A UNIVERSAL PROCEDURE FOR CAPACITY DETERMINATION AT UNSIGNALIZED (PRIORITY-CONTROLLED) INTERSECTIONS

Ning Wu
Institute for Transportation and Traffic Engineering, Ruhr-University, 44780 Bochum, Germany
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ning.wu@ruhr-uni-bochum.de
http://ningwu.verkehr.bi.ruhr-uni-bochum.de

ABSTRACT

This paper introduces a universal procedure for calculating the capacity at unsignalized (priority-controlled) intersections with an arbitrary number of streams of arbitrary priority ranks. This procedure can handle all possible stream and lane configurations (e.g., number of lanes and ranks of streams, etc.) at unsignalized intersections.

First, a overview of all common approaches for the simplest configuration with one major stream and one minor stream is given. From the basic idea of Heidemann and Wegmann (1997) some new explicit capacity formulae are derived considering the distributions of critical gaps, move-up times, and minimum time headways between two vehicles going in succession. Relationships between the common approaches can be derived.

A new universal formula for the simplest configuration with one major stream and one minor stream is then introduced. The formula is based on the idea that the time scale of the major stream can be divided into four regimes according to the relative positions between the vehicles in the major stream: 1) that of free space (no vehicle), 2) that of single vehicle, 3) that of bunching, and 4) that of queuing. The probability of these regimes can be calculated according to the queuing theory. Therefore, the capacity of the minor stream that depends predominantly on the probability of the state that no vehicle blocks the major streams (state of free space) can also be calculated.

Starting from the universal formula for the simplest configuration with one major stream and one minor stream, a universal calculation procedure for configurations with arbitrarily many streams of arbitrary ranks is derived. This procedure is built up according to the parallel or serial configurations of the streams. For serial configurations of streams of different ranks, a special procedure for estimating the total probability of the state of free space within the major streams is developed.

The present procedure is derived mathematically using queuing theory. It generalizes all of the known procedures for calculation capacities at unsignalized intersection. The model is calibrated and verified by measurements at roundabouts and by intensive simulations. Based on the theoretical background, the model can easily be extended to other priority systems with arbitrary priority ranks. The results of the present procedure are already incorporated into the 2000 German Highway Capacity Manual.

1 INTRODUCTION

Unsignalized intersections (priority-controlled intersections) are the mostly used type of road junctions in highway transportation systems. The capacity at these intersections is thereby one of the most researched topics in traffic science and engineering. The capacity of a traffic facility describes the maximum possible throughput of the facility under predefined conditions. Starting from the capacity, further traffic parameters which represent traffic quality can be calculated.

At unsignalized intersections, there are traffic streams which have different ranks in the priority hierarchy. Depending on which stream is considered different queuing systems result. For calculating the capacity of these queuing systems different procedures should be used.

The procedures for calculating the capacity can basically be divided into two groups:

- Calculation of the capacity of a simple queuing system with two streams: one major stream and one minor stream.
- Calculation of the capacity of a comprehensive queuing system with more than two streams of different rank in the priority regulation.

In the group "queuing systems with one major stream and one minor stream", a large variety of calculation methods which yield the corresponding accuracy depending on the assumed traffic conditions exists. Here, there exist firstly mathematical solutions that based on the theory of stochastic processes and gap-acceptance. In the group "queuing systems with more than two streams", only one pragmatic procedure exits for practice uses. This procedure was developed in Germany and has also found broad applications in other countries.

In this paper, most of the known procedures in the category "queuing systems with one major stream and one minor stream" are compiled. They are divided according to their properties into groups. The relationships between these procedures are depicted. The available procedures are then extended and generalized to include further parameters. A new procedure which represents a generalization of the procedures for "queuing systems with one major stream and one minor stream" and "queuing systems with more than two streams" is presented.

The new development corrects and completes the procedure for the calculation of capacities at unsignalized intersections. It is derived conclusively and it can be applied simply in practice. This procedure has a systematic structure which allows extension to more complicated systems.

In this paper, the following notations and symbols are used:

$L(t(q))$	=	notation for Laplace transform of t at q	
$E(x)$	=	notation for expected value of x	
Pr()	=	notation for probability	
—	=	notation for mean value	
|	=	notation for "under condition"	
C	=	capacity of the minor stream	[veh/s]
C_s	=	capacity of the minor stream in the Free-space state	[veh/s]
f(t)	=	distribution intensity of gaps t in the major stream	[-]
F(t)	=	distribution function of gaps t in the major stream	[-]
g(t)	=	function for the number of vehicles which can depart during t	[veh/s]
N	=	mean queue length in the M/M/1 queuing system	[veh]

p_n	=	probability that n vehicle arrive during t	[-]
$p_{0,S}$	=	probability for the state of Queuing-free	[-]
$p_{0,B}$	=	probability for the state of Bunching-free | Queuing-free	[-]
$p_{0,F}$	=	probability for the state of Vehicle-free | (Bunching-free | Queuing-free)	[-]
q_f	=	$\frac{\varphi \cdot q_p}{1 - q_p \cdot \tau}$ = traffic intensity within the portion of free traffic	[veh/s]
q_p	=	traffic intensity in the major stream	[veh/s]
t	=	length of a time headway in major stream (gap)	[s]
t_0	=	$t_g - \frac{t_f}{2}$ = zero-gap	[s]
t_f	=	move-up time	[s]
t_g	=	critical gap	[s]
x	=	saturation degree of the queuing system	[-]
α	=	parameter of the Erlang-distribution	[-]
φ	=	portion of free traffic in the major stream	[-]
τ	=	minimum gap between two vehicles going in succession	[s]
τ_{tf}	=	minimum for t_f	[s]
τ_{tg}	=	minimum for t_g	[s]
τ_τ	=	minimum for τ	[s]

2 QUEUING SYSTEM WITH ONE MAJOR STREAM AND ONE MINOR STREAM

The queuing system with only one major stream and one minor stream is a so-called M/G2/1 queuing system. For this system, many mathematical approaches have been developed. These approaches have their validity under different predefined traffic conditions. These conditions are:

- free and bunched traffic
- discrete and continuous departure
- const. and distributed critical gap t_g, move-up time t_f , and minimum gap τ
- consistent and inconsistent driver behavior

For these different conditions, deferent different formulae can be obtained.

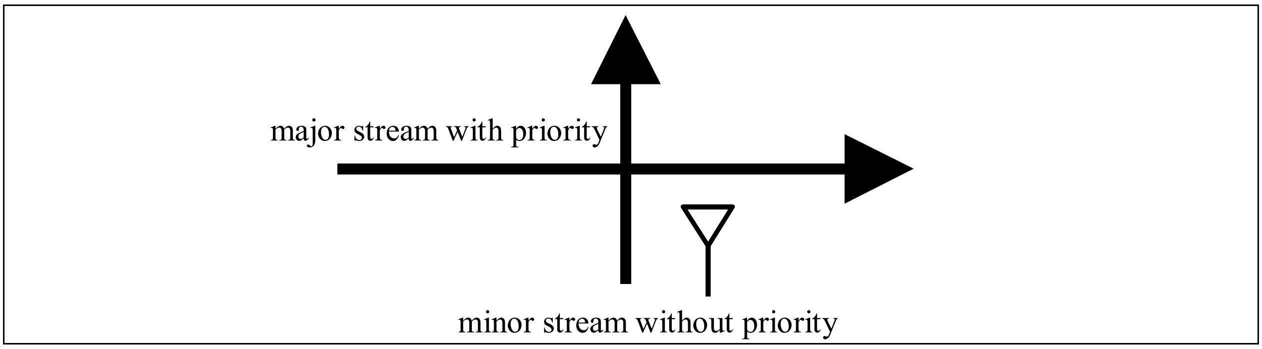


Fig. 1 - System with a major stream and a minor stream

A queuing system with two crossing streams (Fig.**1**) is now considered. The major stream has the right of priority and can drive through without stopping at the intersections. The minor stream has to give way to the major stream and stop appropriately. A vehicle from the minor stream can only depart crossing the major stream (or merging into the major stream), when a large time headway (gap) is offered between two vehicles in the major stream. The classic procedure for the determination of capacity is based on the calculation of the distribution of gaps in the major stream and on the calculation of the number of vehicles which can depart during a gap within the major stream. Accordingly, the capacity of the minor stream, C, is given by

$$C = q_p \cdot \int_0^\infty f(t) \cdot g(t) \cdot dt \qquad (1)$$

(cf. Siegloch, 1973).

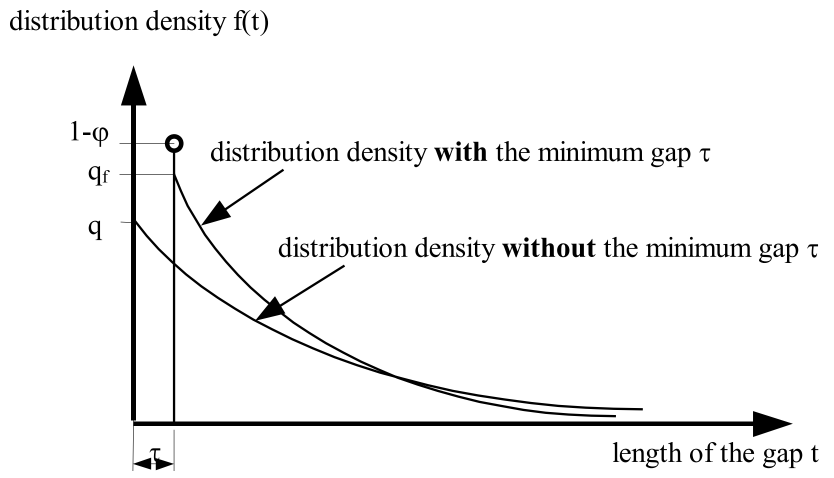


Fig.2 - Shape of the probability density of gaps in the major stream f(t)

Here, f(t) is the probability density of gaps t in the major stream, g(t) is the function for the number of vehicles which can depart during a gap of the length t, and q_p is the traffic intensity per unit of time in the major stream. Eq.(1) indicates the sum of vehicles departing during all gaps in the major stream: the capacity C in vehicle per unit of time. Depending on which function for f(t) and g(t) is used, different formulae for the determination of capacity C can result.

2.1 Free and bunched traffic flow in the major stream

For choice of functions of the probability density of gaps, f(t), two assumptions modeling the traffic flow in the major stream are presupposed:

- free traffic flow in the major stream
  Under free traffic flow it is assumed that a vehicle does not influence the vehicles going behind him. Mathematically means: the arrivals of vehicles which go in succession are by chance and absolutely independent of each other; the gaps between two vehicles can also take the value of zero.
- bunched traffic

Under bunched traffic flow it is assumed that between two vehicles which go in succession a minimum gap has to be held. From this assumption a different distribution of gaps, compared to that for the free traffic flow, can be obtained.

Clearly, these assumptions are only true under the predefined conditions.

