

# **EQUILIBRIUM OF LANE FLOW-DISTRIBUTION ON MOTORWAYS**

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## **EQUILIBRIUM OF LANE FLOW-DISTRIBUTION AND ITS IMPACT ON CAPACITY ON MOTORWAYS**

### **ABSTRACT**

The distribution of traffic flow in the individual lanes of multilane motorways is an important investigation task in traffic engineering, because the lane flow-distribution affects directly the capacity of the motorway section under investigation. Based on the basic idea of the previous work from Heidemann, the flow-distribution in the individual lanes is re-derived and extended to carriageways with more than three lanes. The Model is then calibrated with field data obtained from Germany. In addition, the proposed model is extended to congested traffic conditions. The proposed model can therefore be applied for entire flow conditions in the reality.

### **Keywords:**

lane flow-distribution, motorway capacity

## INTRODUCTION

The distribution of traffic flow in the individual lanes of multilane motorways is an important parameter in traffic engineering. The lane flow-distribution can directly influence the capacity of the freeway section under investigation. Different works regarding this problem were given by Breuer and Beckmann (1), Hotop (2), Sparmann (3), Leutzbach and Busch (4), Hall and Lam (5), Heidemann (6), and other authors. These investigations revealed that in the presence of high volumes more vehicles travel on the median lane rather than on the shoulder lane. Thus, it is interesting to develop a method to measure and improve the capacity of multilane carriageways and to explain the phenomena of traffic congestion. It is well-known that higher total capacity could be possible if more balanced lane utilization could be established.

Compared to the other works, that from Heidemann (6) has a better theoretical background. In his work, the distribution of the total traffic flow in individual lanes was derived mathematically. Results for two-lane and three-lane carriageways were achieved. The frequency of lane changing was also considered in his work. Unfortunately, the mathematical derivation in his work contains a minor error, although this does not significantly affect the numerical results in his work.

In the presented paper, the flow-distribution in the individual lanes is re-derived and extended to carriageways with more than three-lanes. The Model is then calibrated with field data obtained from Germany.

## LANE FLOW-DISTRIBUTION AND GAP DISTRIBUTION

### Gap distribution in traffic flow

In order to derive the lane flow-distribution, we should take into account the distribution of time gaps between vehicles in the traffic flow. In general, the gaps in a traffic flow can be described with a distribution function  $f(t) = f(t, q)$ , where  $t$  is the length of the gap and  $q$  the traffic flow rate. For example, the probability distribution function and the probability density function for the partially bunched traffic flow is given by Cowan's M3 model (Cowan, 7, 8).

Cowan's M3 model is a dichotomized distribution as shown in cumulative form in Equation (1):

$$F(t) = \begin{cases} 1 - \alpha e^{-\lambda(t-\Delta)} & \text{for } t \geq \Delta \\ 0 & \text{for } t < \Delta \end{cases} \quad (-) \quad (1)$$

where:  $t$  is the sample gap (s)

$\Delta$  is the minimum gap within bunches (s)

$\alpha$  is the proportion of non-bunched vehicles (-)

$$\lambda = \frac{\alpha q}{1 - \Delta q} \text{ is the apparent flow rate (veh/s)} \quad (2)$$

$q$  is the flow rate (veh/s)

The value  $\lambda$  represents the flow rate within the non-bunched vehicles.

The parameter  $\alpha$  has values between 0 and 1. It represents the proportion of non-bunched vehicles in the traffic flow. Between these vehicles, the time gaps are always larger than  $\Delta$ . The remaining vehicles of proportion  $(1 - \alpha)$  are bunched and between these vehicles the time gaps are always equal to  $\Delta$ . For example, we can use  $\alpha = 1 - q \cdot \Delta$  and  $\lambda = q$  (cf. Tanner, 9) for normal traffic conditions without impedance from traffic signals. In this case Equation (1) yields

$$F(t) = \begin{cases} 1 - (1 - q \cdot \Delta) e^{-q(t-\Delta)} & \text{for } t \geq \Delta \\ 0 & \text{for } t < \Delta \end{cases} \quad (-) \quad (3)$$

Usually, for simplification, we can also use  $\alpha = 1$  (i.e., between all vehicles, the time gaps are larger than  $\Delta$ ). Then we have the negative shifted-exponential distribution for the gap-distribution:

$$F(t) = \begin{cases} 1 - e^{\frac{-q}{1-\Delta q}(t-\Delta)} & \text{for } t \geq \Delta \\ 0 & \text{for } t < \Delta \end{cases} \quad (-) \quad (4)$$

Equation (3) and Equation (4) are illustrated in Figure 1. We can see that both equations deliver nearly the same values for  $t > 2s$ . Because only gaps larger than 2s are relevant for traffic flow analysis, we can consider both approaches as nearly equivalent. In the following, we use Equation (4) for describing the gap-distribution on motorways.

### Lane flow-distribution on motorways

Given the total flow rate of a motorway,  $q_{\text{sum}}$ , the relationship between the proportion of traffic flow rates,  $p_1, p_2, \dots$ , on the different traffic lanes 1, 2 ... can be calculated as a function of the total flow rate  $q_{\text{sum}}$ . Hence, the following relationship always holds:

$$\begin{cases} p_1 = q_1 / q_{\text{sum}} \\ p_2 = q_2 / q_{\text{sum}} \\ \dots \\ p_1 + p_2 + \dots = 1 \end{cases} \quad (-) \quad (5)$$

Where the values of  $p_1, p_2, \dots$  are functions of  $q_{\text{sum}}$ .

Let  $P_q(i)$  be the probability that a vehicle is in lane  $i$  at traffic flow rate  $q$  ( $i = 1$  refers to the shoulder lane), i.e.  $P_q(i)$  is the proportion of vehicles in lane  $i$  at flow rate  $q$ . Let  $P_q(i, j)$  be the transition probability that a vehicle after a sufficient long section travels in lane  $j$  if it had been in lane  $i$  at the beginning of the section.

If the section is infinitely long, the following condition of equilibrium must be satisfied (cf. Heidemann, 6) for arbitrary two adjacent lanes:

$$P_q(i)P_q(i, i+1) = P_q(i+1)P_q(i+1, i) \quad (-) \quad (6)$$

or

$$P_q(i) = \frac{P_q(i+1, i)}{P_q(i, i+1)} P_q(i+1) = Q_q(i, i+1)P_q(i+1) \quad (-) \quad (7)$$

with

$$Q_q(i, i+1) = \frac{P_q(i)}{P_q(i+1)} = \frac{P_q(i+1, i)}{P_q(i, i+1)} \quad (-) \quad (8)$$

For all lanes together, the following restraint holds:

$$\sum P_q(i) = 1 \quad (-) \quad (9)$$

In general, for a carriageway with  $n$  lanes, the following equation is true

$$\begin{aligned}
 P_q(i) &= \frac{P_q(i+1, i)}{P_q(i, i+1)} P_q(i+1) \\
 &= \frac{P_q(i+1, i)}{P_q(i, i+1)} \frac{P_q(i+2, i+1)}{P_q(i+1, i+2)} P_q(i+2) \\
 &= \dots \\
 &= \left( \prod_{k=0}^{n_l-1} Q_q(i+k, i+k+1) \right) P_q(i+n_l+1)
 \end{aligned} \tag{10}$$

Where  $n_l$  is the number of the neighbor lanes on the left-hand side (for the right-hand traffic).

