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A Stochastic Model for Reliability Analysis in Freeway Networks

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Abstract

The capacity of a freeway is traditionally considered as a constant value in traffic engineering. In reality, capacities vary according to external conditions such as dry or wet road surfaces, daylight or darkness, and to the prevailing travel purpose of drivers on the freeway. Even under constant external conditions different capacities can be observed on freeways because of variations in driver behaviors. A capacity in this sense is no longer a constant value. Empirical analyses of traffic flow patterns show that this type of capacity can be treated as Weibull distributed. Using the distribution function of capacities, the probability of traffic breakdowns and thus the reliability of the freeway can be estimated. Up to now stochastic capacities have been mainly analyzed at specific points which are considered as bottlenecks. The stochastic relationship between the adjacent bottlenecks has not been taken into account. Furthermore, if a long segment of a freeway without clearly defined bottlenecks is analyzed, no methods are available for estimating the distributed capacities of several combined bottlenecks along a freeway. This paper introduces a concept dealing with the stochastic interpretation of capacity and breakdown probability within a larger freeway network. The stochastic methodology presented delivers a theoretical average capacity and the probability of breakdowns for freeway segments with different lengths. The methodology can also be used to identify the effects of consecutive freeway segments and bottlenecks such as on-ramps, off-ramps, and weaving areas with different characteristics. Using the proposed method, it is possible to determine the probability distribution function of breaking down from free flow into congested flow for a freeway segment as a function of the average volume or density. Using the methodology presented in this paper, the risk of disturbance of traffic flow along a freeway segment or within a freeway network can be analyzed.

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Keywords: Stochastic capacity; Reliability; Freeway; Tandem queue; Continuum theory

1. Introduction

Capacities of freeways are traditionally considered as constant values in traffic engineering guidelines around the world, for example in *Highway Capacity Manual* (Transportation Research Board, 2000), hereafter referred

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to as HCM. Treating capacities as constant values was questioned by many researchers such as Ponzlet (1996) who demonstrated that capacities vary according to external conditions such as dry or wet road surfaces, daylight or darkness, and on the prevailing purpose of the freeway (long distance or metropolitan commuter traffic). Several authors affirmed that even under constant external conditions, different capacities can be observed on freeways (Elefteriadou et al., 1995; Minderhoud et al., 1997; Persaud et al., 1998; Kuehne and Anstett, 1999; Lorenz and Elefteriadou, 2000; Okamura et al., 2000; Dong and Mahmassani, 2009a). Most of these authors only observed traffic breakdowns at different flow volumes to demonstrate the variability of flows preceding a breakdown.

A theoretical concept for a stochastic capacity analysis was proposed by Brilon et al. (2005) based on ideas from Minderhoud et al. (1997) and Toorenburg (1986). This approach has meanwhile been applied in a couple of circumstances. Dong and Mahmasani (2009a/b) used this concept to improve travel time predictions for route choice models with real-time traveller information. Elefteriadou et al. (2009) applied probabilities for flow breakdown on freeways to develop pro-active ramp metering strategies. Brilon et al. (2010) furnished a program system for large scale freeway network performance assessment applying the stochastic capacity concept.

Thus, the stochastic understanding of capacity and the corresponding concept for the reliability of freeways becomes an important topic in the area of theoretical freeway capacity analysis including application in practice. Here capacity is understood as the traffic volume below which the traffic is free (fluent) and above which the flow breaks down into a congested (stop-and-go or even standing) traffic condition. The capacity in this sense is not a constant value. Empirical analysis of traffic flow patterns shows that this type of pre-breakdown capacity can be treated as Weibull distributed with a nearly constant shape parameter representing the variance. The distribution of pre-breakdown capacity can be identified using the so-called product limit method (PLM) or by maximum likelihood estimation techniques. Using the distribution function of pre-breakdown capacities, the probability of traffic breakdowns and thus the reliability of the freeway can be estimated.

Stochastic pre-breakdown capacity has mainly been analysed at specific points along the freeway which are considered as bottlenecks. The stochastic relationship between the adjacent bottlenecks cannot be taken into account. Furthermore, if a long segment of a freeway without clearly defined bottlenecks is analysed, no methods are available for estimating the distributed pre-breakdown capacities of combined bottlenecks along a freeway. Thus, a stochastic capacity analysis in a freeway network consisting of several freeway segments and series of bottlenecks has not been possible. In order to overcome this problem, this paper introduces a model dealing with a stochastic interpretation of pre-breakdown capacity and breakdown probability in a freeway network with long freeway segments and series of bottlenecks.

The model is based on the theory of continuity. Using the fundamental relationship of traffic flow (volume = density · speed), the probability distribution function of breakdowns from free flow into congested flow at a given traffic density can be estimated if the probability distribution function of the pre-breakdown capacity and the probability distribution function of the pre-breakdown critical speed is given. The distribution function of breakdowns as a function of the pre-breakdown traffic density can be estimated numerically for an arbitrarily distributed pre-breakdown capacity and pre-breakdown critical speed.

