An approximation for the distribution of queue lengths at unsignalized intersections

Ning Wu

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Abstract

This paper presents a new theoretical-empirical formula for estimating the distribution of the queue length at unsignalized intersections under stationary and nonstationary traffic conditions. The formula for stationary traffic is based on the data of the M/G2/1 queue system and is nearly as exact as a M/G2/1 queue system. But it can be very easily applied, similar to the formula from the M/M/1 queue system. The formula for estimating the distribution of queue length under nonstationary traffic conditions is then derived from the theoretical-empirical formula for stationary traffic conditions. This can be done by using the transformation technique of Kimber and Hollis (1979).

For the practical applications, graphical nomographs for calculating the 95% and 99% (also possible for other percentiles) queue lengths are produced under stationary as well as nonstationary traffic conditions. They can be used for proving the traffic quality (in the analysis module) or for determining the necessary queueing spaces (in the planning module).

Author's address:
Dr. Ning Wu
Institute of Traffic Engineering
Ruhr-University Bochum
IA 2/126
44780 Bochum
Germany

Tel.: ++49/234/3226557
Fax: ++49/234/3214151
ning.wu@rub.de
http://homepage.rub.de/ning.wu

1. Introduction

The queue length of waiting vehicles at intersections in the street network is an important parameter for proving (determining) the quality of the traffic control. This is valid for both the signalized intersections and unsignalized intersections. The calculations of the average queue length and the percentiles of the queue lengths are in this sense of special importance if the waiting space is limited for the queueing vehicles. For example, these queue lengths above can be used for the design of the lane length for the left-turn stream. It is desirable that the length of a left-turn lane is so dimensioned that oversaturation of the lane can be avoided, so the blockage of the through traffic could be held in limit. Normally, oversaturation probability of the left-turn lanes should be limited to 1% or 5%. In other words, the length of the left-turn lanes should not be shorter than the 99 or 95 percentile of the queue lengths. Several approximation formulas have been given by Wu /10/ for calculating the 95 and 99 percentile of the queue lengths at signalized intersections. These formulas can be used under many different traffic conditions.

In the case of unsignalized intersections no simple formulas for calculating the 95 and 99 percentiles of the queue lengths exist. There are a few theoretical approaches for calculating the distribution of the queue lengths at unsignalized intersections under stationary traffic conditions /2//4/. These approaches are mathematical exact under the corresponding assumptions. However, they contain very complex recursive operations, so that the solution of the 95 and 99 percentile of the queue lengths is very difficult (by computation). Under nonstationary traffic conditions one cannot find in the literatures any approaches for calculating the 95 and 99 percentiles of the queue lengths at unsignalized intersections.

In this paper, a theoretical-empirical approach for calculating approximately the distribution of the queue lengths at unsignalized intersections is presented. This approach gives a description of the exact but complex theoretical approach under stationary traffic conditions and it can easily be used in practical applications. The deviations between the exact theoretical approach and the approximation are so small that they could be ignored in practice. With this approximation for calculating the distribution, the 95 and 99 percentiles of the queue lengths under stationary traffic conditions can easily be calculated. The distribution of the queue lengths under nonstationary traffic conditions can then be obtained with the help of the well-known "transformation" technique /5//11/.

The following symbols will be used in this paper:

q_h = traffic flow of the major stream (main stream) (veh/s)

q_n = traffic flow of the minor stream (side stream) (veh/s)

Q_h = traffic flow of the major stream (main stream) (veh/h)

Q_n = traffic flow of the minor stream (side stream) (veh/h)

t_g = critical time headway (s)

t_f = move-up time (s)

$q_{n,max}$ = maximal traffic flow (capacity) of the minor stream

$$= \frac{q_h}{\exp(q_h \cdot (t_g - t_f)) \cdot (\exp(t_f \cdot q_h) - 1)} \quad \text{(after Harders)} \qquad \text{(veh/s)}$$

x = saturation degree = $q_n/q_{n,max}$ (-)

T = length of the considered peak period (time interval) under nonstationary traffic conditions (s)

QT = $\bar{q}_{n,max} \bullet T$

= sum of the capacity in the considered time interval T (veh)

(for $QT=\infty \Rightarrow$ stationary traffic)

$\bar{q}_h$ = average traffic flow of the major stream during T (veh/s)

$\bar{q}_n$ = average traffic flow of the minor stream during T (veh/s)

$\bar{q}_{n,max}$ = average maximal traffic flow (average capacity) of the minor stream during T

$$= \frac{\bar{q}_h}{\exp(\bar{q}_h \cdot (t_g - t_f)) \cdot (\exp(t_f \cdot \bar{q}_h) - 1)}$$ (after Harders) (veh/s)

$\bar{x}$ = average saturation degree during T

$= \bar{q}_n / \bar{q}_{n,max}$ (-)

x_∞ = saturation degree before and after the considered time interval T (-)

N_∞ = queue length before and after the considered time interval T (veh)

N_0 = average queue length (veh)

W = average delay (s/veh)

N_{95} = 95 percentile of the queue lengths (veh)

N_{99} = 99 percentile of the queue lengths (veh)

N_α = α percentile of the queue lengths (veh)

$p(n)$ = probability of the queue lengths n

= probability of finding queue length = n (-)

$P(n)$ = probability distribution function of the queue lengths n (-)

$P_{os}(n)$ = 1-$P(n)$: probability of oversaturation with n waiting positions

= probability of queue length > n (-)

P_{os} = $P_{os}(0)$: probability of queue length >0 (-)

N_0 = average queue length (veh)

W = average delay (s/veh)

N_{95} = 95 percentile of the queue lengths (veh)

N_{99} = 99 percentile of the queue lengths (veh)

N_α = α percentile of the queue lengths (veh)

2. Theoretical Foundations

The 95 percentile of the queue lengths N_{95} and the 99 percentile of the queue lengths N_{99} can be obtained if the distribution of the queue lengths is known. In addition, the distribution must be solvable for the parameters N_{95} and N_{99} . Fig.1 shows a distribution of queue lengths, which was simulated by the program KNOSIMO /7/. This Figure shows the typical development of the distribution of the queue lengths at unsignalized intersections. For the M/M/1 queueing system (i.e., the queueing system has only one counter, both the arriving time headways and service times are negative-exponential distributed) - which has often been used as an approximation of the queueing system at unsignalized intersections - the following probability functions are valid (cf. /6/):

- probability of the queue lengths n for the M/M/1 queueing system:

$$p(0) = 1 - x \quad (1a)$$

$$p(n) = (1-x) \cdot x^n \quad (1b)$$

- probability distribution function for the M/M/1 queueing system:

$$P(n) = \sum_{i=0}^{n} p(i) = 1 - x \cdot x^n = 1 - x^{n+1} \quad (2)$$

From the distribution function (Eq.(2)) the probability of oversaturation (probability of queue length > n) for the M/M/1 queueing system can be obtained:

$$P_{os}(n) = x^{n+1} \tag{3}$$

The corresponding percentile of the queue lengths n is:

$$N_\alpha = \frac{\ln(P_{os}(n_\alpha))}{\ln(x)} - 1 = \frac{\ln(1 - P(n_\alpha))}{\ln(x)} - 1 = \frac{\ln(1 - \frac{\alpha}{100})}{\ln(x)} - 1 \tag{4a}$$

Thus, the 95 percentile can be represented as

$$N_{95} = \frac{\ln(1 - 0.95)}{\ln(x)} - 1 = \frac{\ln(0.05)}{\ln(x)} - 1 \tag{4b}$$

Eq.(4) shows the first - and the simplest - approximation for establishing the percentile of the queue lengths at unsignalized intersections if the queueing system at unsignalized intersections is assumed as an M/M/1 queueing system. Under this assumption, the vehicle arrivals in the major stream are Poission distributed and the service times for the vehicles in the minor stream are negative-exponential distributed. The Poisson distribution of the arrivals is appropriate under normal traffic conditions (free traffic), but the negative-exponential distribution of the service times has been proved to be incorrect.