Unfortunately, the further derivation in the work Heidemann (Heidemann, 6) is incorrect because a term which is always equal to zero is divided by in his derivation (cf. Wu, 2005). The corrected derivation is given as following.

In case of a 2-lane carriageway we have always

$$P_q(1) + P_q(2) = 1 \tag{11}$$

$$P_q(1)P_q(1,2) = (1 - P_q(1))P_q(2,1) = P_q(2,1) - P_q(1)P_q(2,1) \tag{12}$$

$$P_q(1)(P_q(1,2) + P_q(2,1)) = P_q(2,1) \tag{13}$$

$$P_q(1) = \frac{P_q(2,1)}{P_q(1,2) + P_q(2,1)} \tag{14}$$

According to Heidemann's derivation is  $P_q(1) = P_q(2,1)$ . Since  $P_q(1,2) + P_q(2,1) \neq 1$ , thus,  $P_q(1) \neq P_q(2,1)$ . Heidemann's result is incorrect.

Thus, we have from eqs. (11) and (14)

$$\begin{cases} P_q(1) = \frac{P_q(2,1)}{P_q(1,2) + P_q(2,1)} \\ P_q(2) = 1 - P_q(1) \end{cases} \tag{15}$$

Or with a better expression

$$\begin{cases} \frac{P_q(2)}{P_q(1)} = \frac{P_q(1,2)}{P_q(2,1)} \\ P_q(2) = 1 - P_q(1) \end{cases} \quad (-) \quad (16)$$

In case of a 3-lane carriageway we have

$$\begin{cases} \frac{P_q(2)}{P_q(1)} = \frac{P_q(1,2)}{P_q(2,1)} \\ \frac{P_q(3)}{P_q(2)} = \frac{P_q(2,3)}{P_q(3,2)} \\ P_q(3) = 1 - P_q(2) - P_q(1) \end{cases} \quad (-) \quad (17)$$

Heidemann gives the following formulation for the 3-lane carriageway:

$$\begin{aligned} \frac{P_q(2)}{P_q(2) + P_q(3)} &= P_q(3,1) \\ \frac{P_q(1)}{P_q(1) + P_q(2)} &= P_q(2,1) \\ P_q(1) + P_q(2) + P_q(3) &= 1 \end{aligned}$$

This result is incorrect too.

For the right-hand traffic, the probability that a vehicle changes from a left-hand lane  $i+1$  to a right-hand lane  $i$  is the probability that there is a gap  $t \geq t_{0,i+1,i}$  in the right-hand lane (commandment of driving on right), where  $t_{0,i+1,i}$  is the net gap for changing maneuver from lane  $i+1$  to lane  $i$ . The probability that a vehicle changes from a right-hand lane  $i$  to a left-hand lane  $i+1$  is the probability that the gap in front of the vehicle  $t < t_{0,i+1,i}$  (demand of overtaking) and there is a gap  $t \geq t_{0,i,i+1}$  (possibility of a lane-changing) in the left-hand lane. The demand of overtaking can be expressed by the predefined gap  $t_{0,i+1,i}$  in front of the vehicle because if the current gap  $t$  is approaching the value  $t_{0,i+1,i}$ , the vehicle must change lane in order to maintain its desired speed and gain speed advantage. Thus, the following conditions are true

$$\begin{aligned}
P_q(i+1, i) &= \Pr(t_i \geq t_{0,i+1,i}) \\
P_q(i, i+1) &= \Pr(t_i < t_{0,i+1,1}) \Pr(t_{i+1} \geq t_{0,i,i+1})
\end{aligned}
\tag{18}$$

Therefore, for M3 distributed gaps, we have

$$\begin{aligned}
P_q(i+1, i) &= \alpha_i \exp(-\lambda_i t_{0,i+1,i}) \\
P_q(i, i+1) &= [1 - \alpha_i \exp(-\lambda_i t_{0,i+1,1})] \alpha_{i+1} \exp(-\lambda_{i+1} t_{0,i,i+1})
\end{aligned}
\tag{19}$$

For the negative-exponentially distributed gaps ( $\alpha_i = 1$ ), we have

$$\begin{aligned}
P_q(i+1, i) &= \exp(-\lambda_i t_{0,i+1,i}) \\
P_q(i, i+1) &= [1 - \exp(-\lambda_i t_{0,i,i+1})] \exp(-\lambda_{i+1} t_{0,i,i+1})
\end{aligned}
\tag{20}$$

Obviously, the values of the proportion of traffic flow rates,  $p_1=P_q(1)$ ,  $p_2=P_q(2)$ ,..., can only be obtained by iteration calculations using the equation systems (16) and (17), because the probabilities  $P_q(i)$  (cf. Equations (19)) are functions of  $P_q(i)$  themselves. The values of  $t_0$  and  $\Delta$  are parameters which should be calibrated based on field data. For sample motorways in Germany, parameters for 2- and 3-lane carriageways are obtained by calibrating the model with field data (here, with  $\alpha_i = 1$ ). These parameters can be considered as representative for the average traffic condition on German long-distance motorways (cf. Table 1 and Table 2).

In Figure 2 and Figure 3, lane flow-distributions for the sample 2- and 3-lane carriageways in Germany are illustrated. We can see that the proposed model represents the field data very well.

For four- and five-lane carriageways, there are no suitable field data available at time (the only one available data set from motorway (A5) with 4 lanes per direction in Germany finds no application because a speed limit of 120 km/h was installed there). To demonstrate the characteristics of these roadways, the parameters for the three-lane carriageways are used as initial values for the first two right-hand lanes and the most left-hand lane. Other lanes in between are considered similar to the second lane of a three-lane carriageway (cf. Table 3 and Table 4).

Using the parameter given in Table 3 and Table 4, the flow-distributions to the individual lanes are presented in Figure 4 and Figure 5. These figures show the schematic shape of the



lane flow-distribution. For more detailed analysis, the parameters need more calibrations with the field data.

Equations (16) and (17) are derived from the gap-acceptance theory. They have a good theoretical background. Unfortunately, the solution of these equations needs iterative computations.

