Geometric Quantization

Seminar

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Description of the module

Title: Geometric Quantization

Code: MS

Credits: 6 CP

Duration: One semester

Language: English

Cycle: Irregular

Format: Seminar 2SWS + Tutoring 2SWS, active and passive participation at talks.

Workload: 180 h, thereof 30h presence and 150h preparation.

Course of study: B.Sc. Mathematics, M.Sc. Mathematics

Goal: Read and research mathematical literature. Work on a mathematical topic in an autonomous way. Explain and present mathematical questions to a small audience in an understandable and engaging way.

Content: Quantization describes the process of assigning a quantum system to a given classical system. Even though there is no general recipe working in all cases, in the last fifty years a successful mathematical approach, known as Geometric Quantization, has been developed. Such an approach entails the following three steps: Prequantization, Polarization, and Metaplectic correction. Prequantization produces a natural Hilbert space and transforms Poisson brackets of functions on the classical side into commutators of operators on the quantum side. Nevertheless, the prequantum Hilbert space is generally "too large" to be physically meaningful. Therefore, the choice of a polarization and, in some cases, the introduction of a metaplectic correction are needed to get the right quantum Hilbert space. Each step will be clarified using concrete examples such as the harmonic oscillator and the spin of a particle. Following the story of geometric quantization we will learn about many fascinating and crucial mathematical concepts such as

1. Symplectic manifolds and Hamiltonian formalism
2. Hermitian line bundles and their characterization via the first chern class
3. Integrable distributions (in particular Lagrangian distributions)
4. Metaplectic group and metaplectic structures
5. Theory of unitary representations

Time permitting we could trace back the origin of geometric quantization to Kirillov's orbit method, compare geometric quantization with other types of quantization (Berezin-Toeplitz, deformation quantization), or discuss how to recover the classical from the quantum world via the semi-classical limit and the WKB approximation.

Prerequisites: Linear Algebra 1-2, Analysis 1-3, Basic knowledge of Differential Geometry and Topology (manifolds, vector fields and differential forms, vector bundles), and Functional Analysis.

Examination: 90min graded talk, active and passive participation in other talks.

Useful literature: *(Individual source for the talk will be provided later)*

A. Echeverría-Enríquez, M. C. Munoz Lecanda, N. Roman-Roy, and C. Victoria-Monge, *Mathematical Foundations of Geometric Quantization*, 1999.

B. C. Hall, *Quantum Theory for Mathematicians*, Springer, 2013.

A.A.Kirillov, *Lectures on the Orbit Method*, AMS, Graduate Studies in Mathematics Volume: 64; 2004

S. Bates and A. Weinstein, *Lectures on the Geometry of Quantization*

N. M. J. Woodhouse, *Geometric Quantization*, Clarendon Press, 1997