Under the assumption that the arrivals of vehicles in the major stream are completely coincidental (free), the probability density of gaps t between two vehicles is

$$f(t) = q_p \cdot e^{-q_p \cdot t} \tag{2}$$

with q_p = mean traffic intensity in the major stream in [veh/time unit]

That is, the gaps t are negative-exponentially distributed.

If the arrivals of vehicles in the major stream are not completely stochastic but depend on the vehicle in the front, then the traffic in the major stream is no more completely free. A vehicle must keep a minimum gap τ to the vehicle in the front and drive in succession. One speaks in this case of bunched traffic. The distribution of gaps in the bunched major stream can be described with the shifted-negative-exponentially distribution. The probability density of the shifted-negative-exponentially distributed gaps t reads (Cowan, 1975):

$$f(t) = q_f \cdot e^{-q_f \cdot (t-\tau)} \tag{3}$$

with φ = portion of free traffic within the major stream

$q_f = \dfrac{\varphi \cdot q_p}{1 - q_p \cdot \tau}$ = traffic density within the portion of free traffic

τ = minimum gap between two vehicles going in succession

The relationship between q and q_f is given by

$$(1-\varphi) \cdot \tau \cdot q_p + \varphi \cdot (\tau + \frac{1}{q_f}) \cdot q_p = (\tau + \frac{\varphi}{q_f}) \cdot q_p = 1 \tag{4}$$

This equation assumes that the mean length of gap t has within the bunched portion of the traffic the value τ and within the free portion of the traffic the value $\tau + 1/q_f$. The portion of free traffic φ within the major stream describes the portion of the vehicles which go in succession with a gap $t > \tau$. φ depends in general on the traffic intensity q_p in the major stream. In the case of free input, i.e., the up-stream traffic in the major stream is considered as absolutely coincidental, bunched traffic is only caused by compliance with the minimum gap τ. Under the assumption that keeping of a minimum gap τ affects the vehicles within the major stream like a M/D/1-queuing system, Tanner (1962) specified the portion of the free traffic by

$$\varphi = 1 - q_p \cdot \tau \tag{5}$$

Jacobs (1980) proposed the estimate of the portion of the free traffic as following

$$\varphi = e^{-k \cdot q_p} \tag{6}$$

In this case, k is a parameter with a value between 4 and 9.

Fig.2 shows a schematic representation of both kinds of the probability density f(t).

2.2 Departure from the minor stream through (or into) the major stream

During choice of functions for the number of departures from the minor stream crossing (or merging into) the major stream two usual models with two different assumptions are available for the function g(t):

- discrete departure from the minor stream
- continuous departure from the minor stream

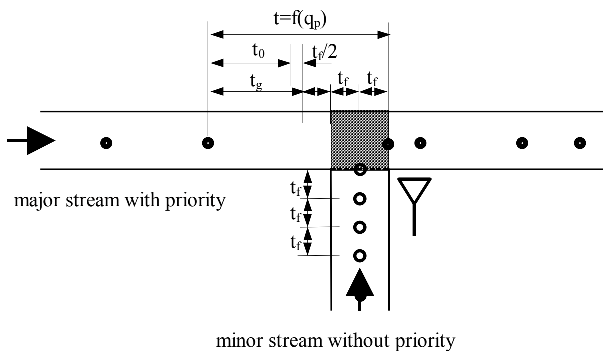


Fig. 3 - Departure mechanism with a free major stream

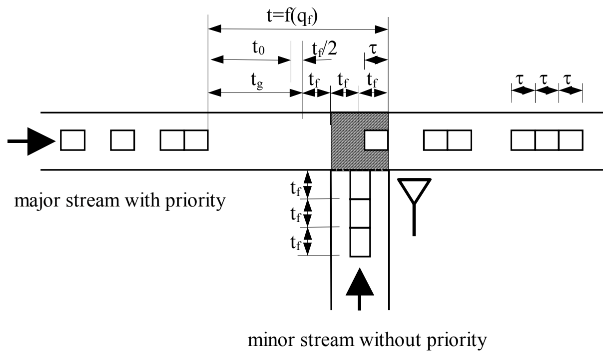


Fig. 4 - Departure mechanism with a bunched major stream

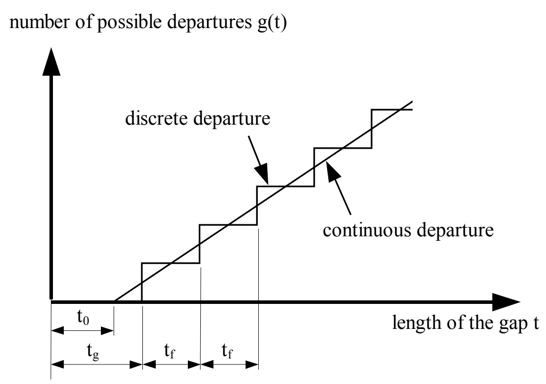


Fig. **5** – Shape of the function for the departure g(t)

For the discrete departure, it is assumed that within the major stream the gap t with the length $t_g \le t \le t_g + t_f$ enables the departure of one vehicle, the gap t with the length $t_g + t_f \le t \le t_g + 2 \cdot t_f$ enables the departure of two vehicles, the gap t with the length $t_g + 2 \cdot t_f \le t \le t_g + 3 \cdot t_f$ enables the departure of three vehicles and so on (cf. Fig.3 and Fig.4). The discrete departure function g(t) reads (cf. Harders, 1976)

$$g(t) = \begin{cases} \mathrm{int}\left(\frac{t - t_g}{t_f}\right) & \text{for } t \ge t_g \\ 0 & \text{for } t < t_g \end{cases} \tag{7}$$

with t_g = critical gap

t_f = move-up time

The corresponding density function for the departure reads

$$g'(t) = \begin{cases} 1 & \text{for } t \ge t_g \text{ and } \mathrm{mod}\left(\frac{t - t_g}{t_f}\right) = 0 \\ 0 & \text{for } t < t_g \end{cases} \tag{8}$$

For the continuous departure the function g(t) reads (cf. Siegloch, 1973)

$$g(t) = \begin{cases} \frac{t - t_0}{t_f} & \text{for } t \ge t_0 \\ 0 & \text{for } t < t_0 \end{cases} \tag{9}$$

and

$$g'(t) = \begin{cases} \frac{1}{t_f} & \text{for } t \ge t_0 \\ 0 & \text{for } t < t_0 \end{cases} \tag{10}$$

with $t_0 = t_g - \frac{t_f}{2}$

Fig.5 shows the shapes of the functions of departure, g(t), mentioned above.

In the following, separate formulae for the determination of capacity with discrete and continuous departures from the minor stream are derived. Under specific conditions, formulae for discrete departure and continuous departure (cf. section 2.6) can be converted pair-wise into each other.

2.3 Capacity of systems with one major stream and one minor stream

Setting eqs.(2), (7), and (9) into eq.(1) or setting eqs.(3), (7), and (9) into eq.(1) one obtains different basic formulae for the determination of capacity of systems with one major stream and one minor stream:

- for discrete departure under free traffic (formula of Harders, 1976, cf. also Daganzo, 1977)

$$C_{free} = q_p \cdot \frac{e^{-q_p \cdot t_g}}{1 - e^{-q_p \cdot t_f}} \tag{11}$$

- for continuous departure under free traffic (formula of Siegloch, 1973)

$$C_{free} = \frac{1}{t_f} \cdot e^{-q_p \cdot t_0} \quad (12)$$

- for discrete departure under bunched traffic (formula of Plank *et al.*, 1984)

$$\begin{aligned} C_{bunch} &= (1 - q_p \cdot \tau) \cdot \frac{q_f \cdot e^{-q_f \cdot (t_g - \tau)}}{1 - e^{-q_f \cdot t_f}} \\ &= \varphi \cdot \frac{q_p \cdot e^{-q_f \cdot (t_g - \tau)}}{1 - e^{-q_f \cdot t_f}} \end{aligned} \quad (13)$$

- and for continuous departure under bunched traffic (formula of Jacobs, 1980)

$$C_{bunch} = \frac{1 - q_p \cdot \tau}{t_f} \cdot e^{-q_f \cdot (t_0 - \tau)} \quad (14)$$

With $\varphi = 1 - q_p \cdot \tau$ (cf. Tanner, 1962)

$$q_f = \frac{\varphi \cdot q_p}{1 - q_p \cdot \tau} = \frac{(1 - q_p \cdot \tau) \cdot q_p}{1 - q_p \cdot \tau} = q_p$$

and correspondingly

$$C_{bunch} = (1 - q_p \cdot \tau) \cdot \frac{q_p \cdot e^{-q_p \cdot (t_g - \tau)}}{1 - e^{-q_p \cdot t_f}} \quad (15)$$

This it is exactly the capacity formula of Tanner (1962).

2.4 Consideration of the distributions of t_g, t_f, and τ

In section 2.3, formulae derived by deferent authors for const. parameters t_g, t_f, and τ are given. In this section, new formulae which consider the distribution of t_g, t_f, τ are presented.

As a assumption, the probability density $f(t_g)$, $f(t_f)$, and $f(\tau)$ for t_g, t_f, and τ can in general be described by an Erlang-function. An Erlang distribution has the density function:

$$erl(t_x) = \frac{\lambda}{(\alpha_{t_x} - 1)!} \cdot (\lambda \cdot t_x)^{\alpha_{t_x} - 1} \cdot e^{-\lambda \cdot t_x} \quad (16)$$

with $\lambda = \frac{\alpha_{t_x}}{\bar{t}_x}$

$\bar{t}_x$ = mean value of t_x

α_{t_x} =parameter of the Erlang-distribution for t_x

For the derivation of the formulae for the determination of capacity with discrete departure under bunched traffic regarding the distributions of t_g, t_f, and τ the result from Heidemann and Wegmann (1997) is used as a initial approach.

One can distinguish another two cases according to the departure behavior during the derivation by choice of a gap: a) inconsistent and b) consistent. In the case of the consistent departure behavior, one assumes that a driver by choice of a gap makes his decision every time, independently of the length of the gaps that he refused before. That is, a driver can accept a gap that is shorter than some gaps he refused before. On the other hand, in the case of the consistent departure behavior one assumes that a driver may accept only a gap that is larger than all gaps he refused before.