The in Figure 2 and Figure 3 illustrated lane flow-distributions represent average traffic conditions on German motorways. In other European countries, the lane volume distribution on motorways may have a comparable characteristic as those in Germany. We can recognize in these figures, that under heavy traffic conditions (near capacity) only 32% (compared to 50% for a uniform distribution) of the total traffic flow is traveling on the first (most right) lane of a two-lane carriageway (Figure 2). For a three-lane carriageway, it is only 21% (Figure 3, compared to 33% for a uniform distribution). According to the model calculation, the most-right lane on a four-lane carriageway has only 16% of the total flow rate (Figure 4, compared to 25% for a uniform distribution). For a five-lane carriageway, it is only 13% (Figure 5, compared to 20% for a uniform distribution).

Assuming a maximum lane capacity for the most occupied lane, the total capacity of a carriageway can be estimated from the lane flow-distribution at capacity. For example, for the lane flow-distributions from Figure 2 through Figure 5, the total and average lane capacities for carriageways with 2 through 5 lanes can be estimated as shown in Table 5 (with  $C_{\max, \text{ln}} = 2400$  veh/h from the most left lane). It can be recognized here, that the average lane capacity increases with increasing number of lanes (cf. also TRB, 2000 and FGSV, 2001).

Of course the lane flow-distribution is different from country to country because different traffic behaviors and regulations. The typical shape of lane flow-distribution on German motorways is a consequence of the strict regulations of "Commandment of Driving on Right" and "Prohibition of Overtaking on Right". In general, the country-related lane flow-distribution should be used for further calculations.

Equations (18) and (19) are derived from the gap-acceptance theory. They have a good theoretical background. Unfortunately, these equations need iterative computations to be solved. Therefore, equations (18) and (19) are very difficult for practical applications. For

simplicity, equations (16) and (17) can be replaced by a generalized regression function (21) for a carriageway with  $n$  lanes. The resulting deviations are not significant.

$$\begin{cases} p_i = a(1 - b \cdot e^{-c \cdot q_{sum}^d}) q_{sum}^{-e} & \text{for } i \neq 1 \\ p_1 = 1 - \sum_{i=2}^n p_i & \text{for } i = 1 \end{cases} \quad (-) \quad (21)$$

The parameter  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$  for the sample motorways are given in Table 6.

Figure 6 through Figure 9 show the shape of the lane flow-distribution from equation (21) for motorways. We can see here that the generalized regression function (21) fits the field data (Figure 6 and Figure 7 for 2- and 3-lane carriageway) and the theoretical model (Figure 8 and Figure 9 for 4- and 5-lane carriageway) perfectly. Thus, this function can be recommended for fitting real lane flow-distributions under arbitrary traffic conditions.

Once the proportion of the traffic flow rates,  $p_1, p_2, \dots$ , are obtained, the traffic flow rate of the different lanes,  $q_1, q_2, \dots$ , can be calculated by the function

$$\begin{cases} q_1 = p_1 \cdot q_{sum} \\ q_2 = p_2 \cdot q_{sum} \\ \dots \\ \dots \end{cases} \quad \text{(veh/s)} \quad (22)$$

Obviously, the lane flow-distribution can only be calculated up to a certain maximum limit of traffic flow rates. Normally, this limit is the capacity of the motorway. Beyond this limit, all functions derived above are not defined because the apparent flow rate,  $\lambda$ , is no more defined. Thus, the lane distribution of flow rate in congested conditions cannot yet be obtained from this model. In order to solve this problem, we can extend the model to the congested traffic flow by redefining the minimum headway  $\Delta$  as a function of the actual average speed  $v$  of the traffic flow and therefore – using the Fundamental Diagram – as a function of the traffic density  $k$ . The minimum headway between two consecutive moving vehicles can be defined as

$$\Delta = t_{ra} + \frac{l_{veh}}{v} \quad , \quad (s) \quad (23)$$

Where  $t_{ra}$  is the reaction time (including a portion of safety time) of a driver and  $l_{veh}$  is length of a model vehicle (including a portion of safety length). Thus, the apparent flow rate is

$$\lambda = \frac{\alpha q}{1 - \Delta q} = \frac{\alpha q}{1 - \left(t_{ra} + \frac{l_{veh}}{v}\right)q} = \frac{\alpha vk}{1 - \left(t_{ra} + \frac{l_{veh}}{v}\right)vk} = \frac{\alpha vk}{1 - (t_{ra}vk + l_{veh}k)} \quad (\text{veh/s}) \quad (24)$$

Now, the parameter  $\lambda$  is defined in the entire range of  $0 \leq k \leq k_{\max}$ .

In Figure 10, the lane flow-distribution for the entire range of traffic – that is, both for the free flow and for the congested condition – is illustrated for the two-lane carriageway. In Figure 11, the lane flow-distribution for the entire range of traffic flow is illustrated for the three-lane carriageway. For this calculation, a Fundament Diagram based on the ideas from van Aerde (11) is used. The lane distribution of flow rate presented in Figure 10 fulfills the necessary boundary conditions of  $p_1=1$  and  $p_2=0$  for  $k=0$  and  $p_1=p_2=0.5$  for  $k=k_{\max}$ . In Figure 11, the necessary boundary conditions of  $p_1=1$ ,  $p_2=0$ , and  $p_3=0$  for  $k=0$  and  $p_1=p_2=p_3=0.33$  for  $k=k_{\max}$  are fulfilled.

## CONCLUSIONS AND OUTLOOK

For analyzing traffic flow on motorways, the approach from Heidemann (6) is re-derived more exactly. Some corrections are given. This approach is extended to motorways with more than three lanes in each direction. It delivers a useful method for flow and capacity analysis. This approach can be used for carriageways with arbitrary many lanes. For selected motorways with up to five lanes, the approach is calibrated or validated. This approach is also extended to congested traffic conditions. For applications in the practice, a generalized regression function is given to represent the theoretical approach, which can only be solved by iteration calculations. This regression function can be recommended for fitting real lane flow-distributions under arbitrary traffic conditions.

The approach is fully compatible to the Cell Transmission model (Daganzo, 12, 13) und to the NetCell simulation package (Daganzo et al, 14) in order to take into account the lane changing behavior. Combined with the proposed model, the NetCell simulation package can be extended to multilane traffic flow.

The proposed model can also be used – in combination with the continuum/hydrological model – in the Finite Elements and Cellular Automaton techniques for taking into account the lane changing behavior.

