way to understand a concept is to learn it with the objective of teaching it to someone else. During the four past years, I got involved in my teaching duties in many ways. First of all, I always come in front of students after a careful preparation of my lectures, because a teacher has to be perfectly sure of what he is doing. Moreover, I never hesitate to give students appointments to answer their questions. Finally I always got involved in the writing of the exam topics and their corrections.

During my studies I always was interested in computer science and acquired skills in this domain, that let me also teach in this field. A part of my teaching duties was practical work in Maple and Matlab, in particular in Ecole de l'Air. On the other hand I also had to learn how to use the Moodle platform for the CTES.

Below is a detailed list of my teachings since 2009. In 2013/2014 I exceptionally have a post-doctorate without teaching.

2012–2013 **Tutor**, *Maths for engineers*, lecturer: Prof. Boissy.
(1st-year Bachelor's, Univ. Aix-Marseille) - 20 hours/semester

Tutor, *Differential calculus*, lecturer: Prof. Boissy.
(1st-year Bachelor's, Univ. Aix-Marseille) - 48 hours/semester

Lecturer & Tutor, *Initiation to mathematical writing and LaTeX*.
(2nd-year Bachelor's in Mathematics, Univ. Aix-Marseille) - 30 hours/semester

Oral interrogations, in "classes préparatoires", to students of Prof. Renaud and Prof. Rouget.
(MPSI and MP (Math. course), Lycée Notre Dame de Sion, Marseille) - 5 hours/week

2011–2012 **Tutor**, *Complex Analysis*, lecturer: Prof. Youssfi.
(3rd-year Bachelor's in Mathematics, Univ. Aix-Marseille) - 36 hours/semester

Lecturer & Tutor, *Linear Algebra*.
(CSEA 1 - École de l'Air [French Air Force School] in Salon de Provence) - 24 hours/semester

Lecturer & Tutor, *Matrix calculus*.
(EMA 1 - École de l'Air [French Air Force School] in Salon de Provence) - 24 hours/semester

2010–2011 **Tutor**, *Complex Analysis*, lecturer: Prof. Youssfi.
(3rd-year Bachelor's in Mathematics, Univ. Aix-Marseille) - 36 hours/semester

Tutor, *Maths for biology*, lecturer: Prof. Aimar.
(3rd-year Bachelor's in Mathematics, Univ. Aix-Marseille) - 20 hours/semester

Tutor, *Probability theory*, lecturer: Prof. Goffaux.
(EA 1 - École de l'Air [French Air Force School] in Salon de Provence) - 16 hours/semester

Tutor, *Matrix calculus*, lecturer: Prof. Barache.
(EMA 1 - École de l'Air [French Air Force School] in Salon de Provence) - 18 hours/semester

2009–2010 **Tutor**, *Complex Analysis*, lecturer: Prof. Youssfi.
(3rd-year Bachelor's in Mathematics, Univ. Aix-Marseille) - 36 hours/semester+8 hrs of support

Tutor, *Maths for biology*, lecturer: Prof. Aimar.
(3rd-year Bachelor's in Mathematics, Univ. Aix-Marseille) - 20 hours/semester

2008–2009 **Support in Mathematics**, in CTES (Univ. d'Aix-Marseille's distance education system, via the Moodle platform.
(1st, 2nd & 3rd-year Bachelor's in Mathematics, Univ. d'Aix-Marseille)

Research activities

Talks

April 2014 **Séminaire de Géométrie**, *Université de Bordeaux*, Bordeaux, France.

April 2014 **Séminaire de Topologie et de Géométrie algébrique**, *Université de Nantes*, Nantes, France.

Feb. 2014 **Séminaire d'Analyse et Géométrie complexe**, *Université Paul Sabatier*, Toulouse, France.

Feb. 2014 **Séminaire d'Algèbre, Topologie et Géométrie**, *Université Nice Sophia Antipolis*, Nice, France.

Jan/Feb 2014 **Series of talks: "Introduction to OT-manifolds"**, *Ruhr-Universität Bochum*.