For the predefined conditions

- bunched traffic with a minimum gap τ and a portion of bunched traffic 1 - φ,
- discrete departure and
- exponentially distributed gaps t within the portion of free traffic φ

Heidemann and Wegmann (1997) recommended in the inconsistent case the formula

$$\begin{aligned} C &= \frac{q_f}{1+q_f\cdot\overline{B}}\cdot\frac{L(t_g'(q_f))}{1-L(t_f(q_f))} \\ &= \frac{q_f}{1+q_f\cdot\overline{B}}\cdot\frac{L(t_g(q_f))\cdot L(\tau(-q_f))}{1-L(t_f(q_f))} \end{aligned} \tag{17}$$

and in the consistent case the formula

$$\begin{aligned} C &= \frac{q_f}{1+q_f\cdot\overline{B}}\cdot\frac{1}{(1-L(t_f(q_f)))\cdot L(t_g'(-q_f))} \\ &= \frac{q_f}{1+q_f\cdot\overline{B}}\cdot\frac{1}{(1-L(t_f(q_f)))\cdot L(t_g(-q_f))\cdot L(\tau(q_f))} \end{aligned} \tag{18}$$

with $$\overline{B} = \frac{\overline{\tau}}{\varphi} \tag{19}$$

$$t_g' = t_g - \tau \tag{20}$$

and $L(t_g'(q_f))$ = Laplace transform of t_g' at q_f

= $L(t_g(q_f))\cdot L(\tau(-q_f))$

$L(t_g(q_f))$ = Laplace transform of t_g at q_f

$L(t_f(q_f))$ = Laplace transform of t_f at q_f

$L(\tau(-q_f))$ = Laplace transform of τ at $-q_f$

for the determination of the capacity at unsignalized intersections.

The eqs. (17) and (18) can be rewritten as

$$\begin{aligned} C &= (1-q_p\cdot\overline{\tau})\cdot q_f\cdot\frac{L(t_g'(q_f))}{1-L(t_f(q_f))} \\ &= \varphi\cdot q_p\cdot\frac{L(t_g(q_f))\cdot L(\tau(-q_f))}{1-L(t_f(q_f))} \end{aligned} \tag{21}$$

and

$$C = (1 - q_p \cdot \bar{\tau}) \cdot q_f \cdot \frac{1}{\left(1 - L(t_f(q_f))\right) \cdot L(t_g(-q_f)) \cdot L(\tau(q_f))}$$
$$= \varphi \cdot q_p \cdot \frac{1}{\left(1 - L(t_f(q_f))\right) \cdot L(t_g(-q_f)) \cdot L(\tau(q_f))} \tag{22}$$

Because the Laplace transform $L(t(q))$ has always a larger value with distributed t(q) than with const. t(q), it is evident according to eq.(17) that in the inconsistent case either the distribution of critical gaps t_g or the distribution of move-up times t_f or the distribution of minimum gaps τ increase the capacity C. In the consistent case (cf. eq. (18)), the capacity C is increased by distributed move-up times t_f and decreased by distributed critical gaps t_g and minimum gaps τ .

According to eq.(16), the Laplace transform of the Erlang-distributed gaps t at q with the probability density $f(t_x)$ is given by

$$L(t_x(q)) = \left(\frac{q \cdot \bar{t}_x}{\alpha_{t_x}} + 1\right)^{-\alpha_{t_x}} \tag{23}$$

Substituting eq.(23) with the corresponding parameters t_g, α_{t_g}, $\bar{t}_f$, α_{t_f}, $\bar{\tau}$, and α_τ into the eqs.(21) and (22), one obtains

- for the inconsistent case

$$C_{bunch} = \varphi \cdot q_p \cdot \frac{\left(\frac{q_f \cdot \bar{t}_g}{\alpha_{t_g}} + 1\right)^{-\alpha_{t_g}} \cdot \left(\frac{-q_f \cdot \bar{\tau}}{\alpha_\tau} + 1\right)^{-\alpha_\tau}}{1 - \left(\frac{q_f \cdot \bar{t}_f}{\alpha_{t_f}} + 1\right)^{-\alpha_{t_f}}} \tag{24}$$

- and for the consistent case

$$C_{bunch} = \varphi \cdot q_p \cdot \frac{\left(\frac{-q_f \cdot \bar{t}_g}{\alpha_{t_g}} + 1\right)^{\alpha_{t_g}} \cdot \left(\frac{q_f \cdot \bar{\tau}}{\alpha_\tau} + 1\right)^{\alpha_\tau}}{1 - \left(\frac{q_f \cdot \bar{t}_f}{\alpha_{t_f}} + 1\right)^{-\alpha_{t_f}}} \tag{25}$$

If the critical gaps t_g, the move-up times t_f, and the minimum gaps τ also have to keep a minimum value of $\tau_{t_g} > \tau_\tau$, $\tau_{t_f} > 0$, and $\tau_\tau > 0$ themselves, the capacity formula then reads for the inconsistent case

$$C_{bunch} = \varphi \cdot q_p \cdot \frac{\left(\frac{q_f \cdot (\bar{t}_g - \tau_{t_g})}{\alpha_{t_g}} + 1\right)^{-\alpha_{t_g}} \cdot e^{-q_f \cdot \tau_{t_g}} \cdot \left(\frac{-q_f \cdot (\bar{\tau} - \tau_\tau)}{\alpha_\tau} + 1\right)^{-\alpha_\tau} \cdot e^{q_f \cdot \tau_\tau}}{1 - \left(\frac{q_f \cdot (\bar{t}_f - \tau_{t_f})}{\alpha_{t_f}} + 1\right)^{-\alpha_{t_f}} \cdot e^{-q_f \cdot \tau_{t_f}}} \tag{26}$$

and for the consistent case

$$C_{bunch} = \varphi \cdot q_p \cdot \frac{\left(\frac{-q_f \cdot (\bar{t}_g - \tau_{t_g})}{\alpha_{t_g}} + 1\right)^{\alpha_{t_g}} \cdot e^{-q_f \cdot \tau_{t_g}} \cdot \left(\frac{q_f \cdot (\bar{\tau} - \tau_\tau)}{\alpha_\tau} + 1\right)^{\alpha_\tau} \cdot e^{q_f \cdot \tau_\tau}}{1 - \left(\frac{q_f \cdot (\bar{t}_f - \tau_{t_f})}{\alpha_{t_f}} + 1\right)^{-\alpha_{t_f}} \cdot e^{-q_f \cdot \tau_{t_f}}} \tag{27}$$

The eqs.(26) and (27) are valid for

- discrete departure from the minor stream,
- shifted-Erlang-distributed gaps t within the major stream,
- shifted-Erlang-distributed critical gaps t_g,
- shifted-Erlang-distributed move-up time t_f, and
- shifted-Erlang-distributed minimum gap τ within the major stream

One obtains analogously for continuous departure with inconsistent behavior

$$\begin{aligned} C &= \frac{1}{1 + q_f \cdot \overline{B}} \cdot \frac{L(t_0'(q_f))}{\bar{t}_f} \\ &= (1 - q_p \cdot \bar{\tau}) \cdot \frac{L(t_0(q_f)) \cdot L(\tau(-q_f))}{\bar{t}_f} \end{aligned} \tag{28}$$

and with consistent behavior

$$\begin{aligned} C &= \frac{1}{1 + q_f \cdot \overline{B}} \cdot \frac{1}{\bar{t}_f \cdot L(t_0'(q_f))} \\ &= (1 - q_p \cdot \bar{\tau}) \cdot q_f \cdot \frac{1}{\bar{t}_f \cdot L(t_0(-q_f)) \cdot L(\tau(q_f))} \end{aligned} \tag{29}$$

with $t_0' = t_0 - \tau$

Here, the distribution of the move-up times t_f has no influence on the capacity. One obtains respectively for the inconsistent case

$$C_{bunch} = (1 - q_p \cdot \bar{\tau}) \cdot \frac{1}{\bar{t}_f} \cdot \left(\frac{q_f \cdot (\bar{t}_0 - \tau_{t_0})}{\alpha_{t_0}} + 1\right)^{-\alpha_{t_0}} \cdot e^{-q_f \cdot \tau_{t_0}} \cdot \left(\frac{-q_f \cdot (\bar{\tau} - \tau_\tau)}{\alpha_\tau} + 1\right)^{-\alpha_\tau} \cdot e^{q_f \cdot \tau_\tau} \tag{30}$$

and for the consistent case

$$C_{bunch} = (1 - q_p \cdot \bar{\tau}) \cdot \frac{1}{\bar{t}_f} \cdot \left(\frac{-q_f \cdot (\bar{t}_0 - \tau_{t_0})}{\alpha_{t_0}} + 1\right)^{\alpha_{t_0}} \cdot e^{-q_f \cdot \tau_{t_0}} \cdot \left(\frac{q_f \cdot (\bar{\tau} - \tau_\tau)}{\alpha_\tau} + 1\right)^{\alpha_{t_0}} \cdot e^{q_f \cdot \tau_\tau} \tag{31}$$

The eqs. (30) and (31) are valid for

- continuous departure from the minor stream,
- shifted-Erlang-distributed gaps t within the major stream,
- shifted-Erlang-distributed zero-gaps t_0,
- arbitrarily distributed move-up times t_f, and
- shifted-Erlang-distributed minimum gaps τ in the major stream

2.5 Evaluation of the effect caused by the distributions of t_g, t_f, and τ

The effect of distributed parameters t_g, t_f, and τ is depicted in Fig.**6**. For the hypothetical calculation the parameters t_g, t_f, and τ are assumed to be shifted-Erlang-distributed. Die parameters of the shifted-Erlang-distributed are taken from the Fig.**8**.

Fig.**6** shows clearly the different effect of distributed parameters t_g, t_f, and τ between the consistent and inconsistent departure behavior.