Heidemann /4/ has derived a function for the probability of the queue lengths at unsignalized intersections (M/G2/1 queueing system) with the help of the generating function from Tanner /9/. This function for the probability of the queue lengths describes exactly the distribution of the queue lengths under the following assumptions:

- the time headways in the major stream (q_h) are negative-exponential distributed, i.e., the vehicle arrivals in the major stream are Poisson distributed,
- the critical time headways t_g and move-up times t_f for the minor stream (q_n) are constant and
- the vehicle arrivals in the minor stream are Poisson distributed.

The probability of the queue lengths n at unsignalized intersections from Heidemann is:

$$p(0) = h_1 \cdot h_3 \cdot (q_h + q_n) \tag{5a}$$

$$p(1) = p(0) \cdot h_3 \cdot q_n \cdot \left[\exp(q_n \cdot t_f) - (t_g - t_f) \cdot h_2\right] - q_n \cdot h_1 \cdot h_3 \tag{5b}$$

$$p(n) = p(n-1) \cdot h_3 \cdot q_n \cdot \left[\exp(q_n \cdot t_f) - (t_g - t_f) \cdot h_2\right] - h_3 \cdot \sum_{m=0}^{n-2} p(m) \cdot \left[h_2 \cdot \frac{((t_g - t_f) \cdot q_n)^{n-m}}{(n-m)!} + \frac{(-q_n \cdot t_f)^{n-m} \cdot \exp(q_n \cdot t_f)}{t_f \cdot (n-m-1)!} \right] \tag{5c}$$

$$for \quad n \geq 2$$

with

$$h_1 = \exp(-q_h \cdot t_g) + (\exp(-t_f \cdot q_h) - 1) \cdot \frac{q_n}{q_h}$$

$$h_2 = q_h \cdot \exp(-q_h \cdot t_g - q_n \cdot (t_g - t_f))$$

$$\frac{1}{h_3} = h_2 \cdot q_n \cdot \exp(-t_f \cdot q_h)$$

(In Heidemann´s paper /4/, there is a misprint in Eq.(4.2) (Eq.(5b) in this paper). The parameter h_3 is missing in the last term of the equation.)

The distribution function can be obtained by summing the probabilities of the queue lengths (Eqs.(5a)-(5c)):

$$P(n) = \sum_{i=0}^{n} p(i) \tag{5d}$$

The Heidemann´s distribution function of the queue lengths (Eq.(5d)) at unsignalized intersections is a very complex recursive function. The solution for the queue length is in general not possible. Moreover, the 95 and 99 percentile of the queue lengths cannot be established directly.

As the simplest solution for approximating the distribution function of the queue lengths one has now only the distribution function of the M/M/1 queueing system. The Fig.2 shows a comparison between the distribution function of the M/M/1 queueing system (Eq.(2)) and the Heidemann´s distribution function (Eq.(5)). The difference between the two distribution functions can easily be recognized in view of the strongly dispersed data points. The M/M/1 queueing system is accordingly not a very good approximation for the queueing system at unsignalized intersections.

In the following paragraph a new approximation function for the distribution of the queue lengths will be determined by means of a regression. This function approximates the Heidemann´s function with high accuracy and it can very easily be used for the practical applications in establishing the 95 and 99 percentiles of the queue lengths at unsignalized intersection.

3. Results of the Regression

The following functions can be used as the basic function for approximating the distribution of the queue lengths at unsignalized intersections:

- probability of the queue lengths *n*

$$p(0) = 1 - x^{a} \tag{6a}$$

$$p(n) = (1 - x^{ab}) \cdot x^{a\cdot(b\cdot(n-1)+1)} \qquad (\sum_{i=0}^{\infty} p(i) = 1) \tag{6b}$$

- probability distribution function:

$$\begin{aligned} P(n) &= \sum_{i=0}^{n} p(i) \\ &= 1 - x^{a} \cdot x^{a\cdot b\cdot n} \\ &= 1 - x^{a\cdot(b\cdot n+1)} \qquad\qquad (\lim_{n\to\infty} P(n) = 1) \end{aligned} \tag{7}$$

a and *b* are parameters to be determined. *a* and *b* are generally functions of t_g, t_f and q_h.

Eq.(6) and Eq.(7) are generalizations of the probability functions of the queue lengths of the M/M/1 queueing system (Eq.(5)) . The parameters *a* and *b* can be varied in accordance to the given conditions. If one replaces *a* and *b* with 1, the probability functions of the queue lengths of the M/M/1 queueing system are obtained again.

With help of the method of the smallest quadrate, the parameters *a* and *b* for the queueing system at unsignalized intersections can be determined as following:

$$\begin{aligned} a &= \frac{1}{1 + k1 \cdot \dfrac{t_g - t_f}{t_f} \cdot q_h} \qquad with \quad k1 = 0.45 \\ b &= \frac{k2}{1 + k3 \cdot \dfrac{t_g}{t_f} \cdot q_h} \qquad with \quad k2 = 1.51,\ k3 = 0.68 \end{aligned} \tag{8a}$$

The functions of the parameters *a* and *b* are pragmatically chosen. If the *a* and *b* are equal to 1, the result becomes identical to the M/M/1 queueing system, and if $t_g=t_f$, the result approach to the M/G/1 queueing system. The factors *k*1, *k*2 and *k*3 are determined by a regression from approximately 30 000 data points within the range Q_h=100-1200 step 50 (veh/h), Q_n=100-800 step 50 (veh/h) and n= 0-10 step 1 (veh), which are calculated from the Heidemann´s equation (Eq.(5)). In Eq.(8a), if one sets

$$\frac{t_g}{t_f} \approx 2$$

one obtains

$$a \approx \frac{1}{1+0.45 \cdot q_h}$$
$$b \approx \frac{1.51}{1+1.36 \cdot q_h} \qquad (8b)$$

Table.1: t_g and t_f after Harders with V= 50 and 100 km/h

	V=50 km/h						V=100 km/h					
	t_g	t_f	n	s^2 •10^{-5}	s •10^{-3}	Δ_{max} •10^{-2}	t_g	t_f	n	s^2 •10^{-5}	s •10^{-3}	Δ_{max} •10^{-2}
LT from MA	5.16	2.07	2930	1.62	4.02	2.58	8.41	3.96	1180	3.80	6.16	2.29
RT from MI	5.71	2.61	2450	1.33	3.65	2.01	9.35	5.00	780	3.84	6.20	2.39
Crossing	5.80	3.39	2070	1.04	3.22	1.79	9.45	6.45	590	2.55	5.05	2.21
LT from MI	6.38	3.29	1930	1.32	3.63	1.48	10.39	6.29	530	3.70	6.08	2.51

LT=left-turn, RT=right-turn, MA=major stream, MI=minor stream

t_g	t_f	n	s^2 •10^{-5}	s •10^{-3}	Δ_{max} •10^{-2}
6	3.2	2090	1.09	4.47	1.66
3.2	3.2	3210	2.65	5.13	2.29
6	6	1250	1.59	4.00	2.28
5	2.8	2620	0.94	3.07	1.96
10	5	710	5.38	7.33	2.75
4	1.5	3450	2.35	4.85	3.22
15	10	160	4.78	6.91	3.01
1	1	3450	0.01	0.08	0.62
11	11	260	2.12	4.60	1.83

Table.2: t_g and t_f free choices

In Tables 1 and 2, the results of the regressions are listed in detail in terms of the value of the critical time headways t_g and the move-up times t_f. There are altogether 30 000 data points in the spot-checks. The standard deviation s of the approximation to the Heidemann´s equation is for all data groups below $7.0 \cdot 10^{-3}$. The maximal deviations Δ_{max} are limited to $3.5 \cdot 10^{-2}$.