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**Table 1- Calibrated parameters for the sample two-lane carriageway in Germany**

Lane Changing from $i$ to $j$		$t_{0,ij}$	
	$\Delta i$	$j=1$	$j=2$
$i=1$	0.00	-	1.15
$i=2$	1.07	3.83	-



**Table 2- Calibrated parameters for the sample three-lane carriageway in Germany**

Lane Changing from $i$ to $j$		$t_{0,ij}$		
	$\Delta i$	$j=1$	$j=2$	$j=3$
$i=1$	0.00	-	2.76	-
$i=2$	1.02	7.71	-	2.25
$i=3$	0.00	-	2.76	-

**Table 3 - Presumed parameters for four-lane carriageways in Germany**

Lane Changing from $i$ to $j$		$T_{0,i,j}$			
	$\Delta i$	$j=1$	$j=2$	$j=3$	$j=4$
$i=1$	0.00	-	2.76	-	-
$i=2$	1.02	7.71	-	2.76	-
$i=3$	1.02	-	2.76	-	2.25
$i=4$	0.00	-	-	2.76	-

**Table 4- Presumed parameters for five-lane carriageways in Germany**

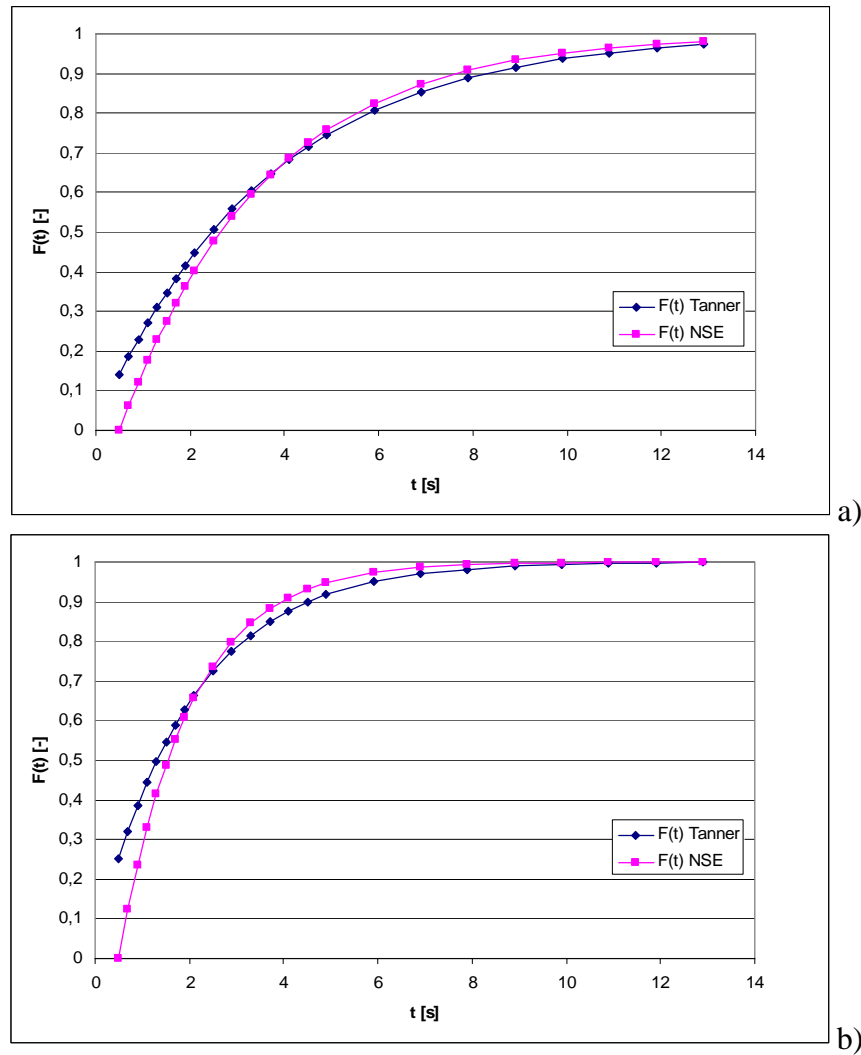
Lane Changing from $i$ to $j$		$t_{0,i,j}$				
		$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
$i=1$	0.00	-	2.76	-	-	-
$i=2$	1.02	7.71	-	2.76	-	-
$i=3$	1.02	-	2.76	-	2.76	-
$i=4$	1.02	-	-	2.76	-	2.25
$i=5$	0.00	-	-	-	2.76	-

**Table 5- Capacity estimation from the lane flow-distribution in Germany**  
**(cf. Figure 2 to Figure 5).**

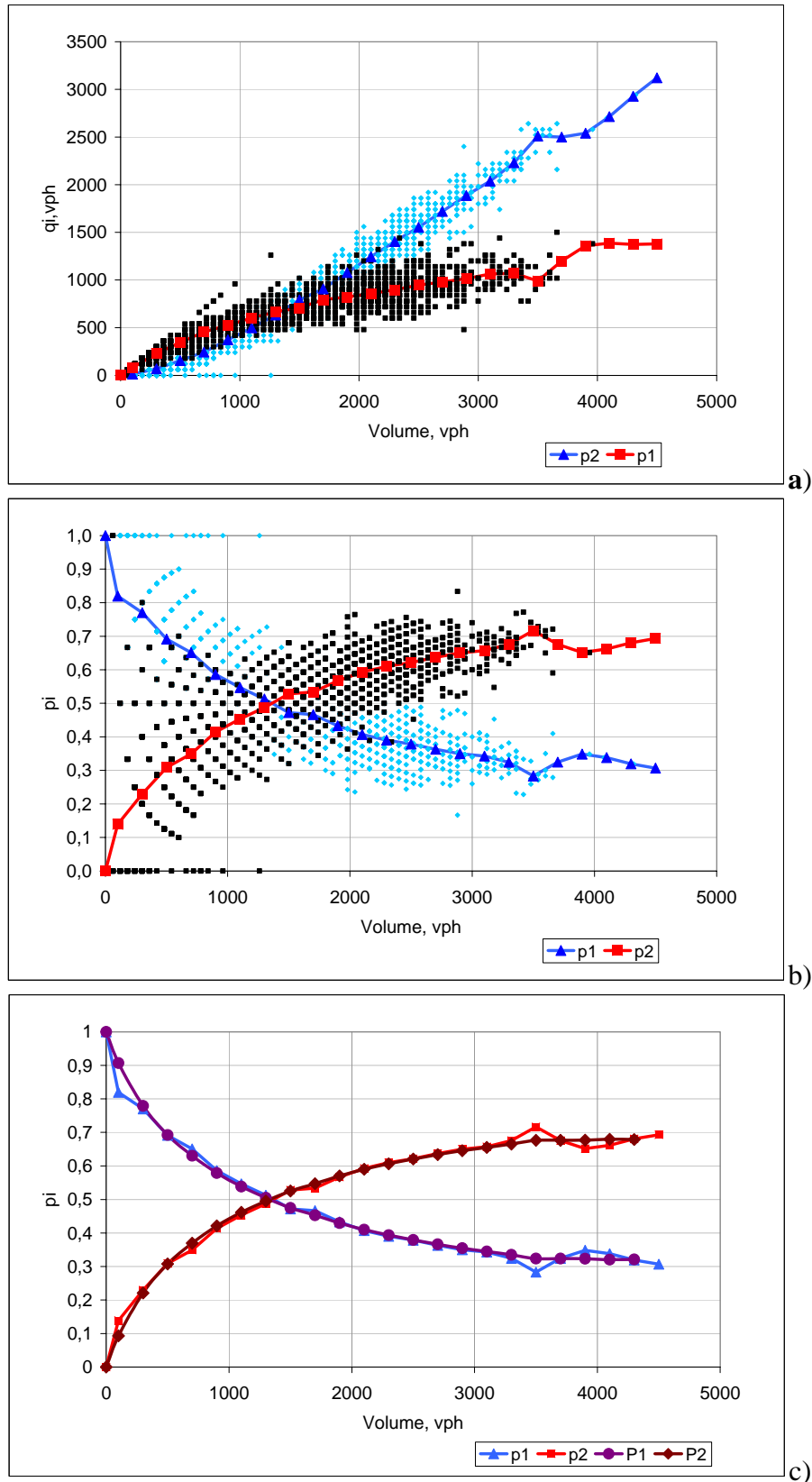
		Proportion of lane flow (%)					Average capacity	
Number of lanes, $n$	lane $C_{\max}$ (veh/h)	$j=1$	$j=2$	$j=3$	$j=4$	$J=5$	$C_{\text{total}}$ (veh/h)	$C_{\text{ln}}$ (veh/h)
2	2400	32	68	-	-	-	3547	1773
3	2400	21	35	44	-	-	5407	1802
4	2400	16	26	25	32	-	7390	1847
5	2400	13	21	20	20	26	9378	1876