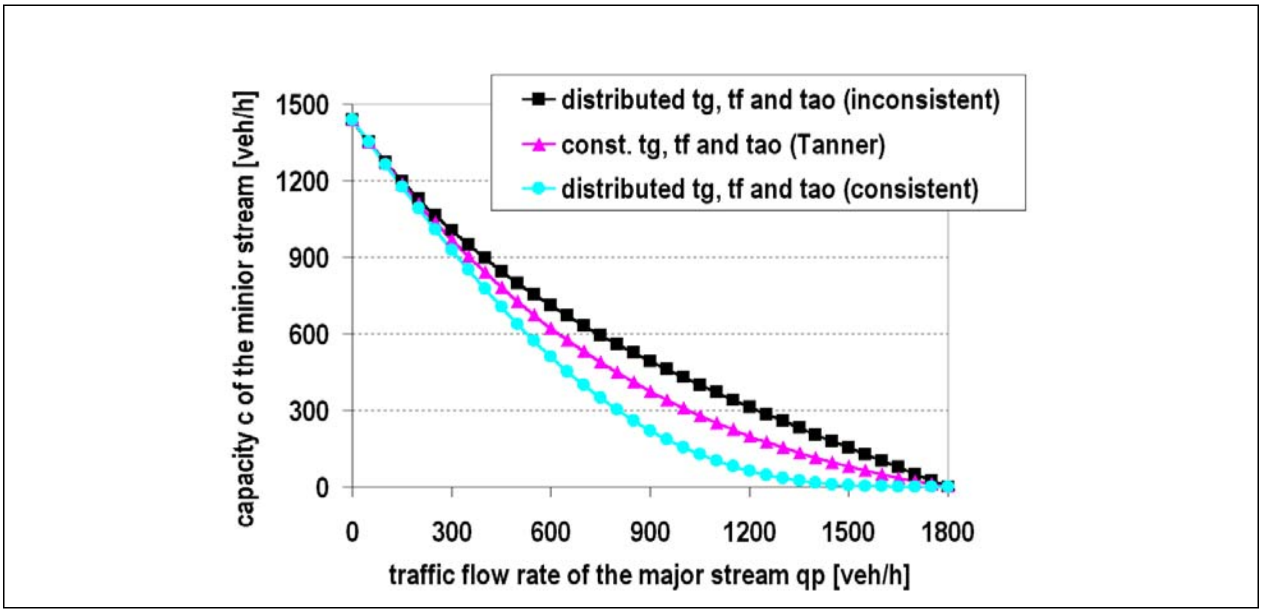


Fig. 6 - Departure mechanism with a free major stream

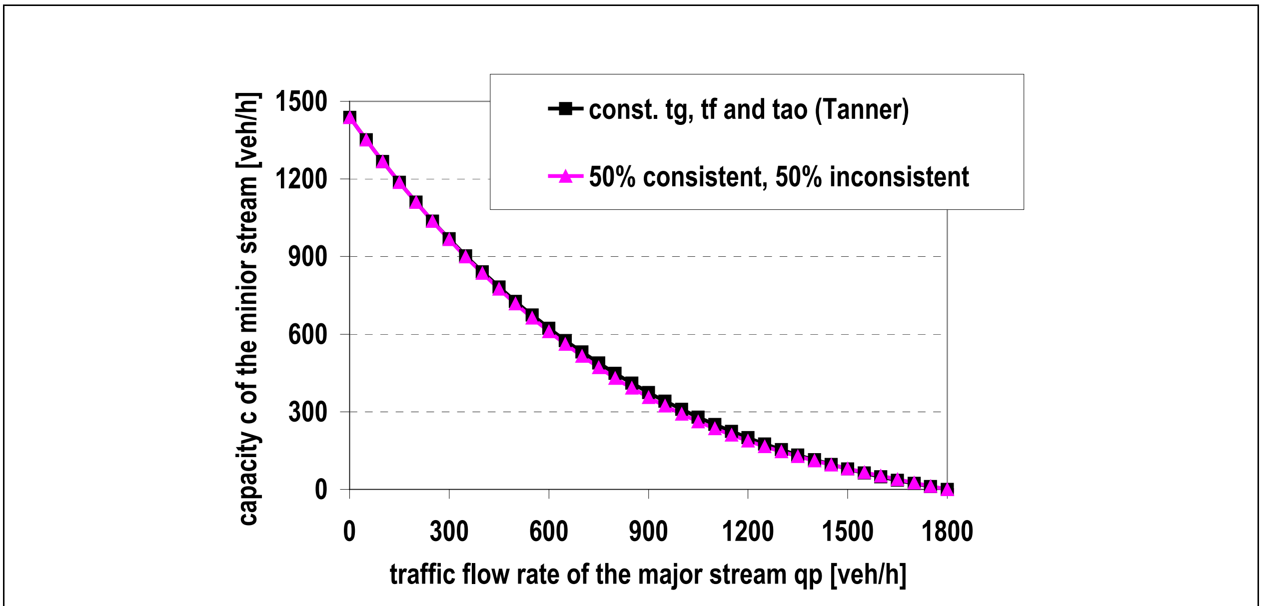


Fig. 7 - Departure mechanism with a bunched major stream

	t_g [s]	t_f [s]	τ [s]
mean	5.8	2.5	2
minimum	2	2	1.4
α	3	3	3

Fig. 8 – Parameters for the shifted-Erlang-distributed t_g, t_f, and τ

In the real world the departure behavior cannot be found exactly. The effect of distributed parameters t_g, t_f, and τ mutually neutralize themselves for consistent and inconsistent behavior. If the drivers behave with 50% consistently and 50% inconsistently, the effect is almost missing (Fig.**7**, cf. Wu, 1997a). Therefore, one can neglect the distribution of the critical gaps t_g, the move-up times t_f, and the minimum gaps τ for calculations in the practice.

2.6 Relationship between the discrete and the continuous departure

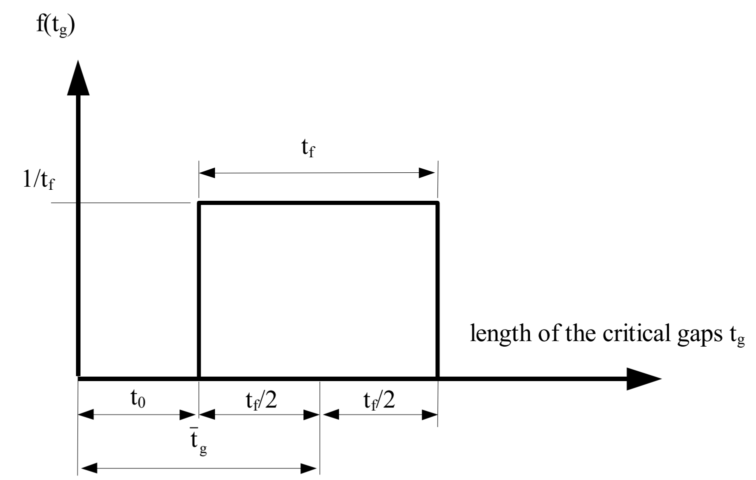


Fig. 9 - Distribution of the critical gaps t_g for the continuous departure

Using a uniform distribution for the critical gaps t_g over the range $\{t_g - t_f/2,\ t_g + t_f/2\}$, i.e., the critical gap t_g has a minimum length of $t_0 = t_g - t_f/2$ and maximum length of $t_0 + t_f$, the capacity formula for the discrete departure can be transformed into the capacity formula for the continuous departure. That is, the function of probability density for the critical gaps t_g

$$f(t_g) = \begin{cases} 0 & \text{for } t_g < \bar{t}_g - t_f/2 \\ \dfrac{1}{t_f} & \text{for } \bar{t}_g - t_f/2 \le t_g \ge \bar{t}_g + t_f/2 \\ 0 & \text{for } t_g > \bar{t}_g + t_f/2 \end{cases}$$

$$= \begin{cases} 0 & \text{for } t_g < t_0 \\ \dfrac{1}{t_f} & \text{for } t_0 \le t_g \le t_0 + t_f \\ 0 & \text{for } t_g > t_0 + t_f \end{cases} \qquad (32)$$

with $\bar{t}_g$ = mean value of the critical gap t_g
is assumed (cf.Fig.9).

Integrating the eq.**(8)** piecewise over the eq.(32), one receives

$$g(t) = \begin{cases} 0 & \text{for } t < t_0 \\ \int_{t_0}^{t} g'(t)\cdot f(t_g)\cdot dt_g + 0 & \text{for } t_0 \le t < t_0 + t_f \\ \int_{t_0+t_f}^{t} g'(t)\cdot f(t_g)\cdot dt_g + \int_{t_0}^{t_0+t_f} g'(t)\cdot f(t_g)\cdot dt_g + 0 & \text{for } t_0 + t_f \le t < t_0 + 2t_f \\ . & . \\ . & . \end{cases}$$

$$= \begin{cases} 0 & \text{for } t < t_0 \\ \dfrac{t-t_0}{t_f} & \text{for } t \ge t_0 \end{cases}$$

Which is exactly the eq. (9).

Using in the eq.(11) the distributed critical gaps t_g according to the eq.(32), one obtains then

$$\begin{aligned} C &= \int_0^{\infty} q_p \cdot \frac{e^{-q_p\cdot t_g}}{1-e^{-q_p\cdot t_f}}\cdot f(t_g)\cdot dt_g \\ &= \frac{1}{t_f}\cdot e^{-q_p\cdot t_0} \end{aligned} \tag{33}$$

Which is exactly the eq.(12).

Also the eq.(17) can be transformed into the eq. (28). Rewriting eq.(17) into

$$C = (1-q_p\cdot\bar{\tau})\cdot\frac{q_f}{1-\mathsf{L}\left(t_f(q_f)\right)}\cdot\mathsf{L}\left(t_0(q_f)\right)\cdot\mathsf{L}\left(\Delta t_0(q_f)\right)\cdot\mathsf{L}\left(\tau(-q_f)\right) \tag{34}$$

with $t_0 + \Delta t_0 = t_g$

and setting

$$\begin{aligned} \mathsf{L}\left(\Delta t_0(q_f)\right) &= \frac{1-\mathsf{L}\left(t_f(q_f)\right)}{q_f\cdot\bar{t}_f} \\ &= \frac{1}{\bar{t}_f}\cdot\left(\frac{1}{q_f}-\frac{\mathsf{L}\left(t_f(q_f)\right)}{q_f}\right) \end{aligned} \tag{35}$$

one obtains

$$C = \frac{(1-q_p\cdot\bar{\tau})\cdot}{\bar{t}_f}\cdot\mathsf{L}\left(t_0(q_f)\right)\cdot\mathsf{L}\left(\tau(-q_f)\right)$$

Which is exactly the capacity formula for continuous departure (eq.(28)) under the same condition.

The eq.(35) leads to the following distribution function for the gaps Δt_0

$$f(\Delta t_0) = \frac{1 - F_{t_f}(\Delta t_0)}{\bar{t}_f} \tag{36}$$

or

$$F(\Delta t_0) = \frac{1}{\bar{t}_f} \cdot \int_0^{\Delta t_0} \left(1 - F_{t_f}(t)\right) \cdot dt \tag{37}$$

That is, the eq.(17) turns into the eq.(28) (note, this is only valid for exponentially distributed gaps within the portion of free traffic), if between the distribution of critical gaps t_g and the distribution of zero-gaps t_0 the relationships

$$t_g = t_0 + \Delta t_0$$

and

$$f(t_g) = f(t_0) \otimes f(\Delta t_0) \tag{38}$$

or

$$\mathsf{L}(t_g(q_f)) = \mathsf{L}(t_0(q_f)) \cdot \mathsf{L}(\Delta t_0(q_f)) \tag{39}$$

state. The probability density $f(\Delta t_0)$ is described by eq.(36). It is in turn a function of the distribution of the move-up times t_f. The mean value of Δt_0 reads according to eq.(35)

$$\begin{aligned}
\Delta\bar{t}_0 &= E(\Delta t_0) = (-1) \cdot \mathsf{L}(\Delta t_0(q_f))'_{|q_f=0} \\
&= \frac{1}{\bar{t}_f} \cdot \left(\frac{(1 - \mathsf{L}(t_f(q_f)))}{q_f^{\,2}} + \frac{\mathsf{L}(t_f(q_f))'}{q_f} \right)_{|q_f=0} \\
&= \frac{1}{\bar{t}_f} \cdot \left(\frac{(1 - \mathsf{L}(t_f(q_f))) + q_f \cdot \mathsf{L}(t_f(q_f))'}{q_f^{\,2}} \right)_{|q_f=0}
\end{aligned}$$

Using the rule of L'hospital, one obtains

$$\begin{aligned}
\Delta\bar{t}_0 &= \left(\frac{\mathsf{L}(t_f(q_f))''}{2 \cdot \bar{t}_f} \right)_{|q_f=0} \\
&= \frac{\bar{t}_f}{2} + \frac{\sigma^2_{t_f}}{2 \cdot \bar{t}_f}
\end{aligned} \tag{40}$$

The eq.(32) is a special case of eq.(36). Setting in the eq.(36) t_f = const., i.e., $\sigma^2_{t_f} = 0$ and

$$F_{t_f}(t) = \begin{cases} 0 & \text{for } t < t_f \\ 1 & \text{for } t \geq t_f \end{cases}$$

one obtains then

$$f(\Delta t_0) = \begin{cases} \dfrac{1}{t_f} & \text{for } \Delta t_0 < t_f \\ 0 & \text{for } \Delta t_0 \geq t_f \end{cases}$$

with $\Delta t_0 = t_f / 2$

Which is exactly the eq.(32).