Similar to the derivation of a theoretical transformation between bottleneck-point-related breakdown probabilities for different interval durations, a transformation between link-related breakdown probabilities for different lengths of freeway segments can be constructed. It can be seen that the average pre-breakdown capacity and the probability of breakdowns are functions of the length L of the freeway segment under consideration. The average pre-breakdown capacity of the freeway segment decreases with an increasing length of the freeway segment under consideration. This decrease is not linear.

In Section 2, a summary of the stochastic capacity analysis at a point considered as a bottleneck is presented. In Section 3, the bottleneck-point-related model of stochastic capacity is extended to link-related models for freeway segments. In Section 4, an approach for estimating reliability of large freeway networks over a longer period is presented. Finally, the main findings and results of the paper are presented in Section 5.

2. Bottleneck-point-related model of stochastic capacities

2.1. Pre-breakdown capacity for an isolated bottleneck

Corresponding to Transportation Research Board (2000), the capacity of a freeway is defined as the maximum flow volume that can be expected at a traffic facility under prevailing roadway, traffic, and control conditions. That is, the maximum flow volume could also be defined as the flow volume below which the performance of the facility is acceptable and above which normal operation is no longer possible. The transition between normal operation and non-acceptable flow conditions is called 'breakdown'. On a freeway, breakdowns occur when the average speed falls below an acceptable speed level and the traffic becomes congested. These transitions usually cause a rather sudden speed reduction.

Based on this definition, the capacity is no longer a constant value. The demand flow volume that causes breakdowns varies in real traffic depending on driver behaviour in conjunction with specific local conditions on the freeway. The breakdown flow volumes, i.e., the pre-breakdown capacities, are random variables. Thus, it is necessary to investigate the pre-breakdown capacity distribution function. Unfortunately, the pre-breakdown capacity itself cannot be easily measured directly in the field. Measurements on freeways deliver only pairs of values of traffic flow volumes and average speeds during predetermined intervals. According to the definition of pre-breakdown capacity, the observed flow volume will be below the pre-breakdown capacity if the average speed is above a certain threshold value (e.g., about 70 km/h). When the average speed is lower than this threshold value, the traffic flow is called congested. Thus, the flow volume must have exceeded the pre-breakdown capacity during the time between two such intervals. Higher flow volumes are less likely to be measured in the field since a breakdown is likely to have happened before. Both effects make it difficult to estimate the pre-breakdown capacity distribution function, which is defined as:

$$F_c(q) = P(c \le q) \tag{1}$$

where $F_c(q)$ = pre-breakdown capacity distribution function [-]
c = pre-breakdown capacity [veh/h]
q = variable = flow volume [veh/h]

A practicable method for estimating $F_c(q)$ was first presented by van Toorenburg (1986) and discussed by Minderhoud et al. (1997) and extended by Brilon et al. (2005). The method is based on the theory of lifetime analysis and renewal theory.

Lifetime distributions are often estimated by measurements of limited durations. Thus, the lifetimes of individuals in the population which exceed the duration of the measurement cannot be measured. It is only possible to state that these lifetimes are longer than the duration of the measurement. This information is valuable. Those data are called 'censored data' (cf., e.g., Lawless, 2003). The 'uncensored data' are directly measured lifetimes.

If a traffic breakdown is considered as a failure event, the method of lifetime data analysis can be used to estimate the pre-breakdown capacity c, which is the analogue of the lifetime t. Thus, the 'censored data' are the measurements where the capacity c is greater than the observed traffic demand q. The 'uncensored data' are directly observed pre-breakdown capacities.

The theory of lifetime data analysis can be used to estimate distribution functions based on samples that include censored data. A non-parametric method to estimate lifetime function is the so-called 'product limit method' (PLM) (Kaplan and Meier, 1958). This method can also be adapted for estimating the pre-breakdown capacity distribution function. For details of the method, readers are referred to Brilon et al. (2005).

The PLM does not need a specific type of distribution function. However, if the type of the distribution is given, then the parameters of the distribution can be estimated by the method of maximum likelihood. Here it is necessary to know the mathematical type of the distribution function $F_c(q)$. By comparing different types of functions based on the maximum value of the likelihood function, the Weibull distribution turned out to be the

function that best fitted the observations on all freeway segments under investigation. The Weibull distribution function for the pre-breakdown capacity c can be expressed as

$$F_c(q) = P(q \le c) = 1 - e^{-(q/\beta_c)^{\alpha_c}} \quad \text{for } q \ge 0 \tag{2}$$

where

α_c = shape parameter of the Weibull distribution [-]
β_c = scale parameter of the Weibull distribution [-]

The function

$$S_c(q) = P(q > c) = 1 - F_c(q) = e^{-\left(\frac{q}{\beta_c}\right)^{\alpha_c}} \quad \text{for } q \ge 0 \tag{3}$$

is called the survival function which describes the probability that the random variable q is larger than a given threshold c.