Sept. 2013 **Oberseminar komplexe Analysis**, *Ruhr-Universität Bochum*, Bochum, Germany.

April 2013 **Géométrie et Systèmes Dynamiques**, *Université de Bourgogne*, Dijon, France.

Dec. 2012 **Analyse et Géométrie Complexe**, *Université de Lorraine*, Nancy, France.

March 2011 **Algèbre, Dynamique, Géométrie, Topologie**, *Université de Provence*, Marseille, France.

Feb. 2011 **PhD students' seminar**, *Université d'Aix-Marseille*, Marseille, France.

Conferences attended (without talking)

March 2014 **Komplex Analysis Winter School**, CIRM, Marseille, France.

Feb. 2014 **Biannual seminar of SFB**, *Workshop*, Langeoog, Germany.

Nov. 2013 **Transformation Groups and Mathematical Physics**, *Workshop*, Jacobs University, Bremen, Germany.

October 2013 **Géométrie birationnelle des variétés algébriques complexes**, *Conference at CIRM*, Marseille, France.

January 2013 **Komplex Analysis Winter School**, Toulouse, France.

July 2011 **Birational Automorphisms of Varieties of General Type**, *Summer school*, Freiburg, Germany.

Feb. 2011 **Géométrie complexe et riemannienne**, *Conference at CIRM*, Marseille, France.

October 2010 **Géométrie des Variétés complexes IV**, *Conference at CIRM*, Marseille, France.

Seminars attended

2013/2014 **Oberseminar komplexe Geometrie**, *Ruhr-Universität Bochum*.

2013/2014 **Oberseminar komplexe Analysis**, *Ruhr-Universität Bochum*.

2013/2014 **Oberseminar über konforme Feldtheorien**, *Ruhr-Universität Bochum*.

2008/2013 **Algèbre, Dynamique, Géométrie, Topologie**, *LATP - University of Aix-Marseille*.

2009/2013 **Séminaire des doctorants**, *University of Aix-Marseille*.

Stays abroad

July 2012 Invitation for a one-week stay at Ruhr-Universität Bochum.

Currently obtained results

Although the field of non-Kähler geometry remains almost completely unknown as of now, there are constant progresses that are made and many new classes of non-Kähler compact complex manifolds have been constructed and studied recently. During the three years of preparation of my PhD and my first year of post-doctorate, I have studied different classes of non-Kähler compact complex manifolds: surfaces of class VII, LVMB manifolds, Oeljeklaus-Toma manifolds, and finally I constructed a new class of such manifolds.

Kato Surfaces

The first class of manifolds I considered were Kato surfaces and this work led to the publication of an article called « *Surfaces de Stein Associées aux Surfaces de Kato Intermédiaires* » (written in French) in the journal « Documenta Mathematica ».

Surfaces of class VII in Kodaira's classification are compact complex surfaces whose first Betti number is 1 ; we call surface of class VII_0 a surface of class VII which is minimal. The case where the second Betti number b_2 is zero is well understood: these are either Hopf surfaces or Inoue surfaces. However, the case $b_2 > 0$ is still explored nowadays. It was conjectured that these surfaces always contain a global spherical shell. The proof of this result would complete the classification of compact complex surfaces.
Surfaces containing a global spherical shell can be constructed with a procedure due to Kato (see [3]). These surfaces are subdivided into three classes : Enoki surfaces, Inoue-Hirzebruch surfaces and intermediate surfaces. Given a minimal Kato surface S, call D the maximal divisor of S made of the rational curves of S and $\varpi : \tilde{S} \to S$ the universal covering of S. I proved that $\tilde{S} \setminus \varpi^{-1}(D)$ is a Stein manifold. This result was known for Enoki and Inoue-Hirzebruch surfaces, but not in the intermediate case. In the last part of the article I gave a characterization for this manifold to be parallelizable.

LVMB manifolds and construction of new manifolds

The continuation of my research was about a class of manifolds called **LVMB manifolds**. My main objective was to generalize this construction by combining it with a method due to Sankaran, and I did it in my thesis. During this study, a new description of the construction of LVMB manifolds also arose, strongly based on the theory of toric varieties, which led me to the publication of a second article, submitted in july 2012 and published in march 2013 in « Mathematische Zeitschrift », entitled « *LVMB manifolds and quotients of toric varieties* ».