2.7 Relationship between the formula for the continuous departure and the linear approach

In the practice, also linear functions are used for calculating capacities at unsignalized intersections, especially at roundabouts (e.g., Kimber, Semmens, 1997). The parameters of this linear function can be then easily determined by regressions. The linear function is a special case of the capacity formula for the continuous departure (eq.(14)). Setting $t_0 = \tau$, i.e., setting $t_g = \tau + t_f/2$ into eq.(14) yields

$$C_{bunch} = \left(1 - \tau \cdot q_p\right) \cdot \frac{1}{t_f} = \frac{1}{t_f} - \frac{\tau}{t_f} \cdot q_p = A - B \cdot q_p$$

For this special case, the capacity, C, is a linear function of the traffic intensity in the major stream, q_p.

3 DETERMINATION OF CAPACITY ACCORDING TO TRAFFIC STATES IN THE MAJOR STREAM

In order to be able to extend the capacity formulae to systems with more than two streams, in this section the capacity in systems with a major stream and a minor stream is considered in another view of point. The resulting formulae correspond to the formulae in the previous section. From this point of view, a transfer of the results from systems with two streams onto systems with more than two stream is entirely possible.

3.1 Systems with continuous departure

First, one concentrates only on a system with continuous departure with arbitrarily distributed t_g, t_f (therefor also arbitrarily distributed t_0) and τ.

On a time axis one can distinguish periods with: queuing, bunching, single vehicle, and no vehicle. One can simply sum all small periods with queuing into one large queuing period without the total length of queuing on the time axis is affected. Similarly, one can also sum the small periods of bunching and single vehicle into large periods. Accordingly, the traffic flow in the major streams can be divided into different states using 3 stages of work steps (cf. Fig. 10):

Stage I:

In this stage, the traffic flow in the major stream is divided into 2 states which excludes each other:

- **Queuing** and **Queuing-free**
  In the state of Queuing, the vehicles in the major stream stay at the stop line or are within discharging operation. Departure from the minor stream is not possible in the state of Queuing (including discharge queuing for the real-life traffic conditions). In the state of Queuing-free all vehicles in the major stream are in motion. Departure from the minor

stream is in the state of Queuing-free possible but dependent on the traffic intensity and bunching situation within the major stream. Denoting the probability for the state of Queuing by

$$p_S = Pr(\text{Queuing}),$$

the probability for the state of Queuing-free is then

$$p_{0,S} = Pr(\text{Queuing-free}) = 1 - p_S.$$

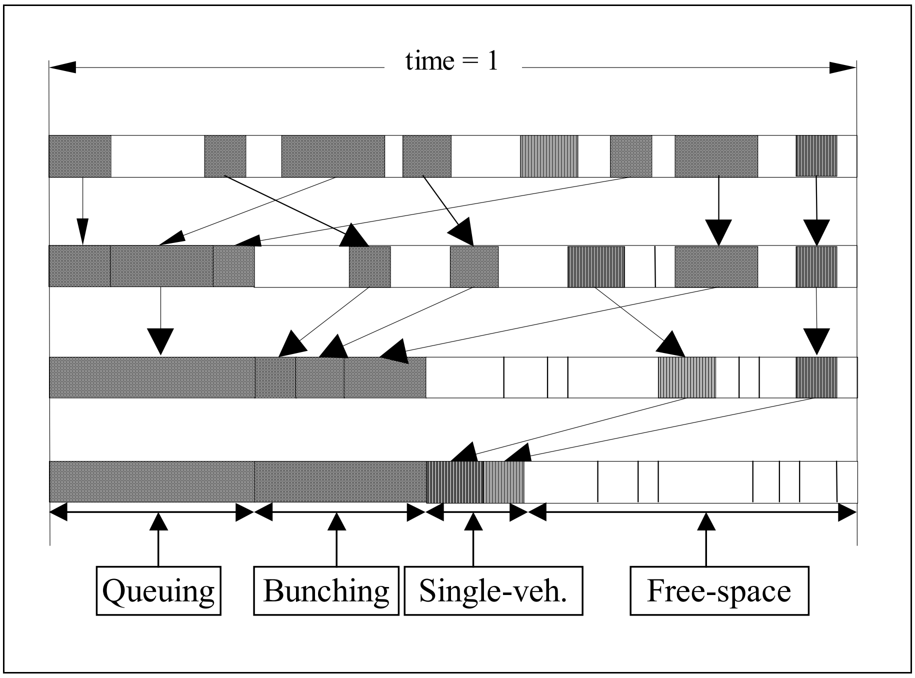


Fig. 10 - States within the major stream.

For further consideration of the probabilities of the single states, the blocking regimes by Queuing, Bunching, Single-vehicle, and the regime of Free-space are pulled together.

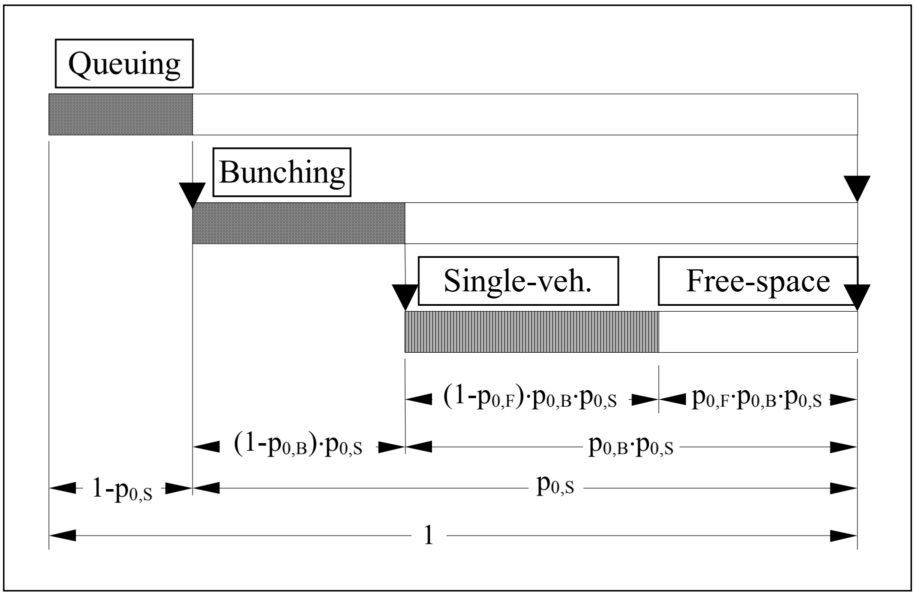


Fig. 11 States in the major stream and their probabilities

Stage II:

In the stage II, the traffic flow in the Queuing-free state is in turn divided into 2 sub-states which excludes each other:

- **Bunching** and **Bunching-free** under the condition of Queuing-free
  In the state of Bunching, the vehicles in the major stream is in motion with the minimum gaps τ. Departure from the minor stream is not possible in the state of Bunching. In the state of Bunching-free the gaps between the vehicles are large than τ and distributed by chance. Departure from the minor stream is in the state of Bunching-free possible but dependent on the traffic intensity within the major stream in this state. Denoting the probability for the state of Bunching under the condition of Queuing-free by
  p_B = Pr(Bunching | Queuing-free),
  the probability for the state of Bunching-free the condition of Queuing-free is then
  $p_{0,B}$ = Pr(Bunching-free | Queuing-free) = 1 - p_B.

Stage III:

In the stage III, the traffic flow in the Bunching-free state under the condition of Queuing-free is divided again into 2 sub-sub-states which excludes each other:

- **Single-vehicle** and **Vehicle-free** (Free-space) under the condition of (Bunching-free | Queuing-free)
  In the state of Single-vehicle, vehicles in the major stream are moving independently from each other. In the front of a vehicle, a time period of the length t_0 is closed for the minor stream. The total closing time by the vehicles in the major stream is the sum of the set $\{t<t_0\}$. Departure from the minor stream is not possible for the state of Single-vehicle. In the state of Free-space there is no vehicle in the major stream. Departure from the minor stream in the state of Free-space is carried out with the saturation capacity $C_s=1/t_f$.
  Denoting the probability for the state of Single-vehicle under the condition of (Bunching-free | Queuing-free) by
  p_F = Pr[Single-vehicle | (Bunching-free | Queuing-free)],
  the probability for the state of Vehicle-free (Free-space) under the condition of (Bunching-free | Queuing-free) is then
  $p_{0,F}$ = Pr[Vehicle-free | (Bunching-free | Queuing-free)] = 1 - p_F.

Thus, the major stream can be divided into four regimes 1) that of state of Free-space (Vehicle-free), 2) that of state of Single-vehicle, 3) that of state of Bunching, and 4) that of state of Queuing. According to the definition of the conditioned probabilities, the probabilities $p_{0,S}$, $p_{0,B}$ and $p_{0,F}$ are completely independent of each other (cf. Figs. 10 and 11). They are to be determined according to the queuing theory.