The mean value of the Weibull distribution is

$$E(c) = \beta_c \cdot \Gamma\left(1 + \frac{1}{\alpha_c}\right) \tag{4}$$

and the variance is

$$\sigma^2(c) = {\beta_c}^2 \cdot \left[\Gamma\left(1 + \frac{2}{\alpha_c}\right) - \Gamma^2\left(1 + \frac{1}{\alpha_c}\right)\right] \tag{5}$$

The median value of the distribution is

$$c_{median} = \beta_c \left[-\ln(0.5)\right]^{1/\alpha_c} = \beta_c \cdot 0.693^{1/\alpha_c} \tag{6}$$

In Figure 1, two examples are illustrated for the pre-breakdown capacity distributions estimated from the PLM and the corresponding Weibull distribution. It can be seen that the Weibull distribution fits very well into the PLM estimation.

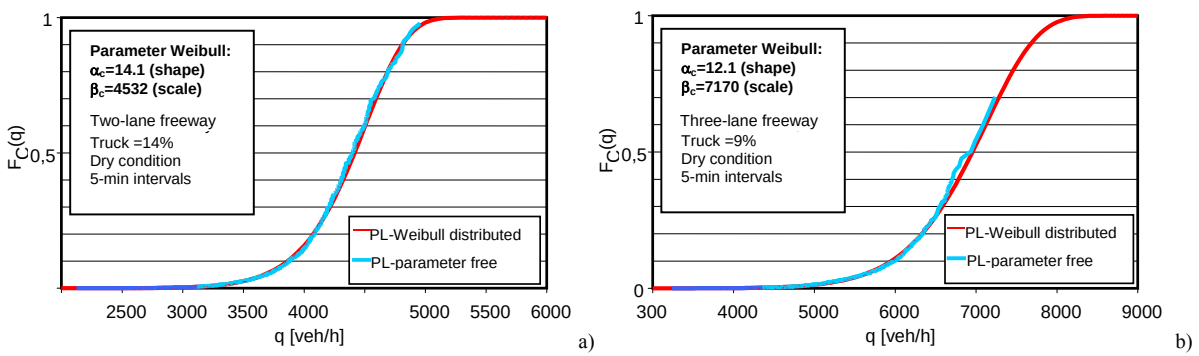

Figure 1 – Estimated pre-breakdown capacity distribution functions for two freeways (each direction) and three lane freeways according to PLM (5-minute intervals and dry roadway conditions, Source Brilon and Zurlinden, 2003).

The shape parameter α_c in the Weibull distribution ranges from 9 to 15 with an average of 13 for German motorways. This magnitude applies both to two-lane freeways (each direction) and to there-lane freeways. This average value is recommended for all types of freeway as a constant (Brilon et al., 2005) in order to ease mathematical derivations. This value, in the subsequent context, is used as an example to demonstrate consequences of this parameter α_c on other characteristic variables.

Considering the shape parameter α_c as a constant, we can transform the pre-breakdown capacity distribution function for different interval durations T (Brilon et al., 2005). According to Eq. (1), we define $F_{c,5}(q)$ as the probability of a breakdown during $\Delta = 5$ minutes at flow volume q. Hence, $p_{5,nbr} = 1 - F_{c,5}(q)$ is the probability of no breakdown occurring in this interval. Assuming an independence between breakdowns in succeeding intervals within an hour (60 minutes = 12 · 5 minutes) yields

$$P_{60,nbr} = \left(P_{5,nbr}\right)^{12} = \left[1 - F_{c,5}(q)\right]^{12} \tag{7}$$

Using the Weibull distribution, i.e., Eq. (2), yields

$$P_{60,nbr} = \left[e^{-\left(\frac{q}{\beta_{c,5}}\right)^{\alpha_c}} \right]^{12} = e^{-12\left(\frac{q}{\beta_{c,5}}\right)^{\alpha_c}} \tag{8}$$

and

$$F_{c,60}(q) = 1 - P_{60,nbr} = 1 - e^{-12\left(\frac{q}{\beta_5}\right)^{\alpha}} = 1 - e^{-\left(\frac{q}{\frac{\beta_5}{\sqrt[\alpha]{12}}}\right)^{\alpha}} \tag{9}$$

which is again a Weibull distribution with an unchanged shape parameter α_c and a scale parameter $\beta_{c,60} = r \cdot \beta_{c,5}$, where $r = 12^{(-1/\alpha c)}$. In general we have the transformation

$$F_{c,T|\Delta}(q) = 1 - e^{-\frac{T}{\Delta}\left(\frac{q}{\beta_{c,\Delta}}\right)^{\alpha_c}} \tag{10}$$

where T is the duration of the output interval and Δ the is duration (cf. Brilon et al., 2005) of the input interval.

The expectation $E(c)$ indicates the mean value of capacities under the condition that the traffic is not broken down. In reality, the expectation $E(c)$, i.e., the mean values of the pre-breakdown capacity estimated from the PLM, cannot be achieved because the traffic flow would already break down at lower flows than $E(c)$ with a certain probability.