LVMB manifolds
In their article [4], López de Medrano and Verjovsky discovered a family of compact complex manifolds, obtained as quotient of a dense open subset U of $\mathbb{P}_n(\mathbb{C})$ under the action of a complex Lie group isomorphic to $\mathbb{C}$. This construction was then extended to the case of an action of $\mathbb{C}^m$ (where m is a positive integer) by Meersseman in [5] and these new manifolds are called **LVM manifolds**.
Finally, Bosio generalized in [1] the construction of Meersseman by allowing other actions of $\mathbb{C}^m$ on some dense open subset of $\mathbb{P}_n(\mathbb{C})$ and the manifolds that he gets are called **LVMB manifolds**. In short, given a family $\mathcal{E}_{m,n}$ of subsets of $\{0, ..., n\}$ each having $2m+1$ elements (we assume that n and m are positive integers with $2m \leqslant n$) and a family $\mathcal{L}$ of $n+1$ linear forms over $\mathbb{C}^m$ with certain technical conditions, Bosio associates to $\mathcal{E}_{m,n}$ an open subset U of $\mathbb{P}_n(\mathbb{C})$ and to $\mathcal{L}$ an action of $\mathbb{C}^m$ on $\mathbb{P}_n(\mathbb{C})$, such that the quotient $U/\mathbb{C}^m$ is a compact complex manifold. We say that the couple $(\mathcal{E}_{m,n}, \mathcal{L})$ is an **LVMB datum** and that it is an **LVM datum** if the manifold that we get is an LVM manifold.

The first step of my work was to reformulate Bosio's construction in terms of toric geometry. For this, one needs to determine a fan Δ of $\mathbb{R}^n$ such that its associated toric manifold is the open set U and then see how the projection of this fan with respect to a $2m$-dimensional linear subspace of $\mathbb{R}^n$ helps to understand the action of $\mathbb{C}^m$ on U. Heuristically, the set $\mathcal{E}_{m,n}$ corresponds to the fan Δ and the choice of the linear forms over $\mathbb{C}^m$ gives the subspace $\mathbb{R}^{2m}$ of $\mathbb{R}^n$. The converse also works, i.e. with suitable conditions on a fan Δ and the choice of a $2m$-dimensional subspace of $\mathbb{R}^n$, one gets an LVMB datum. Finally, I showed:

Theorem 1. *i) Let $(\mathcal{L}, \mathcal{E}_{m,n})$ be an LVMB datum. Then there is a pair (E, Δ) where E is a $2m$-dimensional linear subspace of $\mathbb{R}^n$ and Δ is a subfan of the fan $\Delta_{\mathbb{P}_n(\mathbb{C})}$ of $\mathbb{P}_n(\mathbb{C})$, satisfying the following two properties:*
$a)$ the projection map $\pi : \mathbb{R}^n \to \mathbb{R}^n/E \cong \mathbb{R}^{n-2m}$ is injective on $|\Delta|$,
$b)$ the fan $\pi(\Delta)$ is complete in $\mathbb{R}^n/E$, i.e. $|\pi(\Delta)| = \mathbb{R}^n/E$.
ii) Conversely, given a pair (E, Δ) having the two properties above, one obtains an LVMB datum.

For the proof of this result, I used the notion of *manifold with corners* that I had to extend to the case of a non-necessarily rational fan.