If one substitutes the parameters *a* and *b* (Eq.(8)) into the Eq.(6) and (7), one obtains the complete form of the approximation equations for the probability functions of the queue lengths at unsignalized intersections:

- probability of the queue lengths *n*:

$$p(0) = P(0) = 1 - x^{\frac{1}{1+0.45 \cdot \frac{t_g - t_f}{t_f} \cdot q_h}} \qquad (9a)$$

$$p(n) = P(n) - P(n-1) \tag{9b}$$

- probability distribution function:

$$P(n) = 1 - x^{\frac{1}{1+0.45\cdot\frac{t_g - t_f}{t_f}\cdot q_h}\cdot\left(\frac{1.51}{1+0.68\cdot\frac{t_g}{t_f}\cdot q_h}\cdot n+1\right)} \tag{10}$$

Eqs.(9) and (10) should only be used within the following data ranges:

$$t_g = \quad 1 \quad to \quad 15s$$

$$\frac{t_f}{t_g} = 0.35 \;\; to \;\; 1 \qquad (\Delta p_{\max} < 0.035)$$

In these data ranges, the maximal expected deviations of the distributions function are not greater than 0.035 (=3.5%) in comparison with the results of the Heidemann´s approach.

Fig.3 shows the development of the distribution function (Eq.(10)) with respect to the saturation degree *x*.

The results from the approximation for the distribution function (Eq.(10)) and the results from Heidemann (Eq.(6)) are compared in Fig.4. The agreement is good. Fig.5 shows the differences between the results according to Eq.(10) and to Eq.(6) on a larger scale.

For checking the results of the approximation (Eq.(10)), it has been compared with the results from the simulations (KNOSIMO /7/) also. Fig.5 shows this comparison and indicates again a good agreement. The relatively larger deviation between the results of the approximation and the results of the simulation can be explained by the facts, that in the simulation a la KNOSIMO

- the time headways in the major stream are hyper-erlang distributed,
- the critical time headways t_g and move-up times t_f for the minor stream are shift-erlang distributed,
- the number and the duration of the simulation are limited and
- the simulation is according to nature subject to stochastic variations.

So far one can distinguish the Heidemann´s mathematical assumptions and the realistic conditions after the model KNOSIMO.

4. Possible Applications of the Approximation Formula

The approximation formula for describing the distribution function of the queue lengths at unsignalized intersections (Eq.(10)) can be used for the following applications:

- Under stationary traffic conditions

1. Probability of oversaturation for the left-turn lane with *n* possible queueing positions $P_{os}(n)$:

$$P_{os}(n) = 1 - P(n) = x^{a\cdot(b\cdot n+1)} \tag{11}$$

2. Saturation degree x with given percentile of the queue lengths N_α:

$$x = (1 - P(N_\alpha))^{\frac{1}{a\cdot(b\cdot N_\alpha+1)}} = (1 - \frac{\alpha}{100})^{\frac{1}{a\cdot(b\cdot N_\alpha+1)}} \tag{12}$$

e.g., if a queue lengths of N=10 vehicles may not be exceeded in 95% of the time, i.e., N_{95}=10 - one must set

$$x \le (1-0.95)^{\frac{1}{a\cdot(b\cdot 10+1)}} = (0.05)^{\frac{1}{a\cdot(b\cdot 10+1)}}$$

3. Percentile of the queue lengths N_α:

$$N_\alpha = (\frac{\ln(P_{os}(n_\alpha))}{a\cdot\ln(x)} - 1)\cdot\frac{1}{b} = (\frac{\ln(1-P(n_\alpha))}{a\cdot\ln(x)} - 1)\cdot\frac{1}{b} = (\frac{\ln(1-\frac{\alpha}{100})}{a\cdot\ln(x)} - 1)\cdot\frac{1}{b} \tag{13a}$$

e.g., the queue length, which will not be exceeded in 95% of the time is

$$N_{95} = (\frac{\ln(1-0.95)}{a\cdot\ln(x)} - 1)\cdot\frac{1}{b} = (\frac{\ln(0.05)}{a\cdot\ln(x)} - 1)\cdot\frac{1}{b} \tag{13b}$$

4. Average queue length N_0 :

$$N_0 = \sum_{n=0}^{\infty} p(n)\cdot n = \frac{x^a}{(1-x^{a\cdot b})} \tag{14}$$

5. Average delays W:

$$W = \frac{N_0}{q_n} = \frac{x^{a\cdot}}{q_n\cdot(1-x^{a\cdot b})} \tag{15}$$

- Under nonstationary traffic conditions

By nonstationary traffic one means the traffic state in which the traffic flow is not always of constant value over the time. Also the queue lengths of all types (average queue length, 95 and 99 percentile of the queue lengths) depend on the time. The consideration of the nonstationarity is limited to the handling of only a certain time section (e.g. the peak period). The average values of the queue lengths shall be established over this time section (time interval T).

The distribution function of the queue lengths at unsignalized intersection can be obtained by using the transformation technique (derivation in appendix G).

0. Probability distribution function $P(n)$:

$$P(n) = \begin{cases} 1-(\bar{x}-\frac{2\cdot n}{QT})^{a\cdot(b\cdot n+1)} & \text{für } \bar{x}-\frac{2\cdot n}{QT} \ge 0 \\ 1 & else \end{cases} \tag{16}$$

1. Probability of oversaturation for the left-turn lane with n possible queueing positions $P_{os}(n)$:

$$P_{os}(n) = 1-P(n) = \begin{cases} (\bar{x}-\frac{2\cdot n}{QT})^{a\cdot(b\cdot n+1)} & \text{für } \bar{x}-\frac{2\cdot n}{QT} \ge 0 \\ 0 & else \end{cases} \tag{17}$$

2. Average saturation degree $\underline{\bar{x}}$ with a given percentile N_α:

$$\bar{x} = \frac{2\cdot N_\alpha}{QT} + (P_{os}(N_\alpha))^{\frac{1}{a\cdot(b\cdot N_\alpha+1)}} = \frac{2\cdot N_\alpha}{QT} + (1-P(N_\alpha))^{\frac{1}{a\cdot(b\cdot N_\alpha+1)}} = f(N_\alpha) \tag{18}$$

3. Percentile of the queue lengths N_α:

$$N_\alpha = f^{-1}(N_\alpha) = g(\bar{x}(N_\alpha)) \tag{19}$$

Eq.(19) is the inverse function of Eq.(18). Since N_α in Eq.(18) is not solvable, it can not be presented as a explicit function of $\underline{\bar{x}}$. But N_α is implicitly and unequivocally

defined. However, with help of the Eq.(18), nomographs for establishing the 95 and 99 percentile of the queue lengths at unsignalized intersection can be produced.