**Table 6- Parameters of the regression function (21) for the sample motorways in Germany**

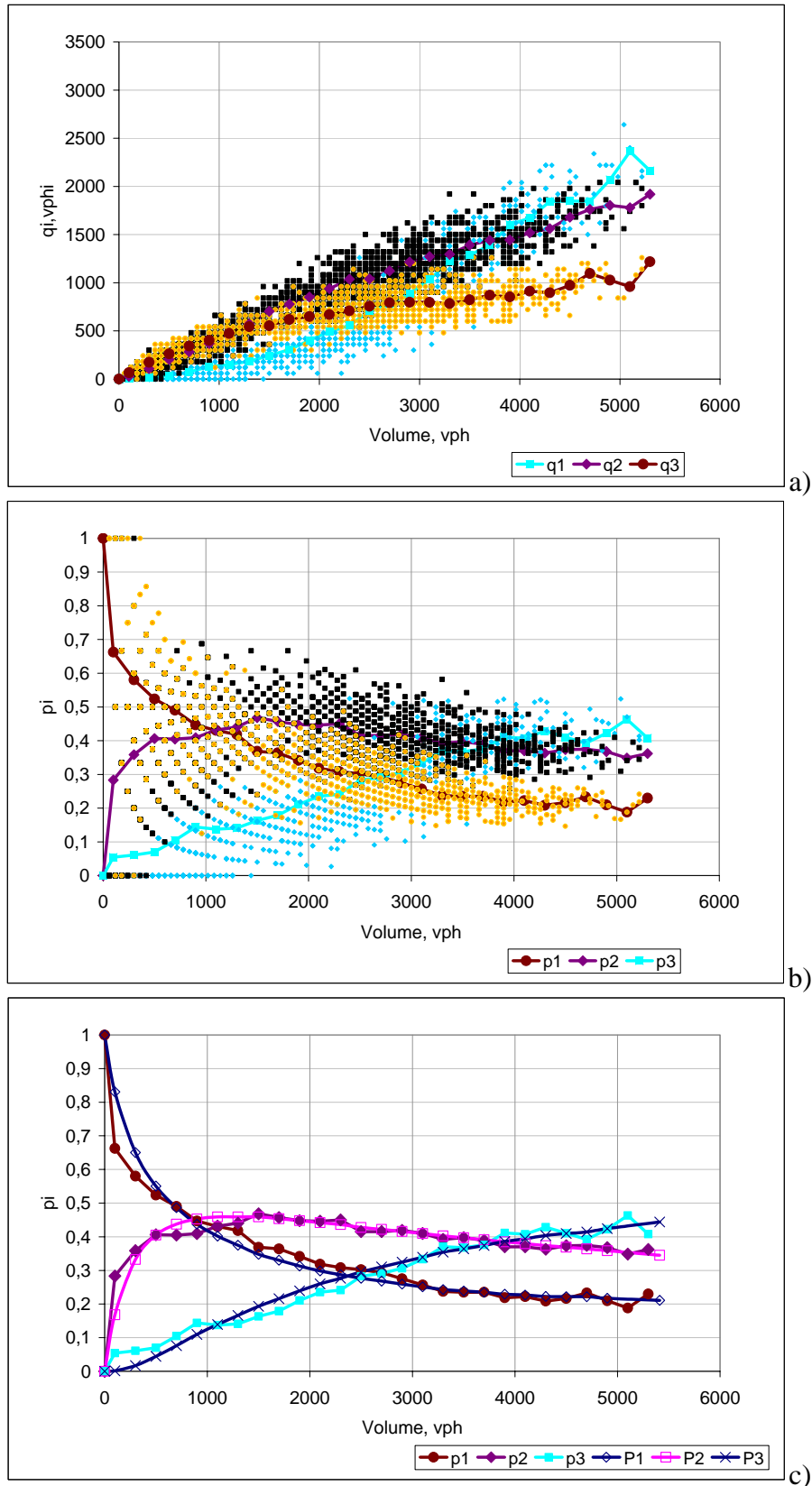
	Germany									
Data	A44	A1		Model			Model			
$n$	2-ln	3-ln		4-ln			5-ln			
parameter	$i=2$	$i=2$	$i=3$	$i=2$	$i=3$	$i=4$	$i=2$	$i=3$	$i=4$	$i=5$
$a$	1,41	0,41	1,67	0,35	0,28	0,20	0,33	0,28	0,68	0,98
$b$	1,00	1,53	1,00	0,99	1,03	1,01	0,00	1,27	1,01	1,00
$c$	0,65	3,87	0,25	6,20	3,28	1,88	2,08	3,35	0,27	0,05
$d$	1,59	0,44	3,35	1,08	1,52	1,96	1,71	1,10	1,81	3,44
$e$	1,02	0,20	2,35	0,40	0,15	-0,65	0,06	0,16	0,84	1,10



**Figure 1 – Comparison of gap distribution functions: Tanner= eq.(3), NSE =eq. (4),  
a) with  $q=1000$  veh/h and  $\Delta=0.5$ s, b) with  $q=1800$  veh/h and  $\Delta=0.5$ s**