2.2. Sequences of bottleneck points

In general, the survival function for an isolated point i which is treated as a bottleneck is

$$S_c(q_i) = 1 - F_c(q_i) = e^{-\left(\frac{q_i}{\beta_{c,i}}\right)^{\alpha_{c,i}}} \tag{11}$$

Now we look at successive sub-segments along the freeway where each sub-segment is treated as one bottleneck point. If we look at this chain of bottleneck points simultaneously, the survival functions for n combined bottleneck points is

$$S_{c,n} = \prod_{i=1}^{n} S_c(q_i) = \prod_{i=1}^{n} e^{-\left(\frac{q_i}{\beta_{c,i}}\right)^{\alpha_{c,i}}} = e^{-\sum_{i=1}^{n}\left(\frac{q_i}{\beta_{c,i}}\right)^{\alpha_{c,i}}} \tag{12}$$

This equation describes the probability that no breakdown occurs at any of the n bottleneck points. The combined survival functions can be used for defining reliability of a network under consideration. Here we assume that the distribution functions and thus the survival functions at different bottlenecks are independent of each other. This assumption is not critical if the bottlenecks are located far enough from each other.

Traffic reliability is an important factor for assessments of the performance of highway segments and systems. In this context, the term 'reliability' mainly refers to the variability of travel times. However, several definitions can be found in the literature. A comprehensive outline of these definitions is given by Shaw (2003). Here, traffic reliability is assessed by analysing the probability that at critical bottlenecks along a freeway link the traffic flow is not congested. However, the stochastic relationship between the adjacent bottlenecks cannot be taken into account.

Generally, the flow volumes q_i at different bottlenecks can have different values. For the special case that all $q_i = q$, for example along a single freeway segment, we obtain

$$S_{c,n}(q) = \prod_{i=1}^{n} S_c(q) = e^{-\sum_{i=1}^{n}\left(\frac{q}{\beta_{c,i}}\right)^{\alpha_{c,i}}} \tag{13}$$

However, the resulting distribution function $F_{c,n}(q)=1-S_{c,n}(q)$ is no longer a Weibull function as long as α_c and β_c are specific for each bottleneck i. But it always has a Weibull-like shape.

Normally, the shape parameters α_c can be considered as constant for all bottlenecks (Brilon et al., 2005). That is $\alpha_{c,i} = \alpha_c$ and

$$S_{c,n}(q) = \prod_{i=1}^{n} S_c(q) = e^{-\sum_{i=1}^{n}\left(\frac{q}{\beta_{c,i}}\right)^{\alpha_{c,i}}} = e^{-\left(q\cdot\sqrt[\alpha_c]{\sum_{i=1}^{n}\left(\frac{1}{\beta_{c,i}}\right)^{\alpha_c}}\right)^{\alpha_c}} = e^{-\left(\frac{q}{\beta_{c,n}}\right)^{\alpha_c}} \tag{14}$$

This combined survival function and the corresponding distribution function has the same shape parameter α_c as for the single bottlenecks. As mentioned above we consider here the stochastic processes at different bottlenecks as independent of each other. This assumption is not always realistic. For two closely adjacent sub-segments, the stochastic processes are expected to be highly dependent on each other. For statistically dependent single bottlenecks the shape parameter has in general a smaller value, that is, the combined distribution has a larger variance (cf. Section 3).

The scale parameter β_c of the corresponding distribution function $F_{c,n}(q)=1-S_{c,n}(q)$ is

$$\beta_{c,n} = \frac{1}{\sqrt[\alpha_c]{\sum_{i=1}^{n}\left(\frac{1}{\beta_{c,i}}\right)^{\alpha_c}}} \tag{15}$$

For a homogeneous freeway segment one can assume additionally $\beta_{c,i} = \beta_c$ and thus,

$$S_{c,n}(q) = \prod_{i=1}^{n} S_c(q) = e^{-\left(\frac{q\cdot\sqrt[\alpha_c]{n}}{\beta_c}\right)^{\alpha_c}} \tag{16}$$

and

$$\beta_{c,n} = \frac{\beta_c}{\sqrt[\alpha_c]{n}} = \beta_c \cdot f_{c,n} \tag{17}$$

where

$$f_{c,n} = \frac{\beta_{c,n}}{\beta_c} = \frac{1}{\sqrt[\alpha_c]{n}} \tag{18}$$

is the capacity reduction factor for n consecutive identical bottlenecks.

3. Link-related model of stochastic capacities

3.1. Pre-breakdown capacity of a single freeway segment

According to the PLM described in the previous section, the distribution function of pre-breakdown capacities at a single bottleneck point can be estimated. The PLM for capacity estimation can also be applied to traffic densities k instead of traffic volumes q. Unfortunately, the density cannot easily be observed in the field. However, for a homogeneous freeway segment under steady-state condition, the distribution function of the critical density k_c (pre-breakdown) corresponds to the distribution function of the pre-breakdown capacity c. The fundamental relationship of traffic flow $q = k \cdot v$, is also valid for $q = c$; thus $c = k_c \cdot v_c$ or $k_c = c / v_c$ where k_c and v_c are the corresponding critical density and critical speed at pre-breakdown capacity c. Therefore, the distribution function of the critical density $F_{kc}(k)$ can be estimated if the distribution function of the pre-breakdown capacity $F_c(q)$ and the probability distribution function of the critical speed $F_{vc}(v)$ is given. That is, the density-related (link-related) probability distribution function of breakdowns for a freeway segment, $P_{br}(k_c \le k) = F_{kc}(k)$, can be transformed from the flow-related (bottleneck-point-related) probability distribution function of breakdowns at an isolated bottleneck, $P_{br}(c \le q) = F_c(q)$. The density-related distribution function of breakdowns then is

$$F_{k_c}(k) = P(k_c \le k) = \oint_{c \le k_c v_c} f_c(q) f_{v_c}(v) dq dv = \int_0^\infty \left(\int_0^{kv} f_c(q) dq \right) f_{v_c}(v) dv = \int_0^\infty F_c(kv) f_{v_c}(v) dv \tag{19}$$