Accordingly, the formula for the determination of capacity of the minor stream reads

C = (saturation capacity | no hindrance)
  × Pr(no hindrance by Queuing)
  × Pr(no hindrance by Bunching | no hindrance by Queuing)
  × Pr[no hindrance by Single -vehicle | (no hindrance by Bunching | no hindrance by Queuing)]
  = Pr[Vehicle-free | (Bunching-free | Queuing-free)]

$$= C_s \cdot p_{0,S} \cdot p_{0,B} \cdot p_{0,F} \quad (41)$$

with C_s = saturation capacity | no hindrance = capacity in the Free-space state

3.1.1 Probability of the Queuing-free state, $p_{0,S}$. The probability for the state of Queuing-free in the major stream p_S can in general (approximately according to the M/G/1 queuing system) be estimated with the degree of saturation, x_p. The probability for the Queuing-free state in the major stream $p_{0,S}$ then reads

$$p_{0,S} = 1 - p_S = 1 - x_p \tag{42}$$

3.1.2 Probability of the Bunching-free state under the condition of Queuing-free, $p_{0,B}$. One can assume that bunching formation in the traffic in motion within the major stream is independent of the queuing saturation (Bunching during discharge queuing belongs to Queuing). Accordingly, it is true

Pr(Bunching-free | Queuing-free) = Pr(Bunching-free)

The probability of Bunching p_B in the major stream is simple the portion of the sum of the minimum gap τ for all vehicles. Thus

$$p_B = \sum_{i=1}^{q_p} \tau_i = q_p \cdot \bar{\tau}$$

The probability for Bunching-free state within the major stream $p_{0,B}$ reads then

$$p_{0,B} = (1 - q_p \cdot \bar{\tau}) \tag{43}$$

3.1.3 Probability of the Vehicle-free (Free-space) state under the condition of (Bunching-free | Queuing-free), $p_{0,F}$. The probability for the state of Free-space under the condition of (Bunching-free | Queuing-free), $p_{0,F}$ (the probability that no hindrance by single vehicles occurs) is only dependent on the traffic intensity q_f. It is identical to the probability that the gap in the major stream t is larger than zero-gap t_0 under the condition that the gap t is larger than the minimum gap τ (cf., Fig.4). That is:

$$\begin{aligned} p_{0,F} &= \Pr(t > t_0 \mid t > \tau) \\ &= \frac{\Pr(t > t_0)}{\Pr(t > \tau)} \end{aligned} \tag{44}$$

3.1.4 Capacity in the Vehicle-free (Free-space) state C_S. The capacity for the minor stream is in the Vehicle-free state the reciprocal of the mean service time of the queuing system. The mean service time in the vehicle free state is equal to the mean move-up time t_f. That is:

$$C_s = \frac{1}{\bar{t}_f} \tag{45}$$

3.1.5 Capacity C. The general formula for the determination of capacity now reads

$$\begin{aligned} C &= C_s \cdot p_{0,S} \cdot p_{0,B} \cdot p_{0,F} \\ &= C_s \cdot (1 - x_p) \cdot (1 - q_p \cdot \bar{\tau}) \cdot \frac{\Pr(t > t_0)}{\Pr(t > \tau)} \end{aligned} \tag{46}$$

With shifted-exponentially distributed gaps t in the major stream it is valid for systems with continuous departure and const. t_g, t_f (and consequently t_0 is also const.) and τ

$$C_s = \frac{1}{t_f} \tag{47}$$

and

$$\frac{\Pr(t > t_0)}{\Pr(t > \tau)} = \frac{e^{-q_f \cdot (t_0 - \tau)}}{1} = e^{-q_f \cdot (t_0 - \tau)} \tag{48}$$

The resulted formula for the determination of capacity reads

$$\begin{aligned} C &= C_s \cdot p_{0,S} \cdot p_{0,B} \cdot p_{0,F} \\ &= (1 - x_p) \cdot (1 - q_p \cdot \bar{\tau}) \cdot \frac{e^{-q_f \cdot (t_0 - \tau)}}{\bar{t}_f} \end{aligned} \tag{49}$$

<u>3.1.6 Generalization.</u> In general it is true

$$C = C_s \cdot p_{0,S} \cdot p_{0,B} \cdot p_{0,F} \tag{50}$$

For systems with continuous departure, one obtains

$$C_s = \frac{1}{\bar{t}_f} \tag{51}$$

$$p_{0,S} = 1 - x_p \tag{52}$$

$$p_{0,B} = (1 - q_p \cdot \bar{\tau}) \tag{53}$$

and

$$p_{0,F} = \frac{\Pr(t > t_0)}{\Pr(t > \tau)} \tag{54}$$

with $\bar{t}_0 = \bar{t}_g - \left(\frac{\bar{t}_f}{2} + \frac{\sigma_{t_f}^2}{2 \cdot \bar{t}_f} \right)$ (cf. eq.(40))

That is:

$$\begin{aligned} C &= C_s \cdot p_{0,S} \cdot p_{0,B} \cdot p_{0,F} \\ &= \frac{1}{\bar{t}_f} \cdot (1 - x_p) \cdot (1 - q_p \cdot \bar{\tau}) \cdot \frac{\Pr(t > t_0)}{\Pr(t > \tau)} \end{aligned} \tag{55}$$

For const. parameters t_f, τ, and t_0, one obtains

$$p_{0,F} = \frac{\Pr(t > t_0)}{\Pr(t > \tau)} = \frac{1 - F(t = t_0)}{1 - F(t = \tau)}$$

Thus, eq. (55) yields

$$\begin{aligned} C &= \frac{1}{t_f} \cdot (1 - x_p) \cdot (1 - q_p \cdot \tau) \cdot \frac{\Pr(t > t_0)}{\Pr(t > \tau)} \\ &= \frac{1}{t_f} \cdot (1 - x_p) \cdot (1 - q_p \cdot \tau) \cdot \frac{1 - F(t = t_0)}{1 - F(t = \tau)} \end{aligned} \tag{56}$$

with $F(t)$ = Distribution function of the gaps t within major stream

According to the eq.(56) one can recognize that the capacity depends only on the distribution function F(t) given the const. parameters $\bar{t}_f$, τ, and t_0. For instance, for the shifted-hyper-Erlang-distributed gaps t within the major stream with const. t_f, t_0 and τ, the capacity reads

$$\begin{aligned} C &= \frac{1}{t_f} \cdot (1-x_p) \cdot (1-q_p \cdot \tau) \cdot \frac{1-F(t=t_0)}{1-F(t=\tau)} \\ &= \frac{1}{t_f} \cdot (1-x_p) \cdot (1-q_p \cdot \tau) \cdot \frac{A \cdot e^{-q_{f,1} \cdot (t_0-\tau)} + (1-A) \cdot e^{-k \cdot q_{f,2} \cdot (t_0-\tau)} \cdot \sum_{i=0}^{k-1} \frac{\left(k \cdot q_{f,2} \cdot (t_0-\tau)\right)^i}{i!}}{1-0} \\ &= \frac{1}{t_f} \cdot (1-x_p) \cdot (1-q_p \cdot \tau) \cdot \left(A \cdot e^{-q_{f,1} \cdot (t_0-\tau)} + (1-A) \cdot e^{-k \cdot q_{f,2} \cdot (t_0-\tau)} \cdot \sum_{i=0}^{k-1} \frac{\left(k \cdot q_{f,2} \cdot (t_0-\tau)\right)^i}{i!} \right) \end{aligned} \quad (57)$$

with A = Portion of the negative-exponentially distributed gaps
$q_{f,1}$ = Traffic intensity in the portion of traffic with negative-exponentially distributed gaps
$q_{f,2}$ = Traffic intensity in the portion of traffic with Erlang-distributed gaps

Here, the relationship

$$\frac{1}{q_f} = A \cdot \frac{1}{q_{f,1}} + (1-A) \cdot \frac{1}{q_{f,2}}$$

is true.

If the gaps t in Bunching-free state are exponentially distributed, the effect of distributed t_0 and τ can be taken into account using the equation

$$p_{0,F} = \mathsf{L}\left(t_0(q_f)\right) \cdot \mathsf{L}\left(\tau(-q_f)\right) \quad (58)$$

for the inconsistent departure behavior and

$$p_{0,F} = \frac{1}{\mathsf{L}\left(t_0(-q_f)\right) \cdot \mathsf{L}\left(\tau(q_f)\right)} \quad (59)$$

for the consistent departure behavior.

For Erlang-distributed t_0 and τ the Laplace transforms can be calculated by eq.(23).

3.2 System with discrete departure

Analogously for systems with discrete departure (cf. eq.(55)), one obtains

$$\begin{aligned} C &= C_s \cdot p_{0,S} \cdot p_{0,B} \cdot p_{0,F} \\ &= C_s \cdot (1-x_p) \cdot (1-q_p \cdot \bar{\tau}) \cdot \frac{Pr(t > t_g)}{Pr(t > \tau)} \end{aligned} \quad (60)$$

Normally, the capacity in Free-space state, C_s, for systems with discrete departure cannot be expressed explicitly. With the assumption that the gaps t in the Bunching-free state are exponentially distributed one receives for inconsistent departure behavior:

$$C_s = \frac{q_f}{1 - L(t_f(q_f))} \tag{61}$$

$$p_{0,S} = 1 - x_p \tag{62}$$

$$p_{0,B} = (1 - q_p \cdot \bar{\tau}) \tag{63}$$

$$p_{0,F} = L(t_g(q_f)) \cdot L(\tau(-q_f)) \tag{64}$$

and for consistent departure behavior:

$$p_{0,F} = \frac{1}{L(t_g(-q_f)) \cdot L(\tau(q_f))} \tag{65}$$

For Erlang-distributed t_g, t_f, and τ the Laplace transforms can be calculated by eq.(23)

For const. t_g, t_f and τ the equation above yield

$$C_s = \frac{q_f}{1 - e^{-q_f \cdot t_f}} \tag{66}$$

and

$$p_{0,F} = e^{-q_f \cdot (t_g - \tau)} \tag{67}$$

The resulted formula for the determination of capacity reads

$$\begin{aligned} C &= C_s \cdot p_{0,S} \cdot p_{0,B} \cdot p_{0,F} \\ &= (1 - x_p) \cdot (1 - q_p \cdot \tau) \cdot \frac{q_f \cdot e^{-q_f \cdot (t_g - \tau)}}{1 - e^{-q_f \cdot t_f}} \end{aligned} \tag{68}$$

4 QUEUING SYSTEM WITH MORE THAN TWO STREAMS (MORE THAN ONE MAJOR STREAMS)

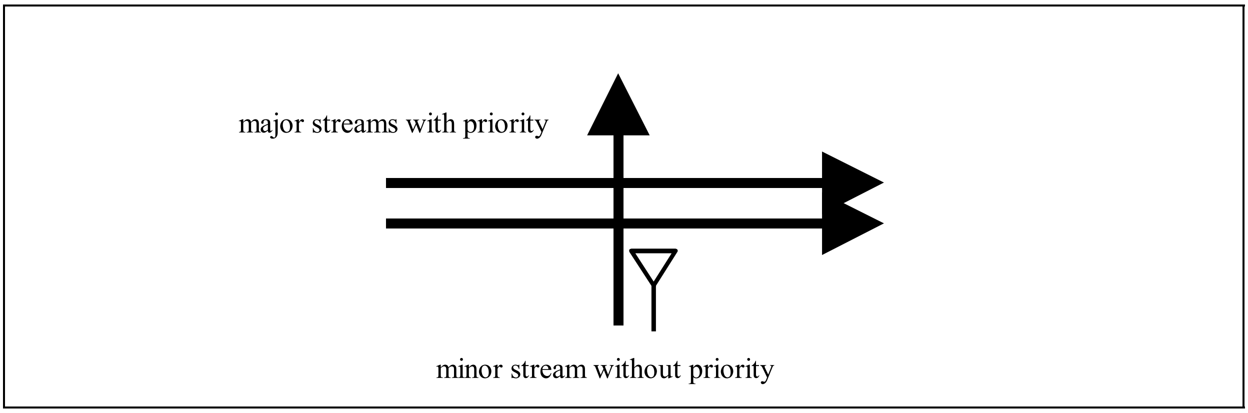


Fig. 12 - System with two major streams with parallel configuration

According to the derivation in the section 3, the traffic flow within the major stream can be divided into four states for which the probabilities can be calculated separately. From these probabilities, the portion of the states in which no hindrance in the major stream occurs to the minor stream can be obtained. The capacity for the minor stream can be calculated from the multiplication of the saturation capacity, C_s, and the portion of the state without hindrance.