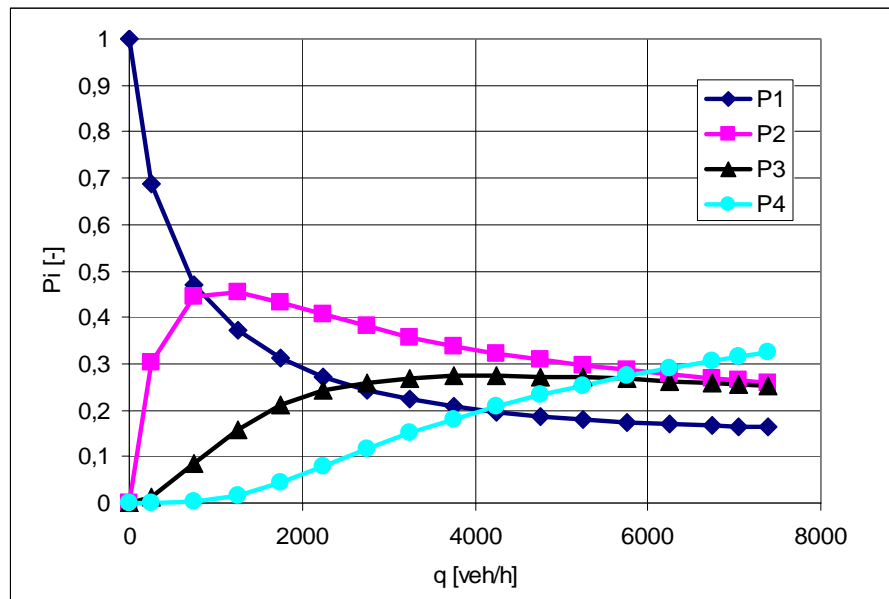


**Figure 2 - Lane flow-distribution for a sample two-lane carriageway in Germany** (eq.(16), Data: German freeway A44, 1-min intervals,  $p$ =class means,  $P$ =theoretical values), a)  $q_i=f(q_{\text{sum}})$ , b)  $p_i=f(q_{\text{sum}})$ , and c)  $P_i=f(q_{\text{sum}})$  with model values

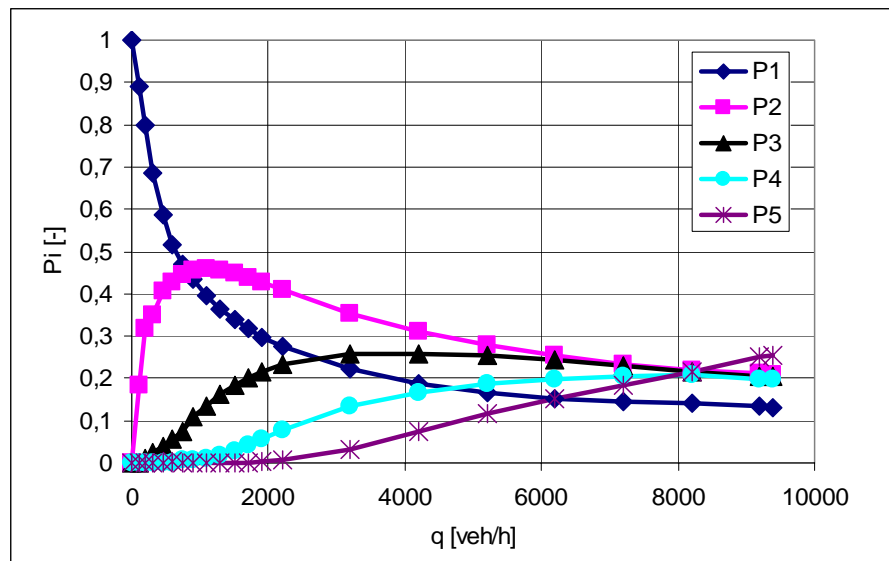


**Figure 3 - Lane flow-distribution for a sample three-lane carriageway in Germany (eq.(17), Data: German freeway A1, 1-min intervals,  $p$ =class means,  $P$ =theoretical values), a)  $q_i=f(q_{sum})$ , b)  $p_i=f(q_{sum})$ , and c)  $p_i=f(q_{sum})$  with model values**

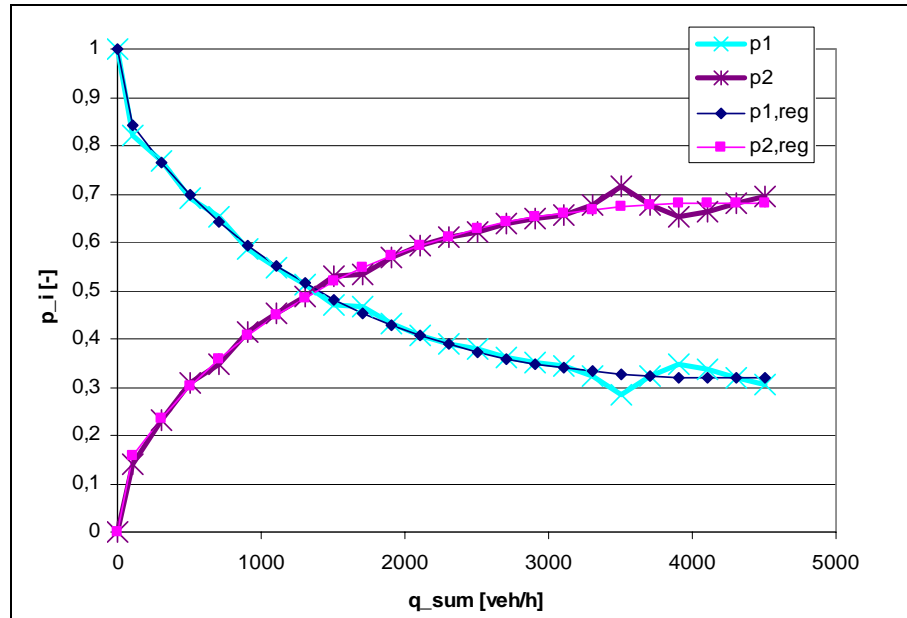




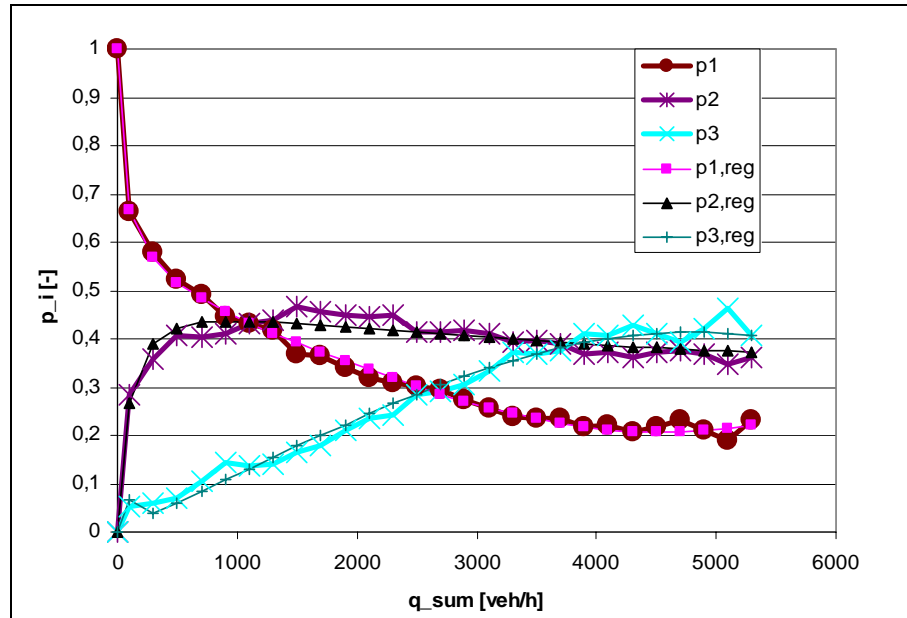
**Figure 4 - Lane flow-distribution for a four-lane carriageway (schematically)**