The flow-related pre-breakdown capacity function $F_c(q)$ is assumed to be a Weibull function (cf. Section 2). Here, any reasonable distributions for the critical speed v_c can be used. For simplicity and in order to ease the derivation we assume the critical speed v_c to be Weibull distributed as an approximation. This assumption seems to be more reasonable than the usual assumption of a Normal distribution for speeds since Weibull is only defined for positive values. Moreover, Weibull reveals significant probabilities only for a narrow range of speed values with a rather sharp lower limit which seems to be an important attribute especially for the critical speed. Thus,

$$\begin{aligned} F_{k_c}(k) &= \int_0^\infty F_c(kv) f_{v_c}(v) dv = \int_0^\infty (1 - e^{-\left(\frac{kv}{\beta_c}\right)^{\alpha_c}}) \cdot \frac{\alpha_{v_c}}{\beta_{v_c}} \left(\frac{v}{\beta_{v_c}} \right)^{\alpha_{v_c}-1} e^{-\left(\frac{v}{\beta_{v_c}}\right)^{\alpha_{v_c}}} dv \\ &= 1 - \frac{\alpha_{v_c}}{\beta_{v_c}} \int_0^\infty \left(\frac{v}{\beta_{v_c}} \right)^{\alpha_{v_c}-1} \cdot e^{-\left[\left(\frac{kv}{\beta_c}\right)^{\alpha_c} + \left(\frac{v}{\beta_{v_c}}\right)^{\alpha_{v_c}} \right]} dv \end{aligned} \tag{20}$$

Using a constant critical speed v_c, the transformation can be carried out analytically. The resulting density-related distribution function $F_{kc}(k)$ is also a Weibull distribution. That is,

$$F_{k_c}(k)_{|v=const} = F_c(kv) = 1 - e^{-\left(\frac{kv}{\beta_c}\right)^{\alpha_c}} = 1 - e^{-\left(\frac{k}{\beta_c / v}\right)^{\alpha_c}} \tag{21}$$

The density-related distribution function $F_{kc}(k)$ can only be estimated numerically for a arbitrarily distributed critical speed v_c. Using a mean value for the critical speed v_c = 80 km/h and a standard deviation $\sigma(v_c)$ = 5 km/h which are common in reality, we have the parameter for a Weibull distributed critical speed β_{vc} = 82 km/h and α_{vc} = 20. Using β_c = 4532 veh/h (from the example freeway, Figure 1a) and α_c = 13 for a two-lane freeway segment, Eq. (20) yields a Weibull-like but not exactly a Weibull distribution. This distribution can be approximated to a Weibull distribution with the parameter β_{kc} = 57 veh/km and α_{kc} = 10.7 (cf. Figure 2a). For a three-lane freeway

segment with β_c = 7170 veh/h (from the example freeway, Figure 1b) and α_c = 13 Eq. (20) yields a Weibull-like distribution with parameters β_{kc} = 89 veh/km and α_{kc} = 10.7 (cf. Figure 2b).

It can be proven that the shape parameter of the critical density α_{kc} only depends on the shape parameters α_c and α_{vc}. The parameter α_{kc} is independent of the scale parameters β_{kc}, β_c, and β_{vc}. In Table 1 the parameters α_{kc} resulting from different combinations of α_{kc} and α_{kc} values are illustrated. Because the shape parameters α_c and α_{vc} can be generally assumed to be constant values (e.g., α_c = 13 and α_{vc} = 20) for all types of freeway segments, the shape parameter for the critical density is also a constant (e.g., α_{kc} = 10.7).