For Eqs.(11)-(19), the parameters a and b are given by the Eq.(8).

Fig.7 shows a comparison between the distribution function of the queue lengths under stationary traffic conditions (Eq.(10)) and the distribution function of the queue lengths under nonstationary traffic conditions (Eq.(16)).

Fig.8 shows a comparison between the 95 percentile of the queue lengths under stationary traffic conditions (Eq.(12) or (13)) and the 95 percentile of the queue lengths under nonstationary traffic conditions (Eq.(18) or (19)).

For establishing the 95 and 99 percentile of the queue lengths, nomographs are produced. These nomographs could be easy used for the practical applications. They are enclosed in the Appendices A to D.

- Streams of higher ranks

The derivations above concentrate on only one major stream and one minor stream. They are only valid for streams of the second rank (in the sense of [8]) at unsignalized intersections. The formulas derived are only applicable for left-turn streams from a major street. It is not possible using these formulas for proving (calculating) the queue lengths in streams of higher ranks (left-turns and right-turns from minor streets, crossings from minor streets) or for share lanes.

Because of the increase of the complexity of the traffic features (more than one major streams, different critical time headways t_g and different move-up times t_f), the queueing system approaches to the M/M/1 queueing system. Therefore, one can approximately use the formulas of the M/M/1 queueing system for calculating the queue lengths in the streams of higher ranks or in the streams of a shared-lane. The resulting expected deviation of the queue lengths are normally not greater than one vehicle. Fig.9 shows the comparison between the 95 percentile of the M/G2/1 queueing system (Heidemann) and of the M/M/1 queueing system. If one replaces the parameters a and b in all prevailing formulas with the value 1, one obtains automatically the simplified formulas for calculating the queue lengths under stationary or nonstationary traffic conditions. Appendices E and F contain the nomographs for establishing the 95 and 99 percentile of the queue lengths from the M/M/1 queueing system. One can obtain the expected percentile of the queue lengths at the y-axis with given saturation degree x and total capacity QT (in veh) of the minor stream in the considered time interval T. Setting $QT=\infty$ (i.e.: $T = \infty$), the result for the stationary traffic condition is obtained.

5. Conclusions and Open Questions

- Conclusions

The approximations of the distribution function of the queue lengths at unsignalized intersections under stationary traffic conditions (Eq.(10)) and nonstationary traffic conditions (Eq.(16)) were determined by regressions. With the help of the approximations, several formulas can be obtained for calculating other traffic parameters. For example, one can calculate the percentile of the queue lengths from Eq.(13) and Eq.(19). Also, approximation formulas for calculating the average queue length and the average delay (only under stationary traffic conditions) can be obtained (Eq.(14) and Eq.(15)).

For establishing the 95 and 99 percentiles of the queue lengths, nomographs were produced for practical applications. Appendices A to D show the nomographs of the 95

and 99 percentile of queue lengths for the left-turn lanes of the major street. These nomographs can be used, e.g., for designing the lengths of the left-turn lanes:

- **Appendix A**: 95-percentile of the queue lengths under **stationary** traffic conditions
- **Appendix B**: 99-percentile of the queue lengths under **stationary** traffic conditions
- **Appendix C**: 95 percentile of the queue lengths under **nonstationary** traffic conditions
- **Appendix D**: 99 percentile of the queue lengths under **nonstationary** traffic conditions

The nomographs were produced separately for the speed limit on the major street V=50, 70 and 90 km/h. The critical time headways t_g and move-up times t_f after [8] were used in depend on the speed limit. The percentile of the queue lengths can be obtained at the y-axis in these nomographs . The input parameters are

- traffic flow of the major stream (Q_h) in veh/h and
- traffic flow of the minor stream (left-turn from the major street) (Q_n) in veh/h.

For calculating 95 and 99 percentile of the higher ranks, the formulas for the M/M/1 queueing system were recommended. These formulas could easily be derived by setting a=1 and b=1 in all the formulas for the M/G2/1 queueing system. Also, nomographs for establishing queue lengths of higher ranks were produced:

- **Appendix E**: 95 percentile of queue lengths under **stationary** and **nonstationary** traffic conditions
- **Appendix F**: 99 percentile of queue lengths under **stationary** and **nonstationary** traffic conditions

The percentile of the queue lengths can be obtained at the y-axis in these nomographs. The input parameters are

- saturation degree x of the minor stream and
- total capacity QT of the minor stream in considered time interval T in veh

Setting $QT=\infty$ (i.e.: $T = \infty$) here, the result under stationary traffic conditions could be obtained.

The nomographs are to be used according to the recommendations in Table 4.

Property of the stream	in peak period	in normal hours
stream of the 2. rank	Appendices C and D	Appendices A and B
stream of higher ranks	Appendices E and F (lines $QT\neq\infty$)	Appendices E and F (line $QT.=\infty$)

Tab. 3 :Recommendations for using the nomographs:

"Peak period " means the traffic flow in the considered time interval T is distinctively larger, i.e., at least 15% larger, than the traffic flow beyond it. "normal hours" means the traffic flow is roughly constant over all of time.

- Open questions

The result of the M/G2/1 queueing system from Heidemann is based on the following assumptions:

- the time headways in the major stream (q_h) are negative-exponential distributed,
- the critical time headways t_g and move-up times t_f for the minor stream (q_n) are constant and
- the vehicle arrivals in the minor stream are Poisson distributed.

However, the following questions are still open for calculating the queue lengths with the M/G2/1 queueing system:

1. the effect of the punching property in the major stream. e.g., the time headways in the major stream are not negative-exponential but hyper-erlang distributed.
2. the effects of the distribution of the critical headways t_g and of the move-up times t_f. e.g., the critical time headways t_g and move-up times t_f are not constant but shift-Erlang distributed.
3. the effect of the punching property in the minor stream. e.g., the time headways are not negative-exponential but hyper-Erlang distributed.

The three questions above can only be answered qualitatively as following:

1. the punching property in the major stream deceases the capacity of the minor stream and therefore increases the queue lengths of all types in the minor stream (cf./4/).
2. the distribution of the critical time headways t_g decreases the capacity of the minor stream and therefore increases the queue lengths of all types; the distribution of the move-up times t_f increases the capacity of the minor stream and therefore decreases the queue lengths of all types (cf./4/).
3. the punching property in the minor stream decreases the average queue length in the minor stream (cf./10/) and therefore decreases the queue lengths of other types.

Considering the three effects together, one finds that they tend to neutralize one another. It can be assumed that together they would affect the queue lengths insignificantly.

The discussion on the calculation of the queue lengths in the streams of higher ranks could not be completed. Only a pragmatic solution can be recommended, which is based on the M/M/1 queueing system. This solution offers sufficient accuracy for proving (determining) the traffic quality in the practices.

Since the nomographs in Appendices E and F are only dependent on the total capacity QT of the minor stream in the considered time interval T and on the saturation degree x of the minor stream, they should also be used for establishing the 95 and 99 percentile of the queue lengths in the streams of the second rank in case only the parameters QT and x in the minor stream are given. An allowance must be made for deviations. These deviations are normally not greater than 1 vehicle.