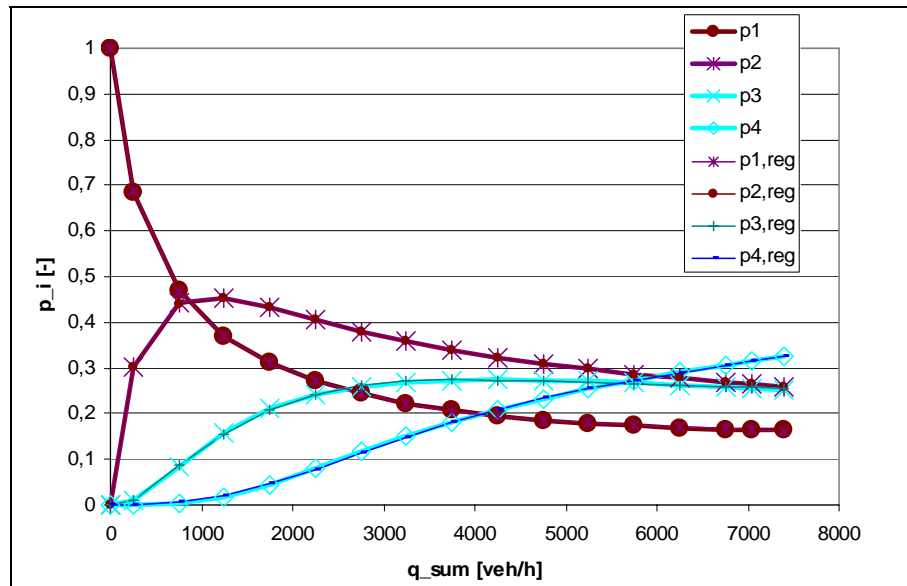
**Figure 5 - Lane flow-distribution for a five-lane carriageway (schematically)**



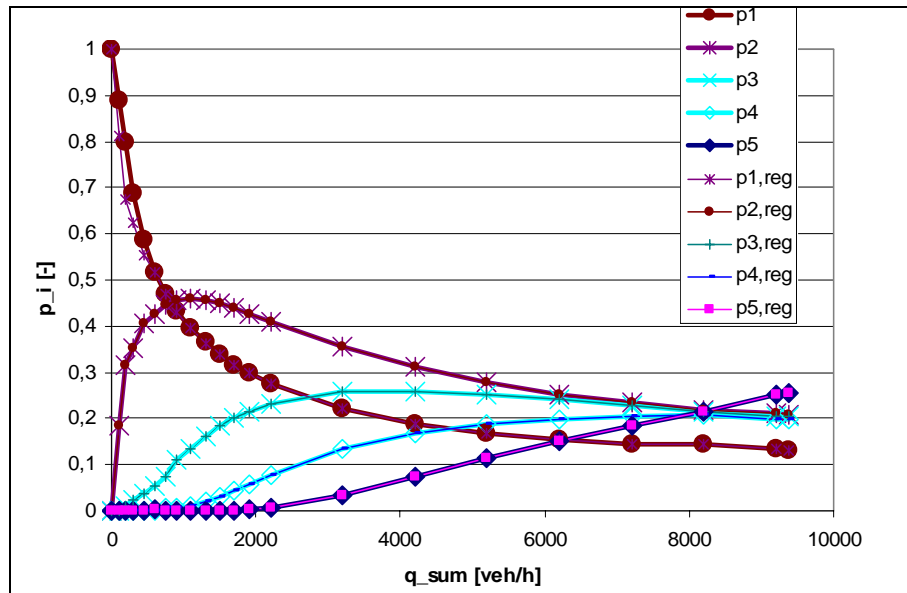
**Figure 6 – Regression function (eq.(21)) of the lane flow-distribution for the sample two-lane motorway in Germany ( $p$ =class means of field data,  $p_{reg}$ =regression)**



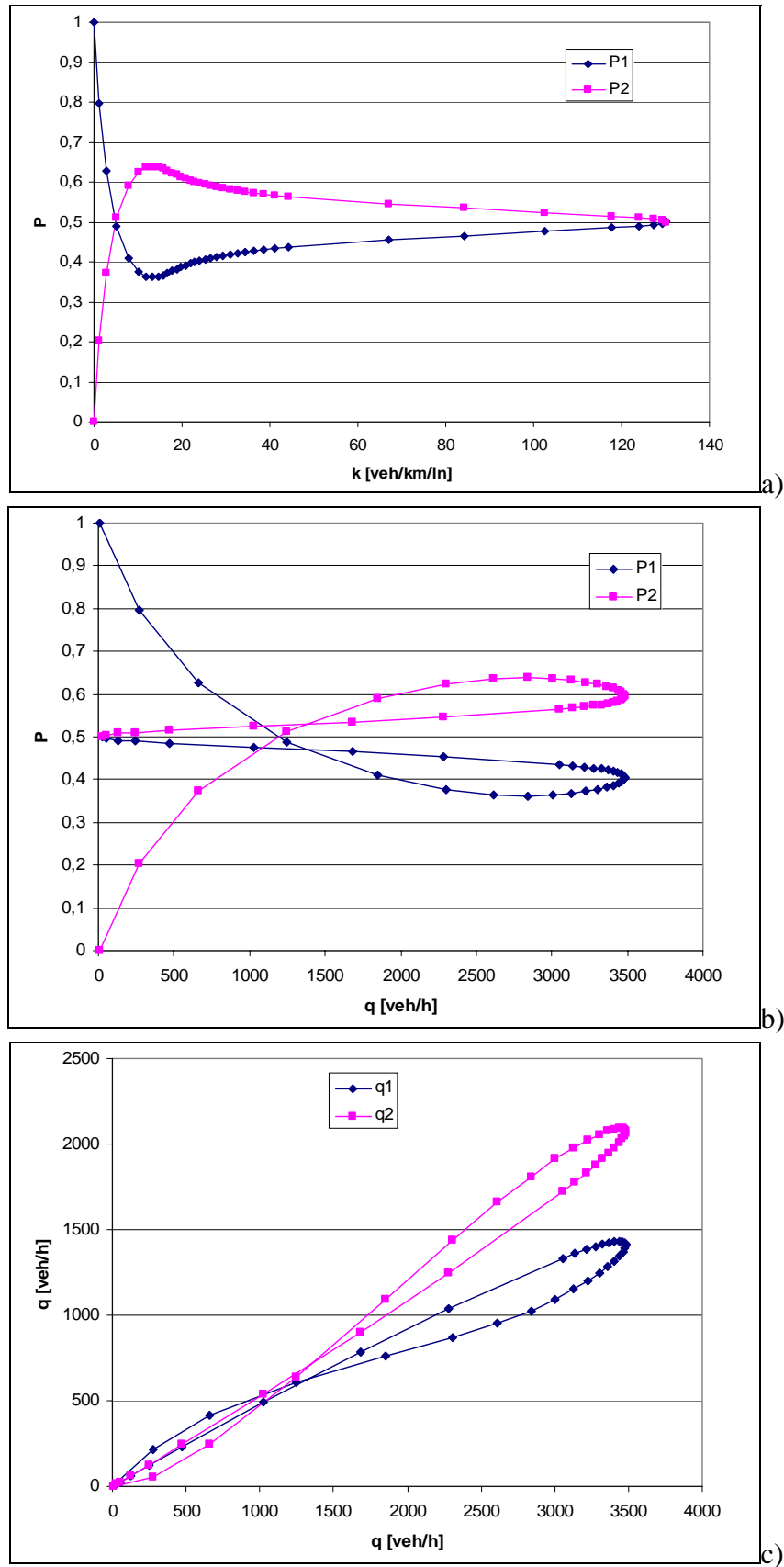
**Figure 7 - Regression function (eq.(21)) of the lane flow-distribution for the sample three-lane motorway in Germany ( $p$ =class means of field data,  $p_{reg}$ =regression)**