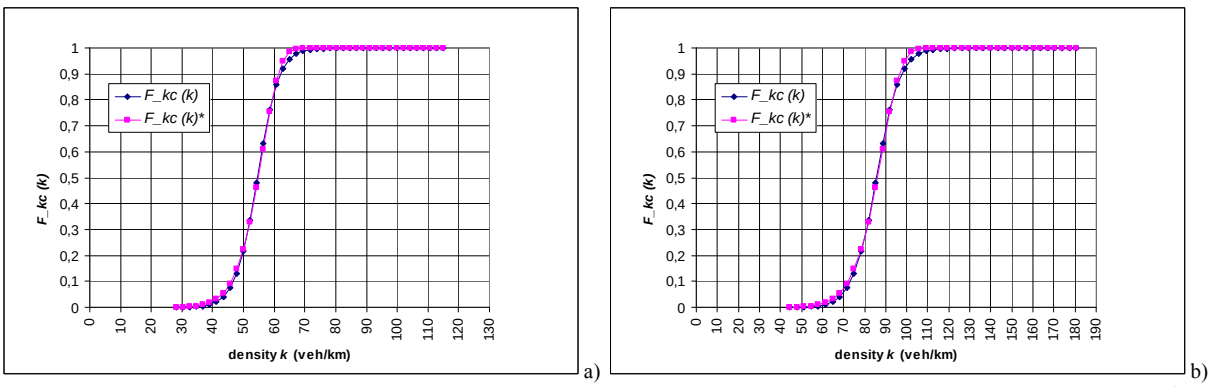

Figure 2 – Numerically estimated distribution function of critical densities ($F_{kc}(k)$) and its approximation as a Weibull distribution ($F_{kc}(k)^*$), a) two-lane freeway segment, b) three-lane freeway segment

Table 1 – Parameters α_{kc} resulting from different combinations of α_{kc} and α_{kc} values

	α_{vc}			
α_c	10	15	20	25
10	6.81	8.1	8.77	9.15
13	7.65	9.56	**10.66**	11.32
16	8.21	10.66	12.21	13.21
σ_{vc} [km/h] (v_c=80 [km/h])	9.6	6.5	5.0	4.0

This result can be verified by real world measurements. Regler (2004) conducted a field measurement using data from three-lane freeway segments. The median of the critical densities ranged from 70 to 90 veh/km with Weibull parameters α_{kc} = 8.4 through 13.2 and β_{kc} = 72 through 92 veh/km for the analysis of 5-minute intervals.

As a result we can state that the distribution of critical densities k_c is approximately Weibull distributed with a shape parameter α_{kc}=10.7. That is,

$$F_{k_c}(k) = 1 - e^{-\left(\frac{k}{\beta_c / \bar{v}_c}\right)^{\alpha_{k_c}}} = 1 - e^{-\left(\frac{k}{\beta_{k_c}}\right)^{\alpha_{k_c}}} \qquad (22)$$

where $\beta_{kc} = \beta_c / \bar{v}_c$, α_{kc} = 10.7, and $\bar{v}_c$ is the mean value of the critical speed v_c.

This transformed distribution function is only valid for a length L_Δ of freeway segment which corresponds to the analysis interval Δ for the pre-breakdown capacity. For example, if the scale parameter $\beta_{c,5}$ for the pre-breakdown capacity is obtained for 5-minute intervals, the resulting distribution function is only valid for a freeway segment of length $L_\Delta = \bar{v}_c \cdot \Delta$ = 80 km/h · 5/60 = 6.67 km.

Similarly to the derivation of the theoretical transformation between bottleneck-point-related breakdown probabilities for different interval durations, a transformation between link-related breakdown probabilities for different lengths of freeway segments can be constructed. The probability function of breakdowns for a freeway segment of length L can be expressed as

$$F_{k_c,L}(k) = 1 - e^{-\left(\frac{k}{\beta_{k_c,L}}\right)^{\alpha_{k_c}}} = 1 - e^{-\frac{L}{L_\Delta}\left(\frac{k}{\beta_{k_c,L_\Delta}}\right)^{\alpha_{k_c}}} = 1 - e^{-\frac{L}{\bar{v}_c \cdot \Delta}\left(\frac{k}{\beta_{c,\Delta}/\bar{v}_c}\right)^{\alpha_{k_c}}} = 1 - e^{-\left(\frac{k}{\beta_{c,\Delta}/\bar{v}_c \Big/ \sqrt[\alpha_{k_c}]{L/L_\Delta}}\right)^{\alpha_{k_c}}} \quad (23)$$

with the scale parameter

$$\beta_{k_c,L} = \frac{\beta_{k_c,L_\Delta}}{\sqrt[\alpha_{k_c}]{L/L_\Delta}} = \frac{\beta_{c,\Delta}/\bar{v}_c}{\sqrt[\alpha_{k_c}]{L/L_\Delta}} \quad (24)$$

Eq. (23) describes the probability that within a time interval of duration Δ no breakdown occurs on a freeway segment of length L. The parameter $\beta_{c,\Delta}$ is the scale parameter of the Weibull distributed pre-breakdown capacities estimated in Δ-minute intervals.

For Δ = 5 min =1/12 h, $\bar{v}_c$=80 km/h, L_Δ = 6.67 km, and α_{kc}=10.7 we have

$$F_{k_c,L}(k) = 1 - e^{-\frac{L}{6.67}\left(\frac{k}{\beta_{c,5}/80}\right)^{10.7}} \quad (25)$$

This equation can be transformed into

$$F_{k_c,L}(k) = 1 - e^{-\frac{L}{6.67}\left(\frac{k}{\beta_{c,5}/80}\right)^{10.7}} = 1 - e^{-\left(\frac{k}{\beta_{c,5}/80 \Big/ \sqrt[10.7]{\frac{L}{6.67}}}\right)^{10.7}} = 1 - e^{-\left(\frac{k}{\frac{\beta_{c,5,L}}{80}}\right)^{10.7}} = 1 - e^{-\left(\frac{k}{\beta_{kc,5,L}}\right)^{10.7}} \quad (26)$$