Finally, one can ascertain that the derivations in this paper offered a useful method for calculating queue lengths and their percentiles at unsignalized intersections. The method can easily be used by the traffic engineers in the practice.

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(No more in use)

Fig.1 - Simulated distribution of the queue lengths from KNOSIMO

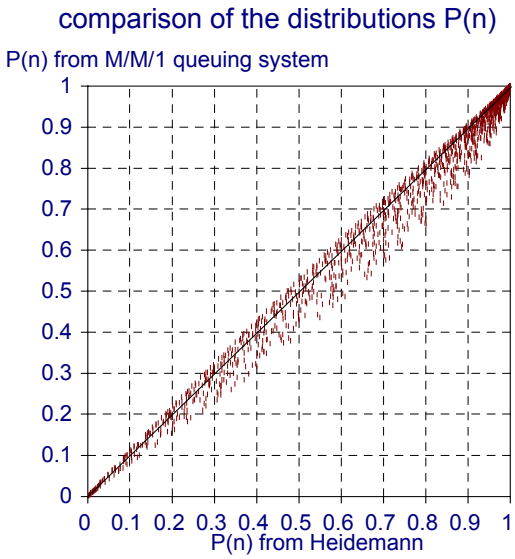


Fig.2 -Comparison M/M/1 vs. Heidemann

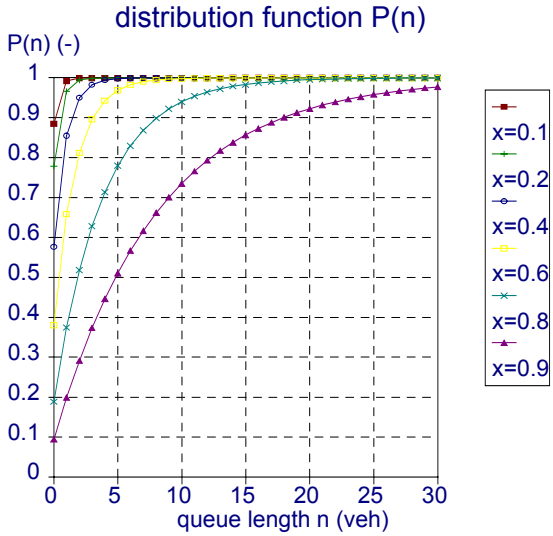
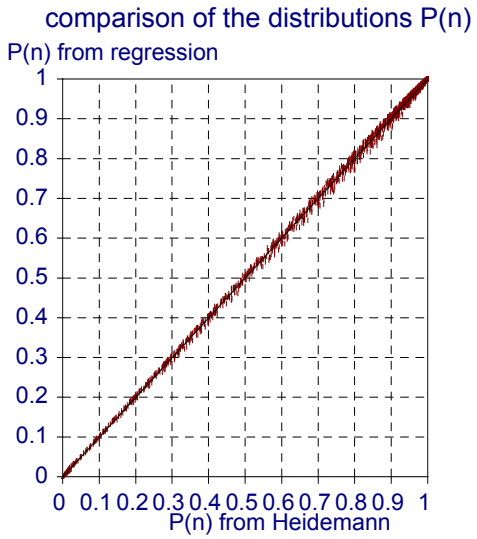


Fig.3 - Distribution functions



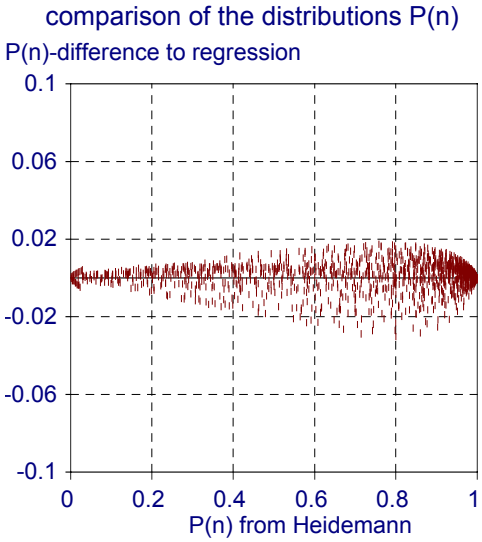
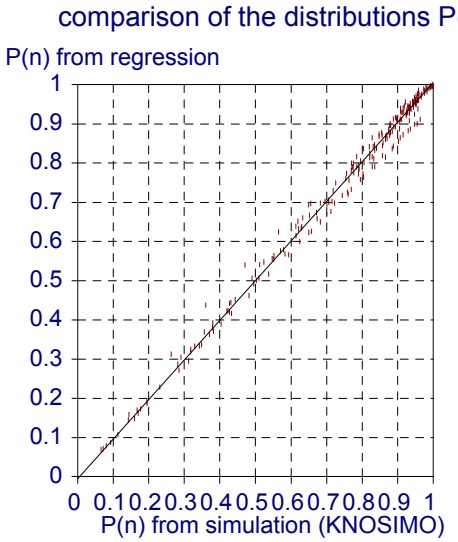


Fig.4 - Comparison Regression vs. Heidemann**Fig.5** - Difference Regression vs. Heidemann



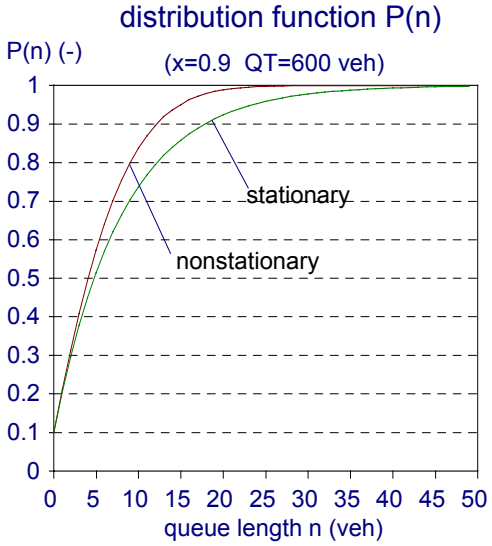
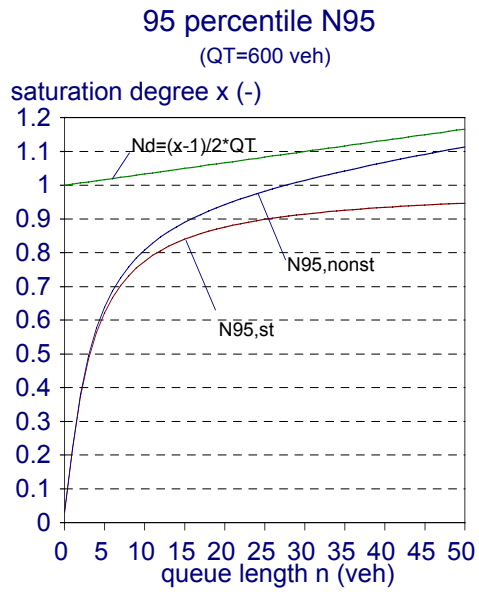


Fig.6 - Comparison Regression vs. KNOSIMO**Fig.7** - Comparison Stationary vs. Nonstationary