In [1], Bosio also gives a criterion to decide whether an LVMB datum is an LVM datum or not. The second objective of my article is to translate this criterion in the toric "language" of the construction, this is the following theorem:

Theorem 2. *Let $(\mathcal{L}, \mathcal{E}_{m,n})$ be an LVMB datum and (E, Δ) its associated pair given by theorem 1. Then, $(\mathcal{L}, \mathcal{E}_{m,n})$ is an LVM datum if and only if the projection by E of the fan Δ is polytopal.*

Finally, it was possible to use this criterion to show that if two LVMB manifolds, say X and Y, are biholomorphic and if X is an LVM manifold, then Y is also an LVM manifold. This question was asked by Cupit-Foutou and Zaffran in [2], where a partial answer was provied (see remark (ii), p. 786, ibid.). I completely answered it with the following statement:

Theorem 3. *Let $(\mathcal{L}_1, \mathcal{E}_{m,n})$ and $(\mathcal{L}_2, \mathcal{E}'_{m',n'})$ be two LVMB data giving two biholomorphic LVMB manifolds, then $n = n'$, $m = m'$ and $(\mathcal{L}_1, \mathcal{E}_{m,n})$ is an LVM datum if and only if $(\mathcal{L}_2, \mathcal{E}'_{m,n})$ is an LVM datum.*

Construction of new manifolds
In my thesis I also described the construction of a new class of non-Kähler compact complex manifolds by a combination of a method of Sankaran (see [7]) and the one giving LVMB manifolds. Sankaran takes an open subset U of a toric manifold whose quotient by a discrete group W is a compact manifold.
Here, we endow some complex manifold Y with the action of a complex Lie subgroup G of $(\mathbb{C}^*)^n$ such that the quotient X of Y by G is a (non-necessarily compact) complex manifold. The second step consists of taking the quotient of an open subset U of X by a discrete group W obtained in a similar fashion to Sankaran's one. Call $Z := U/W$ this quotient.

The complex manifold X has an open dense subset which is a Cousin group (i.e. a complex Lie group having no non-constant holomorphic function), this is the quotient of $(\mathbb{C}^*)^n$ by G. This information is crucial to understand the geometry of the manifold Z. One proves that it is non-Kähler and that it has Kodaira dimension equal to $-\infty$. Recently, I was also able to compute other invariants such as the second Betti number of this manifold and with K. Oeljeklaus we showed that the algebraic dimension is zero thanks to this information. This is the subject of an article I am finishing with K. Oeljeklaus, that we are submitting soon.

Thanks to the use of "generalized" manifold with corners, I could also exhibit a fundamental compact domain in U for the action of the group W. Sankaran talks about this possibilty in [7] to show that his manifolds are compact, but that didn't seem practical to him (he proved the compacity with a cohomological argument). As a consequence, one can also concretely write a compact fundamental domain in Sankaran's case, which can be seen as a progress.

OT manifolds

Finally, I focused on OT manifolds. These manifolds are another class of non-Kähler compact complex manifolds, for which I established that their algebraic dimension is zero. OT manifolds were described by Oeljeklaus and Toma in [6]. They are constructed the following way: first choose a number field K of degree $n = s + 2t$ over $\mathbb{Q}$ (with $s > 0$ and $2t > 0$ being respectively the number of real and complex embeddings of K). We call $\sigma_1, ..., \sigma_s$ the real embeddings of K and $\sigma_{s+1}, ..., \sigma_n$ the complex ones, ordered in such a way that $\sigma_{s+j} = \overline{\sigma_{s+t+j}}$ for all $j \in \{1, ..., t\}$ and we define $\sigma : K \to \mathbb{C}^m$ (where $m := s + t$) by $\sigma(k) := (\sigma_1(k), ..., \sigma_m(k))$. Take the quotient of $\mathbb{H}^s \times \mathbb{C}^t$ by the semi-direct product of the lattice $\sigma(\mathcal{O}_K)$ of integers of K and a suitable subgroup of $\mathcal{O}_K^*$.

In order to prove that their algebraic dimension is zero, I first extended results due to Vogt (see [8]) concerning holomorphic line bundles over Cousin groups. Some of these results generalize to an open subset of a Cousin group, invariant under the action of the maximal compact subgroup. These results then apply to $(\mathbb{H}^s \times \mathbb{C}^t)/\sigma(\mathcal{O}_K)$, which is such an open set.