For two freeway segments of lengths L_1 and L_2 we obtain the relationship

$$\frac{\beta_{kc,L_1}}{\beta_{kc,L_2}} = \frac{\beta_{c,5}/80}{\sqrt[10.7]{\frac{L_1}{6.67}}} \Bigg/ \frac{\beta_{c,5}/80}{\sqrt[10.7]{\frac{L_2}{6.67}}} = \sqrt[10.7]{\frac{L_2}{L_1}} \quad (27)$$

This means e.g.: if the length L_2 of the freeway segment is double the length of L_1, the scale parameter β_{kc} of the density (and capacity) is reduced by the factor $1/\sqrt[10.7]{2} = 0.937$.

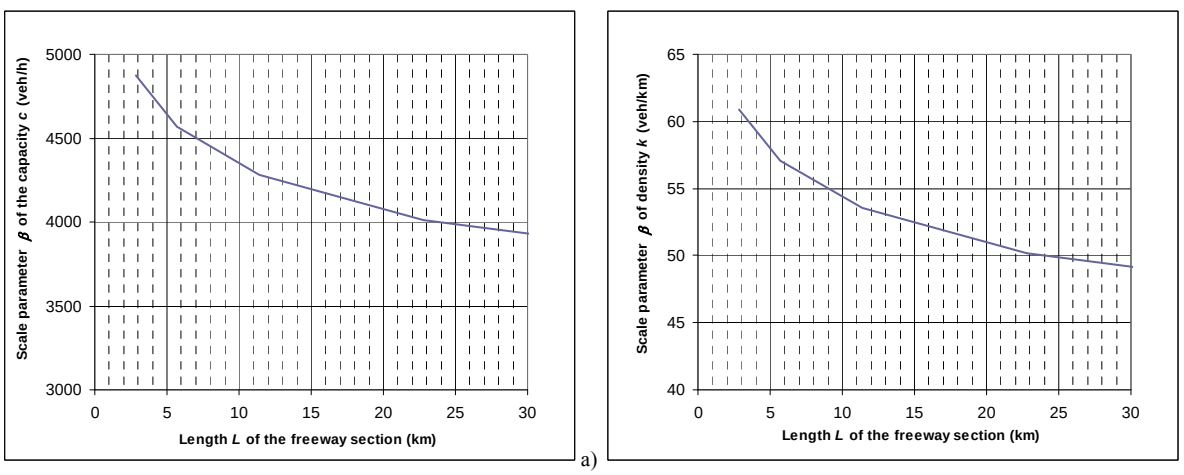

Figure 3 – Scale parameter β as a function of the length of the freeway segment L, a) scale parameter $\beta_{kc,L}$ for critical density k_c, b) scale parameter $\beta_{c,L}$ for the corresponding pre-breakdown capacity c.

Figure 3 shows the parameters $\beta_{kc,L}$ and $\beta_{c,L}$ which indicate the critical density and the corresponding pre-breakdown capacity as a function of length L for a two-lane freeway segment. It can be seen that the scale parameter β for the critical density k_c and the corresponding scale parameter β for the pre-breakdown capacity c of the freeway segments decrease with increasing length of the freeway segment.

For $\alpha_{kc} = 10.7$ we have the capacity which can be expected conditional on no-breakdown occurring $C_{nbr} \approx 0.94E(c) = 0.89\beta_c$, cf. Eq. (4). For the example freeway segment of length L = 25 km, the expected capacity under no-breakdown condition is ca. 4000 · 0.89 = 3560 veh/h.

3.2. Sequence of consecutive freeway segments

The survival function for a single freeway segment j is

$$S_{k_c}(k_j, L_j) = 1 - F_{k_c,L_j}(k_j) = e^{-\frac{L_j}{L_{\Delta,j}}\left(\frac{k_j}{\beta_{c,\Delta,j}/\bar{v}_{c,j}}\right)^{\alpha k_{c,j}}} = e^{-\left(\frac{k_j}{\beta_{c,\Delta,j}/\bar{v}_{c,j}\sqrt[\alpha k_{c,j}]{\frac{L_i}{L_\Delta}}}\right)^{\alpha k_{c,j}}} \quad (28)$$

For Δ = 5 min =5/12 h, $\bar{v}_{c,j} = \bar{v}_c$ = 80 km/h, $L_{\Delta,j} = L_\Delta$ = 6.67 km, and $\alpha_{kc,j} = \alpha_{kc}$ = 10.7 we get

$$S_{k_c}(k_j, L_j) = e^{-\frac{L_j}{6.67}\left(\frac{k_j}{\beta_{c,5,j}/80}\right)^{10,7}} \quad (29)$$

The survival function for m combined freeway segments then is

$$S_{k_c,m} = \prod_{j=1}^{m} S_{k_c}(k_j, L_j) = \prod_{j=1}^{m} e^{-\frac{L_j}{6.67}\left(\frac{k_j}{\beta_{c,5,j}/80}\right)^{10,7}} = e^{-\sum_{j=1}^{m}\frac{L_j}{6.67}\left(\frac{k_j}{\beta_{c,5,j}/80}\right)^{10,7}} \quad (30)$$

This equation describes the probability that in the interval of 5 minutes no breakdown occurs on any of the m freeway segments. The freeway segments can have different values of scale parameter capacity β_c (or β_{kc}), density k, and length L. This can also be used for defining the reliability of a network. Here we assume again the distribution functions and thus also the survival functions at different freeway segments are independent of each other. Normally, for long freeway segments, this independence is given.