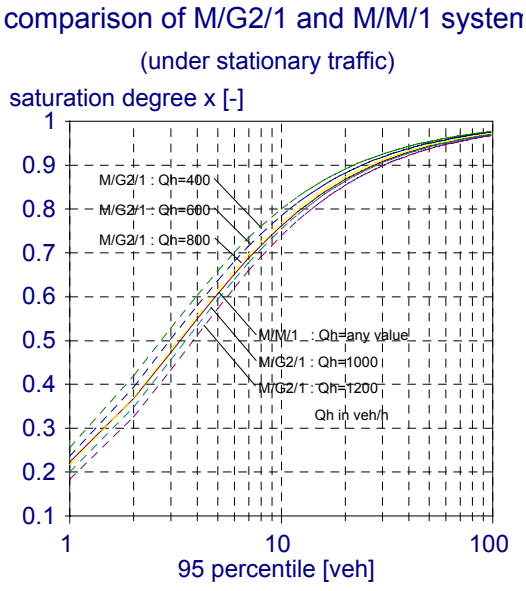


Fig.8 - Comparison of the N95

Fig.9 - Comparison M/G2/1 vs. M/M/1

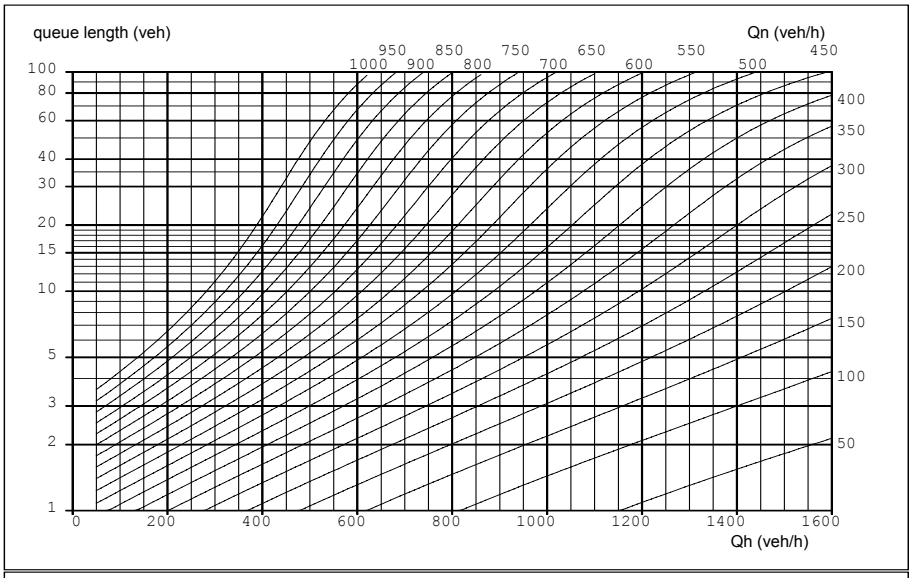
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Appendix A - 95 percentile of the queue lengths for **Streams of the second rank** under **stationary** traffic conditions (Recommendation : for **normal hours)**

(No more in use)

Appendix B - 99 percentile of the queue lengths for **Streams of the second rank** under **stationary** traffic conditions (Recommendation : for **normal hours)**

(No more in use)



for:

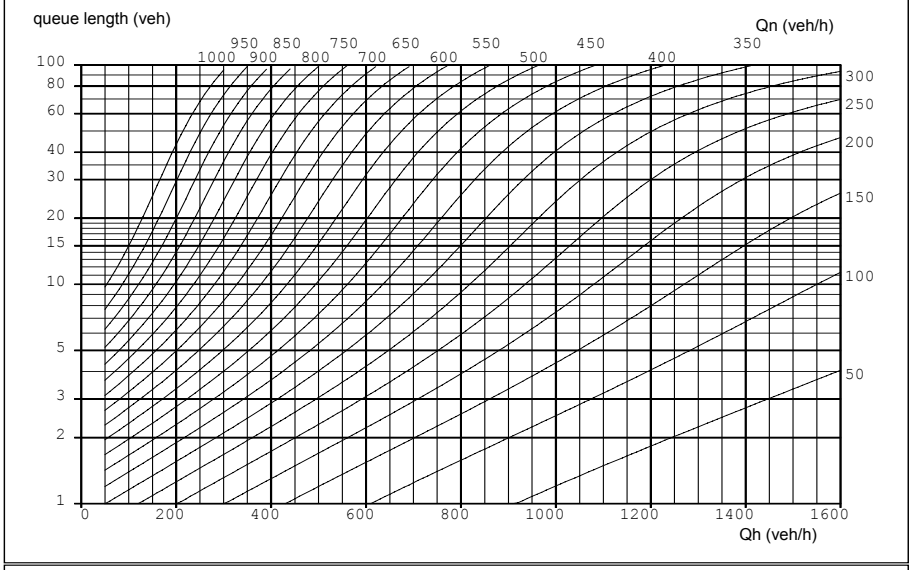
$V = 50$ km / h

$t_g = 5.2$ s

$t_f = 2.1$ s

$T = 1$ h

$x_\infty \leq 0.85$



for:

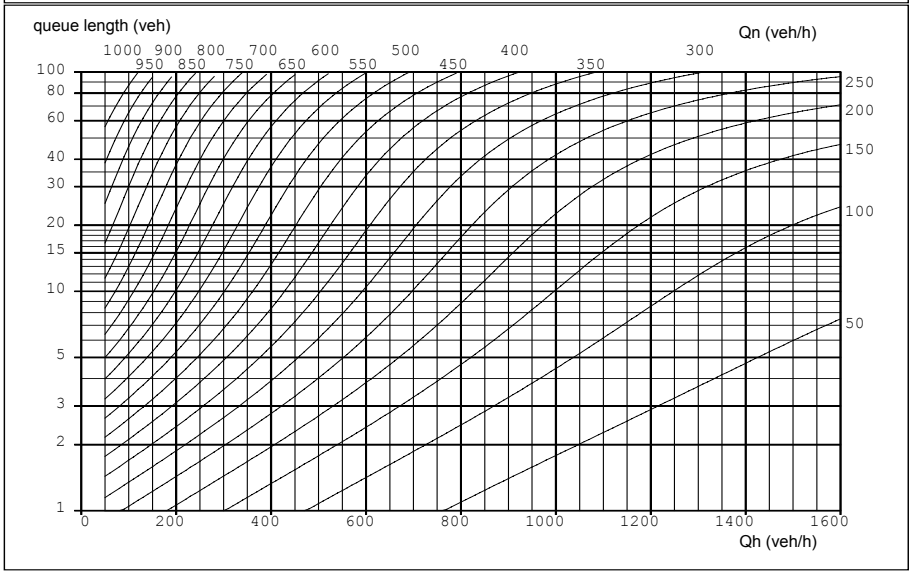
$V = 70$ km / h

$t_g = 6.6$ s

$t_f = 2.8$ s

$T = 1$ h

$x_\infty \leq 0.85$



for:

$V = 90$ km / h

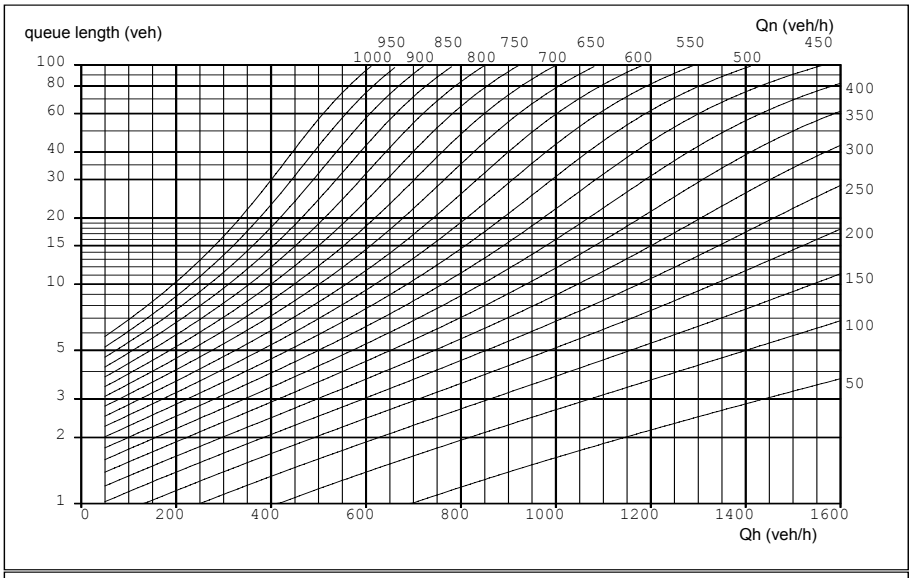
$t_g = 7.8$ s

$t_f = 3.6$ s

$T = 1$ h

$x_\infty \leq 0.85$

Appendix C - 95 percentile of the queue lengths for **Streams of the second rank** under **nonstationary** traffic conditions (Recommendation : for **peak period)**



for:

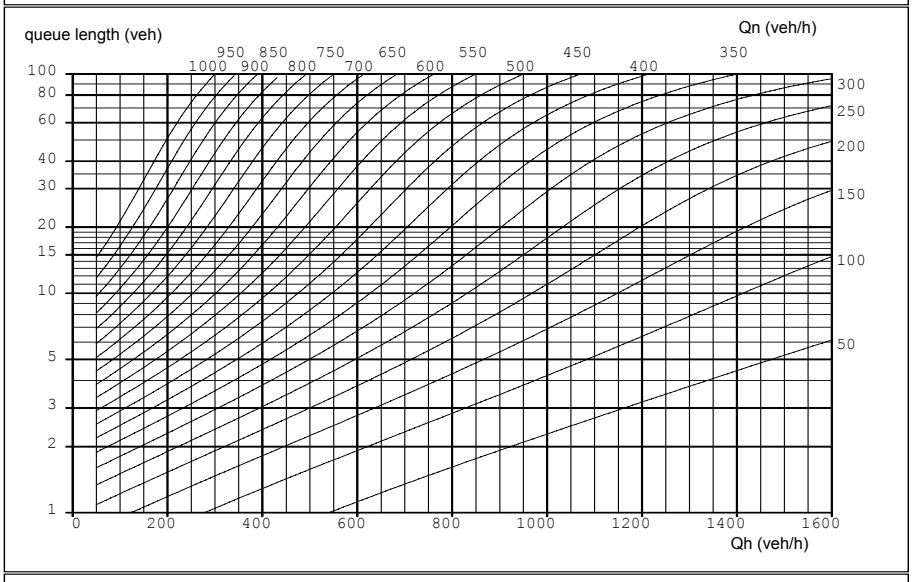
$V = 50$ km / h

$t_g = 5.2$ s

$t_f = 2.1$ s

$T = 1$ h

$x_\infty \leq 0.85$



for:

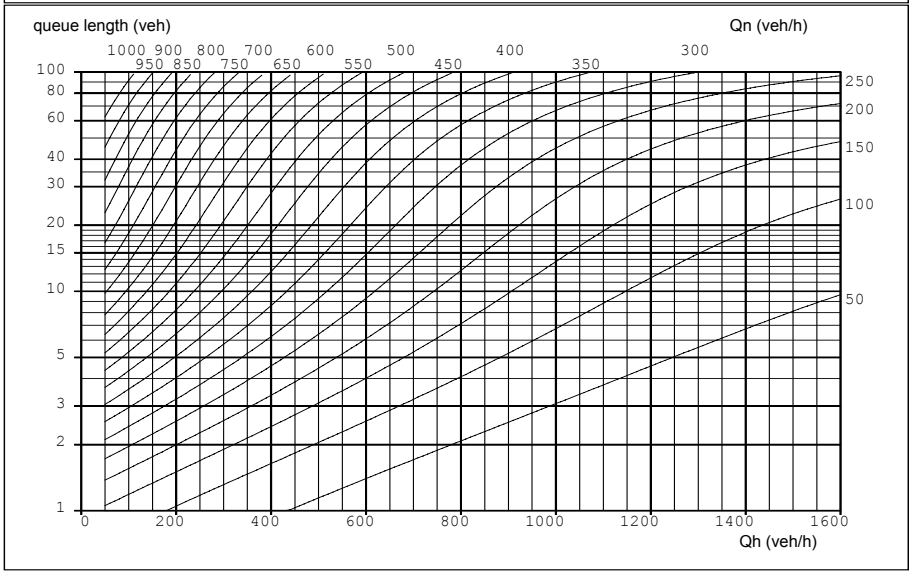
$V = 70$ km / h

$t_g = 6.6$ s

$t_f = 2.8$ s

$T = 1$ h

$x_\infty \leq 0.85$



for:

$V = 90$ km / h

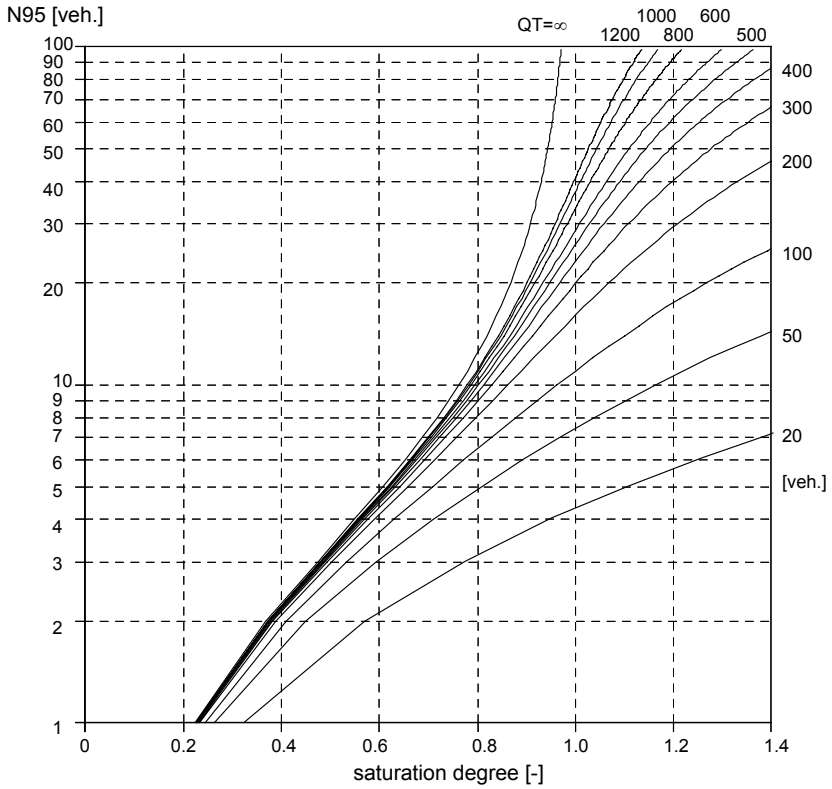
$t_g = 7.8$ s

$t_f = 3.6$ s

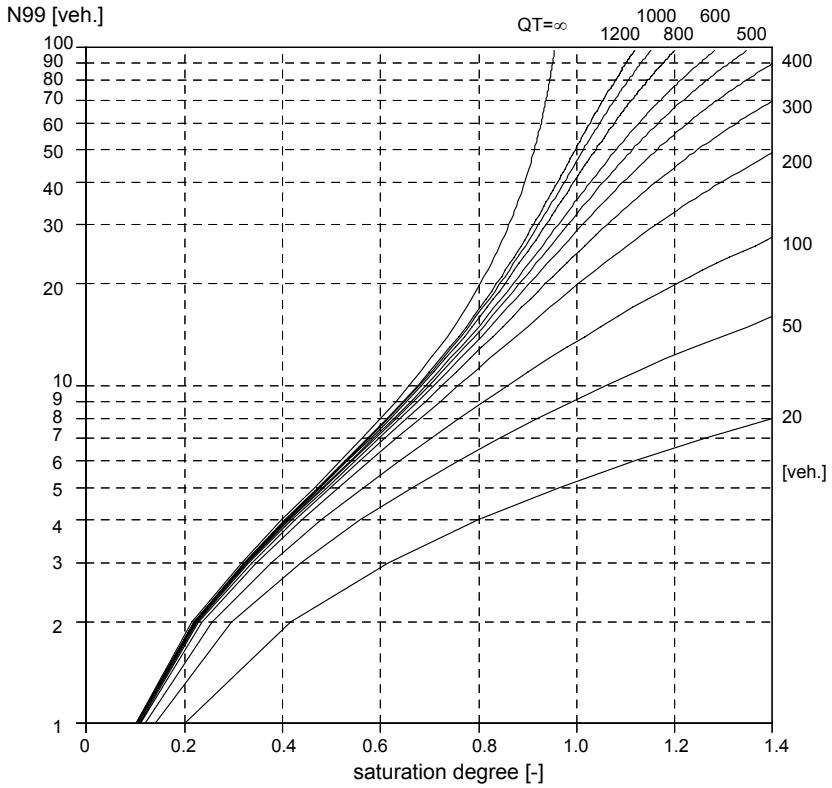
$T = 1$ h

$x_\infty \leq 0.85$

Appendix D - 99 percentile of the queue lengths for **Streams of the second rank** under **nonstationary** traffic conditions (Recommendation : for **peak period)**



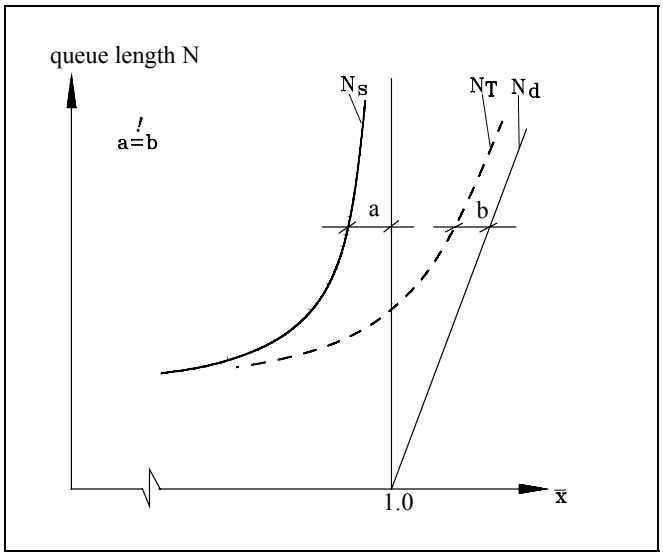
Appendix E - 95 percentile of the queue lengths for **Streams of higher ranks** under **stationary** and **nonstationary** (for $x_\infty \leq 0.85$) traffic conditions



Appendix F - 99 percentile of the queue lengths for **Streams of higher ranks** under **stationary** and **nonstationary** (for $x_\infty \leq 0.85$) traffic conditions

Appendix G : Derivation of Eq.(16) and (17)

The transformation of a equation under **nonstationary and stochastic** arrivals N_T can obtained by transiting the equation under **stationary and stochastic** arrivals N_s and the equation under **nonstationary but deterministic** arrivals N_d. The principle of the transformation can be shown in the Fig. below (see also Fig.8, cf./5//11/).



The key of the transformation is the postulation **distance a = distance b** for the equal queue lengths

$$n=N_s=N_T=N_d \tag{G.0}$$

e.g.,

$$1-\bar{x}_s(N_s)=\bar{x}_d(N_d)-\bar{x}_T(N_T) \tag{G.1}$$

or

$$\bar{x}_T(N_T)=\bar{x}_d(N_d)-(1-\bar{x}_s(N_s)) \tag{G.2}$$

From the Eq.(12) one has

$$\bar{x}_s(N_s)=(P_{\ddot{u}}(N_s))^{\frac{1}{a\cdot(b\cdot N_s+1)}} \tag{G.3}$$

Since no distribution under deterministic conditions exist, all queue lengths (average queue length, percentiles of the queue lengths etc.) in the considered time interval T are always the same (mean value over the time T)

$$N_d=\frac{(\bar{x}_d-1)\cdot QT}{2}+N_\infty \tag{G.4}$$

With the assumption that

$$x_\infty \le 0.85 \quad \text{and} \quad N_\infty \approx 0$$

one obtains

$$\bar{x}_d(N_d)=\frac{2\cdot N_d}{QT}+1 \tag{G.5}$$

Substituting (G.5) and (G.3) in (G.1), one obtains

$$1-(P_{os}(N_s))^{\frac{1}{a\cdot(b\cdot N_s+1)}} = \frac{2\cdot N_d}{QT}+1-\bar{x}_T(N_T) \qquad \text{(G.6)}$$

Solving (G.6) for $P_{os}(n)$, one obtains

$$P_{os}(N_s) = (\bar{x}_T(N_T) - \frac{2\cdot N_d}{QT})^{a\cdot(b\cdot N_s+1)} \qquad \text{(G.7)}$$

This equation is only then meaningful, if and only if

$$\bar{x}_T(N_T) - \frac{2\cdot N_d}{QT} \geq 0 \qquad \text{(G.8)}$$

Because of the necessary condition (G.0) that all of the queue lengths N_s, N_T, and N_d are equal, one can replace them in (G.7) with a general symbol n . Replacing there the $\underline{\bar{x}}_T$ with the general symbol $\underline{\bar{x}}$ also, one obtains

$$P_{os}(n) = 1-P(n)$$
$$= \begin{cases} (\bar{x}-\frac{2\cdot n}{QT})^{a\cdot(b\cdot n+1)} & for\ \bar{x}-\frac{2\cdot n}{QT}\geq 0 \\ 0 & else \end{cases} \qquad \text{(G.9)}$$

That is the Eq.(17). Also, with the relation

$$P(n) = 1-P_{os}(n) \qquad \text{(G.10)}$$

one obtains the Eq.(16)

$$P(n) = \begin{cases} 1-(\bar{x}-\frac{2\cdot n}{QT})^{a\cdot(b\cdot n+1)} & for\ \bar{x}-\frac{2\cdot n}{QT}\geq 0 \\ 1 & else \end{cases} \qquad \text{(G.11)}$$