According to the same principle, the capacity for systems with more than one major streams can also be determined.

4.1 Queuing system with more than one major streams in parallel configuration

The traffic states in major streams with parallel configuration are completely independent of each other. Therefore, one obtains for systems with n major streams (in parallel configuration) of rank r for the minor stream of rank r+1 (cf. Fig.**12**):

$$p_{0,S}^{*} = \prod_{i=1}^{n} p_{0,S,i} \tag{69}$$

$$p_{0,B}^{*} = \prod_{i=1}^{n} p_{0,B,i} \tag{70}$$

$$p_{0,F}^{*} = \prod_{i=1}^{n} p_{0,F,i} \tag{71}$$

and accordingly,

$$\begin{aligned} C &= C_s \cdot p_{0,S}^{*} \cdot p_{0,B}^{*} \cdot p_{0,F}^{*} \\ &= C_s \cdot \prod_{i=1}^{n} p_{0,S,i} \cdot \prod_{i=1}^{n} p_{0,B,i} \cdot \prod_{i=1}^{n} p_{0,F,i} \end{aligned} \tag{72}$$

with

$$C_s = \frac{1}{\bar{t}_f} \tag{73}$$

for continuous departure and

$$C_s = \frac{q_f^{*}}{1 - L\left(t_f(q_f^{*})\right)} = C_s(q_f^{*}) \tag{74}$$

$$q_f^{*} = \sum_{i=1}^{n} q_{f,i}$$

for discrete departure.

In the equations, $p_{0,X,i}$ is the probability that the state X in stream i does not occur.

For const. t_f, eq.(74) turns to

$$C_s = \frac{q_f^{*}}{1 - e^{-q_f^{*} \cdot t_f}} \tag{75}$$

4.2 Queuing system with more than one major streams Queuing system in serial configuration

The Queuing-free states in major streams with serial configuration are not independent of each other (cf. Fig.**13**). The total Queuing-free state of all major streams can not be determined with the simple multiplication of the Queuing-free states in the individual major streams. In order to determine this total Queuing-free state in the major streams the following approximations are introduced (Wu, 1998):

1. The queuing systems in the individual major streams are assumed (simplified) as M/M/1 queuing systems.
2. The queues in all major streams with serial configuration are considered together as a large queuing system. The large queue is also assumed (simplified) as a M/M/1 queuing system (cf. Fig.**14**).

Accordingly, for the sub-queuing systems in the individual major streams (M/M/1) the mean queue lengths reads

$$N = \frac{x}{1-x} = \frac{1-p_{0,S}}{p_{0,S}} \tag{76}$$

with N = mean queue length in the M/M/1 queuing system

x = saturation degree of the queuing system

This leads to

$$p_{0,S} = \frac{1}{1+N} \tag{77}$$

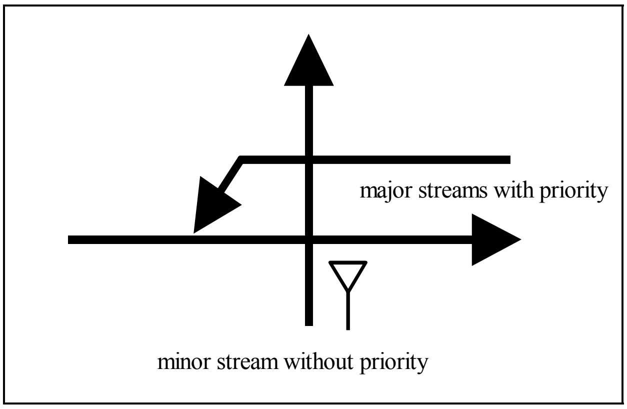


Fig. -13 System with two major streams in serial configuration

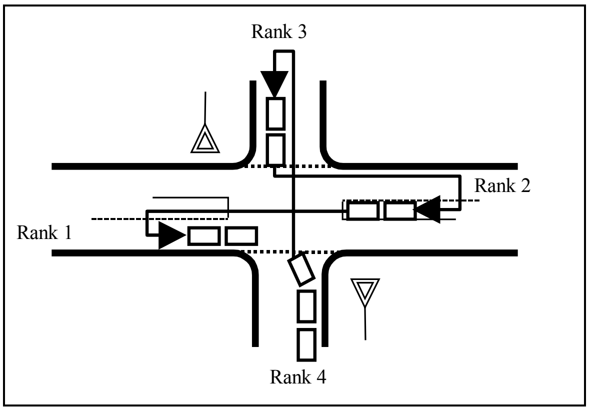


Fig. - 14 System with serial configuration and its departure sequence

The eq.(77) indicates that the probability for the Queuing-free state in a M/M/1 queuing system is equal to the reverse value of the mean queue length N in the queuing system plus 1. Correspondingly, for a total system with waiting streams in serial configuration (cf. Figs.**13** and **14**)

$$N^* = \frac{1-p_{0,S}{}^*}{p_{0,S}{}^*} \tag{78}$$

and

$$p_{0,S}{}^{*} = \frac{1}{1+N^{*}} \tag{79}$$

are valid. Here, N* is the mean queue length in the total system (in the large queue). It is equal to the sum of all mean queue lengths within the individual major streams. That is, for a total system with m major streams in serial configuration

$$N^{*} = \sum_{j=1}^{m} N_j \tag{80}$$

is valid, and accordingly

$$p_{0,S}{}^{*} = \frac{1}{1+N^{*}} = \frac{1}{1+\sum_{j=1}^{m} N_j} = \frac{1}{1+\sum_{j=1}^{m}\frac{1-p_{0,S,j}}{p_{0,S,j}}} \tag{81}$$

The Bunching-free and Free-space states within the different major streams are independent of each other, since the vehicles in motion (within different streams) do not influence on each other. As a result

$$p_{0,B}{}^{*} = \prod_{j=1}^{m} p_{0,B,j} \tag{82}$$

and

$$p_{0,F}{}^{*} = \prod_{j=1}^{m} p_{0,F,j} \tag{83}$$

are obtained. Summarizing all the derivations above, one obtains by systems with m major streams (in serial configuration) of ranks r, r-1, r-2,.,r-m for the minor stream of rank r+1 (cf. Fig.**14**)

$$C = C_s \cdot \frac{1}{1+\sum_{j=1}^{m}\frac{1-p_{0,S,j}}{p_{0,S,j}}} \cdot \prod_{j=1}^{m} p_{0,B,j} \cdot \prod_{j=1}^{m} p_{0,F,j} \tag{84}$$

C_s is expressed by eqs.(73), (74), and (75).

4.3 Queuing system with combinations of major streams in parallel and serial configurations

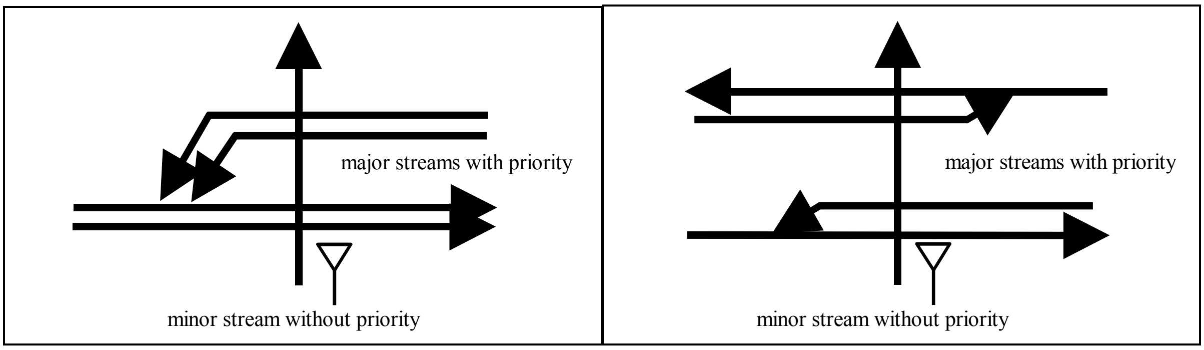


Fig. 15 - System with combination of parallel and serial major streams

Summarizing the results of the sections 4.1 and 4.2, for systems with m serial major stream groups of ranks 1, 2, ..., m, which themselves in turn consist of n_1, n_2,.., n_m parallel major streams of the same rank, one obtains for the minor stream of rank m+1 the capacity (cf. Fig.**15**a)

$$C = C_s \cdot \frac{1}{1 + \sum_{j=1}^{m} \frac{1 - \prod_{i=1}^{n_j} p_{0,S,i,j}}{\prod_{i=1}^{n_j} p_{0,S,i,j}}} \cdot \prod_{j=1}^{m} \prod_{i=1}^{n_j} p_{0,B,i,j} \cdot \prod_{j=1}^{m} \prod_{i=1}^{n_j} p_{0,F,i,j} \qquad (85)$$

and for systems with n parallel major streams groups, which themselves in turn consist of m_1, m_2,.., m_n serial major streams of ranks (1, 2,.., m_1), (1, 2,.., m_2),.., (1, 2,.., m_n), one obtains for the minor stream of ranks r+1 the capacity (cf., Fig.**15**b)

$$C = C_s \cdot \prod_{i=1}^{n} \left(\frac{1}{1 + \sum_{j=1}^{m_i} \frac{1 - p_{0,S,i,j}}{p_{0,S,i,j}}} \right) \cdot \prod_{i=1}^{n} \prod_{j=1}^{m_i} p_{0,B,i,j} \cdot \prod_{i=1}^{n} \prod_{j=1}^{m_i} p_{0,F,i,j} \qquad (86)$$

C_s is expressed by eqs.(73), (74), and (75).

Here, $P_{0,X,i,j}$ is the probability that the state X in stream i of the group j and/or in stream j of the group i does not occur.

The eqs.(85) and (86) are also valid for major streams with different t_g (or t_0) and τ. t_f is always homogeneous, since here only one minor stream is considered.