I am currently still working on OT-manifolds. First, I recently gave a characterization of the existence of LCK metrics on these manifolds, formulated as a number-theoretic condition. This led to a discussion with Professor Dubickas who wrote an article on it, and the criterion of existence of LCK metrics appears in the appendix of his article, along with a reformulation of one of his proofs using geometrical arguments.
Recently, I also computed the second Betti number of OT manifolds, which was only known for a particular class of these manifolds since 2005.

Research project

The research work I carried until now leads to the following questions, that I am already looking for, and that I would like to be able to pursue in the years to come:

Manifolds with corners
An important part of my thesis is devoted to the extension of the notion of the"manifold with corners" associated to a fan, by removing the rationality condition on the fan.

First, it is possible to extend many results about the topology of these generalized manifolds with corners by looking at what is happening in the toric case. It would be interesting to give as many such informations as possible, because I believe that this tool can be of use for further studies of quotients of complex manifolds under the action of complex Lie groups.
Moreover, these spaces carry a semi-algebraic structure, which I must study in details.

LVMB manifolds
In [1] and [2], the authors characterized LVM manifolds among LVMB ones, with a technical hypothesis called "condition (K)". Thanks to the new description of LVMB manifolds given in my thesis, I was able to keep one of these characterizations to be true without this condition. I plan to extend the other characterizations again without this condition, but the approaches seem different. In particular, the existence of a transversely Kähler foliation is such a criterion.
I also introduced a "generalization" of LVMB manifolds (it is the same construction, but one replaces $\mathbb{P}_n(\mathbb{C})$ by any other compact toric manifold); one can ask what are these manifolds' properties and if they are already known spaces.

New manifolds and OT manifolds
As for the new manifolds constructed in my thesis, there are many invariants left to find (dimension of $H^1(\mathcal{O})$, cohomology groups...) and other geometrical and topological properties to explore (existence of foliations, metrics, deformations...). As it has already happened, the techniques developed will certainly adapt to LVMB, OT and Sankaran manifolds, with a strong dependence on the fan of the toric manifold that appears in each construction.

OT manifolds still have properties that remain mysterious. For instance I gave a criterion for the existence of an LCK metric on these manifolds, but it is not known yet whether there exist non-trivial examples. The answer seems to be negative according to the partial results obtained by A. Dubickas, but the problem is always open.

Kato surfaces
Finally, the study of Stein surfaces arising in the construction of Kato surfaces gave a class of parallelizable Stein manifolds. It is conjectured that every such manifold is a Riemann domain. These examples need to be studied, as it is not decided yet if they are Riemann domains or not. They could either be counterexamples to this conjecture or give a hint for the proof of the general case, which would be of utmost importance in both cases.

References

[1] F. Bosio, *Variétés complexes compactes : une généralisation de la construction de Meersseman et López de Medrano-Verjovsky*, Ann. Inst. Fourier (Grenoble) **51** (2001), no. 5, 1259–1297.

[2] S. Cupit-Foutou and D. Zaffran, *Non-Kähler manifolds and GIT-quotients*, Math. Z. **257** (2007), no. 4, 783–797.

[3] Ma. Kato, *Compact complex manifolds containing "global" spherical shells*, Proceedings of the International Symposium on Algebraic Geometry (Kyoto Univ., Kyoto, 1977), Kinokuniya Book Store, Tokyo (1978), 45–84.

[4] S. López de Medrano and A. Verjovsky, *A new family of complex, compact, non-symplectic manifolds*, Bol. Soc. Brasil. Mat. (N.S.) **28** (1997), no. 2, 253–269.

[5] L. Meersseman, *A new geometric construction of compact complex manifolds in any dimension*, Math. Ann. **317** (2000), no. 1, 79–115.

[6] K. Oeljeklaus and M. Toma, *Non-Kähler compact complex manifolds associated to number fields*, Ann. Inst. Fourier (Grenoble) **55** (2005), no. 1, 161–171.

[7] G. K. Sankaran, *A class of non-Kähler complex manifolds*, Tohoku Math. J. (2) **41** (1989), no. 1, 43–64.

[8] C. Vogt, *Line bundles on toroidal groups*, J. Reine Angew. Math. **335** (1982), 197–215.