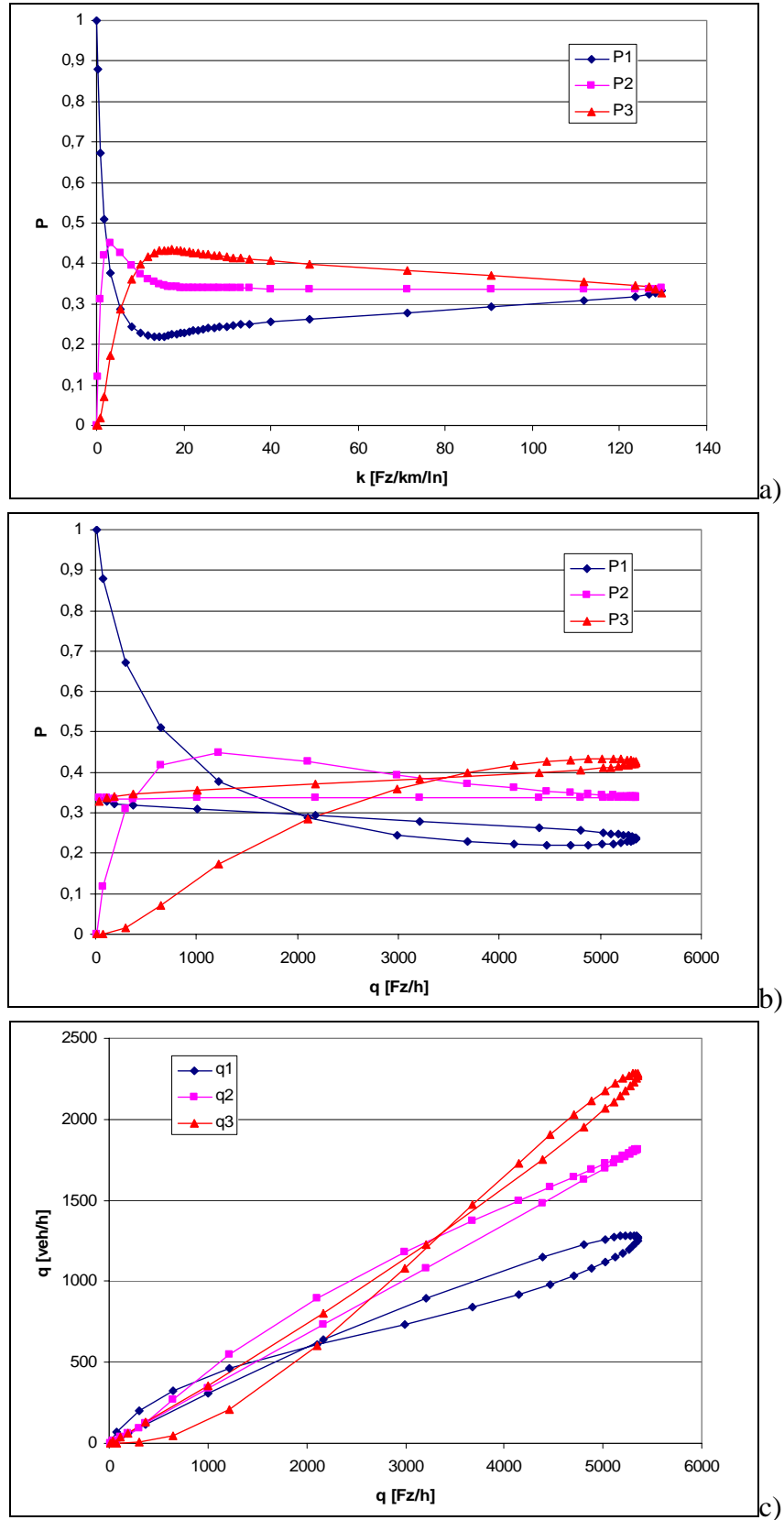
**Figure 8 – Regression function (eq.(21)) of the lane flow-distribution for the model four-lane motorway in Germany ( $p$ =model values,  $p_{reg}$ =regression)**



**Figure 9 - Regression function (eq.(21)) of the lane flow-distribution for the model five-lane motorway in Germany ( $p$ = model values,  $p_{reg}$ =regression)**



**Figure 10 - Lane flow-distribution for a two-lane carriageway (schematically) both for the free flow and for congested condition, a)  $p_i$  as a function of the traffic density, b)  $p_i$  as a function of the flow rate, and c)  $q_i = p_i * q_{\text{sum}}$  as a function of the flow rate**



**Figure 11 - Lane flow-distribution for a three-lane carriageway (schematically) both for the free flow and for congested condition, a)  $p_i$  as a function of the traffic density, b)  $p_i$  as a function of the flow rate, and c)  $q_i = p_i \cdot q_{\text{sum}}$  as a function of the flow rate**