4. Reliability analysis of large freeway networks over long time periods

Using this approach for sequences of freeway segments the reliability of a larger freeway network can be estimated over a longer time period. All parameters used in this section are link-related parameters, that is, they are parameters for the freeway segments according to Section 3. However, these link-related parameters can be transformed from bottleneck-point-related parameters.

The reliability of a larger freeway network can be defined as the probability that on any freeway segment within the network and at any time no breakdown occurs. According to this definition, the reliability can be expressed as the combined survival function of the pre-breakdown capacity over time and space.

The survival function of the pre-breakdown capacity for a single freeway segment j over a time period i of duration T_i and a space-link of length L_j is (cf. Eqs. (10) and (23))

$$S_{T_i+L_j}(k_{ij}(q_{ij}), L_j, T_i) = e^{-\frac{L_j}{L_{\Delta,j}}\frac{T_i}{\Delta}\left(\frac{k_{ij}}{\beta_{c,\Delta,ij}/\bar{v}_{c,ij}}\right)^{\alpha k_{c,j}}} = e^{-\frac{L_j}{L_{\Delta,j}}\frac{T_i}{\Delta}\left(\frac{q_{ij}}{\beta_{c,\Delta,ij}}\right)^{\alpha k_{c,j}}} \quad (31)$$

For $\Delta = 5$ min = 5/12 h, $\bar{v}_{c,ij} = \bar{v}_c = 80$ km/h, $L_{\Delta,j} = L_\Delta = 6.67$ km, and $\alpha_{kc,j} = \alpha_{kc}$=10.7 we get

$$S_{T_i+L_j}(k_{ij}(q_{ij}), L_j, T_i) = e^{-\frac{L_j}{6.67}\cdot\frac{T_i}{5/12}\left(\frac{k_{ij}}{\beta_{c,5,ij}/80}\right)^{10,7}} = e^{-0.36L_jT_i\left(\frac{k_{ij}}{\beta_{c,5,ij}/80}\right)^{10,7}} = e^{-\frac{L_j}{6.67}\cdot\frac{T_i}{5/12}\left(\frac{q_{ij}}{\beta_{c,5,ij}}\right)^{10,7}} = e^{-0.36L_jT_i\left(\frac{q_{ij}}{\beta_{c,5,ij}}\right)^{10,7}} \quad (32)$$

The survival function for *m* combined freeway segments and *n* intervals is

$$S_{T+L,m\times n} = \prod_{j=1}^{m}\prod_{i=1}^{n} e^{-0.36L_jT_i\left(\frac{k_{ij}}{\beta_{c,5,ij}/80}\right)^{10,7}} = e^{-\sum_{j=1}^{m}\sum_{i=1}^{n}-0.36L_jT_i\left(\frac{q_{ij}}{\beta_{c,5,ij}}\right)^{10,7}} \quad (33)$$

This equation describes the probability that in the time period of duration $T = \Sigma T_i$ and within a network of a total length $L = \Sigma L_j$ no breakdown occurs. The freeway segments *j* can have different values of scale parameter β for capacity, density *k*, and length L_j for different time period T_i. According to this formulation, a quantitative assessment of the reliability in a large network over a long period can be conducted.

5. Conclusions

Using a theoretical approach a methodology for the assessment of reliability within a freeway network was introduced. The stochastic methodology presented allows for a derivation of a theoretical average pre-breakdown capacity and the probability of breakdowns for freeway segments with different lengths. This link-related methodology can also be used to identify the effects of consecutive freeway segments and bottlenecks such as on-ramps, off-ramps, and weaving areas with different characteristics. As a result, the stochastic relationship between several adjacent bottlenecks can be taken into account. Furthermore, a long segment of a freeway without clearly defined bottlenecks can be analysed.

Using this method it is possible to determine the probability distribution function of breakdowns from free flow condition into a congested flow condition for a freeway segment as a function of the average pre-breakdown density. This link-related probability distribution of breakdowns can be estimated by transforming the distribution function of pre-breakdown capacities measured at isolated bottleneck points. It turns out that the link-related pre-breakdown capacity distribution (a Weibull-like distribution) has a smaller scale parameter and, thus, a lager variance than the bottleneck-point-related capacity distribution.

Using the methodology presented in this paper, the risk of disturbance of traffic flow (breakdowns from free flow into congested flow) along a freeway segment and within a freeway network can be estimated and analysed. The reliability of a freeway network can be estimated quantitatively. The paper demonstrates basic probabilistic considerations which – for practical application – must be based on breakdown probability functions calibrated for the important parts of the network.

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