5 Possible Application of the New Procedure

5.1 Example for of multi-lane major roads

The traffic lanes on a major road can be considered as major streams in parallel configuration. According to eqs. (72) to (75) one obtains the capacity of the minor stream, C, for a major road containing n traffic lanes with continuous departure and const. parameters t_g, t_f and τ

$$
\begin{aligned}
C &= \prod_{i=1}^{n}\left(1-\tau\cdot q_{p,i}\right)\cdot\frac{1}{t_f}\cdot\prod_{i=1}^{n}\exp\left(-q_{f,i}\cdot(t_{0,i}-\tau_i)\right)\\
&= \prod_{i=1}^{n}\left(1-\tau\cdot q_{p,i}\right)\cdot\frac{1}{t_f}\cdot\exp\left(-\sum_{i=1}^{n}\left(q_{f,i}\cdot(t_{0,i}-\tau_i)\right)\right)
\end{aligned}
\tag{87}
$$

with $t_{0,i} = t_{g,i} - \frac{t_f}{2}$

In the case of discrete departure one obtains

$$
\begin{aligned}
C &= \prod_{i=1}^{n}\left(1-\tau\cdot q_{p,i}\right)\cdot\prod_{i=1}^{n}\exp\left(-q_{f,i}\cdot(t_{g,i}-\tau_i)\right)\cdot\frac{\sum_{i=1}^{n}\left(q_{f,i}\right)}{1-\exp\left(-\sum_{i=1}^{n}\left(q_{f,i}\right)\cdot t_f\right)}\\
&= \prod_{i=1}^{n}\left(1-\tau\cdot q_{p,i}\right)\cdot\frac{\sum_{i=1}^{n}\left(q_{f,i}\right)\cdot\exp\left(-\sum_{i=1}^{n}\left(q_{f,i}\cdot(t_{g,i}-\tau_i)\right)\right)}{1-\exp\left(-\sum_{i=1}^{n}\left(q_{f,i}\right)\cdot t_f\right)}
\end{aligned}
\tag{88}
$$

5.2 Example for roundabouts

A approach with n_e traffic lanes to a roundabout with n_c circulation lanes is considered.

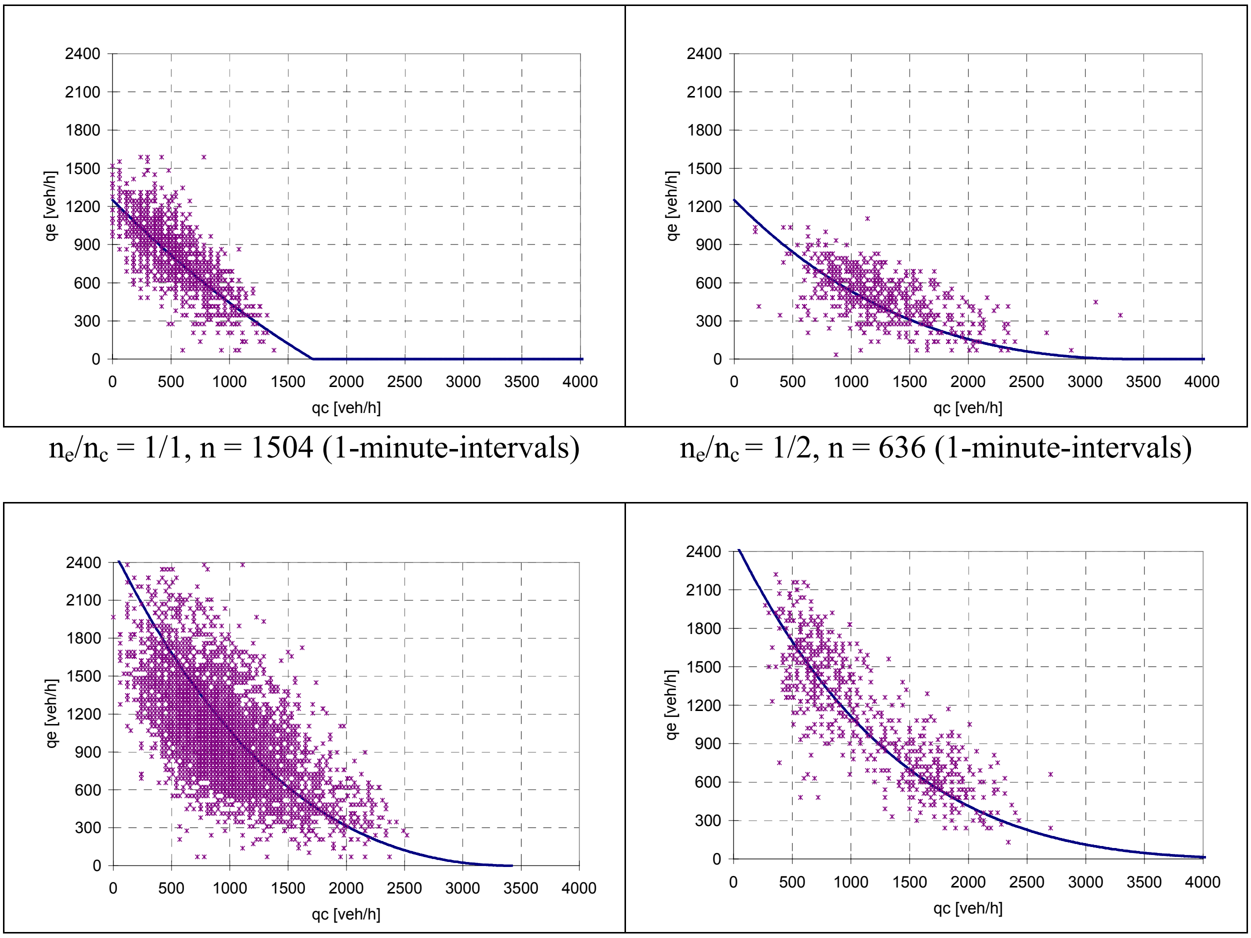
Setting $q_{p,i}=q_c/n_c$, $t_{g,i}=t_g$, $\tau_i=\tau$, and $\varphi_i = 1-\tau_i\cdot q_{p,i}$ for all major streams, one obtains for the approach at roundabouts the universal capacity formula (cf. Wu, 1997b)

$$
q_e = n_e\cdot\left(1-\frac{\tau\cdot q_c}{n_c}\right)^{n_c}\cdot\frac{1}{t_f}\cdot\exp\left(-q_c\cdot(t_0-\tau)\right)
\tag{89}
$$

with q_e = total capacity of the approach (=C)
q_c = total traffic intensity in the circulation lanes
t_0 = t_g - $t_f/2$

The values t_g=4.12s, t_f=2.88s,and τ=2.10s are measured at roundabouts in Germany.

The eq.(89) agrees with the measurements very well (Fig.**16**, cf. Wu, 1997b). This equation is also incorporated into the German HCM 2000.



$n_e/n_c = 1/1$, n = 1504 (1-minute-intervals)

$n_e/n_c = 1/2$, n = 636 (1-minute-intervals)

$n_e/n_c = 2/2$, n = 4574 (1-minute-intervals)

$n_e/n_c = 2/3$, n = 295 (1-minute-intervals)

Fig. 16 – Comparison between eq. (89) and measurements, — eq. (89), ∗∗ measurements

5.3 Example for streams of higher ranks at standard crossroads

In general, the expression for the probability for the state that in all major streams no queuing occurs may have the following form:

$$p_{0,S}^{*} = \prod_{k=1}^{l}\left(\frac{1}{1+\sum_{j=1}^{m_k}\frac{1-\prod_{i=1}^{n_{j,k}} p_{0,S,i,j,k}}{\prod_{i=1}^{n_{j,k}} p_{0,S,i,j,k}}}\right).....\qquad(90)$$

For instance, for the configuration that every stream at a standard crossroad has exactly one traffic lane (cf. Fig.**17**), the ranks of minor streams and their major streams can be depicted in Fig.**18** in the order of the priority of the streams (cf. Wu, 1997a). Here also the pedestrian streams crossing the minor road are taken into account.

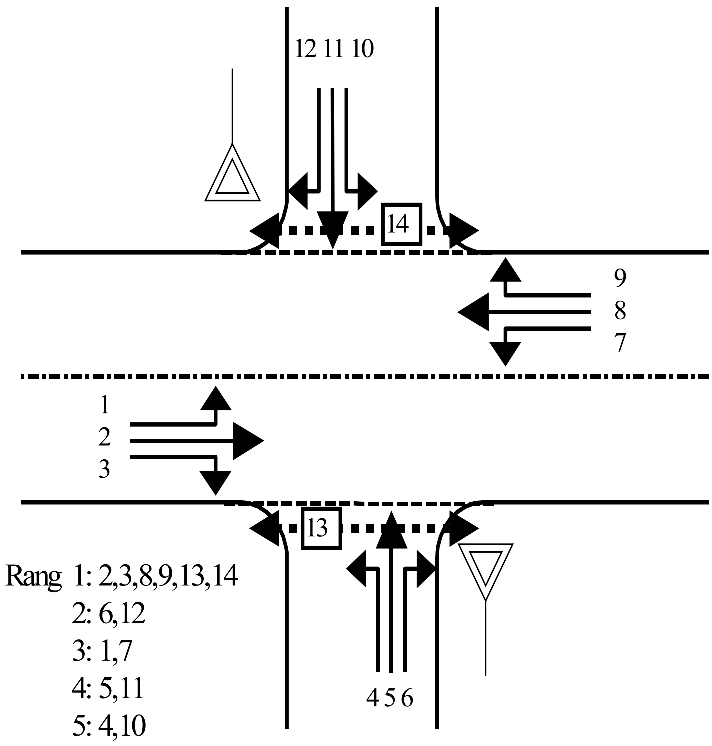


Fig. 17 – Streams of a priority-controlled crossroad

subject stream	rank r	Streams of streams r-1	Streams of streams r-2	Streams of streams r-3	Streams of streams r-4	Note
1	3	9	8+14			MLT
2	1					MTH
3	2	13				MRT
4	5	11	1,7	12	2,8,13	LT
5	4	1,7	9	2,8,13,14		TH
6	2	2,13				RT
7	3	3	2,13			MLT
8	1					MTH
9	2	14				MRT
10	5	5	1,7	6	2,8,14	LT
11	4	1,7	3	2,8,13,14		TH
12	2	8,14				RT
13	1					Pd.S
14	1					Pd.N

Fig. 18 - Composition of the major streams for the minors at the standard crossroad

The relationship between the Queuing-free states within the major streams (in parallel or serial configuration) is specified in Fig.**19**.

The capacity of the streams has to be calculated according to the precedence of ranks. That is, at first for all streams of first rank (if it is necessary), then of second rank and so on.

For example, for the stream 4 (cf. Fig.**18**, row 4) at the priority-controlled standard crossroad the capacities for streams 2, 8, and 13 have to be calculated at first then for the stream 12, and then for the streams 1 and 7, and then for the stream 11 and finally for the stream 4 (cf. Fig. **19**).

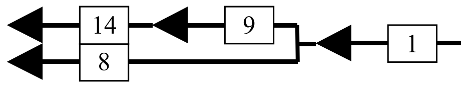
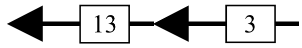
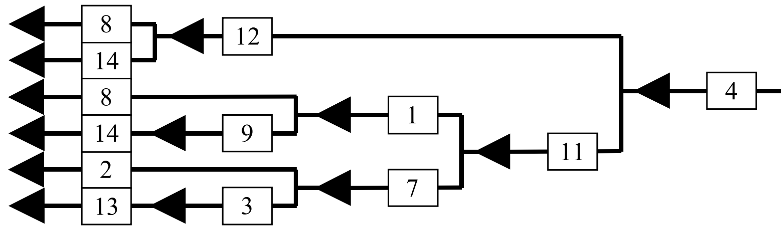
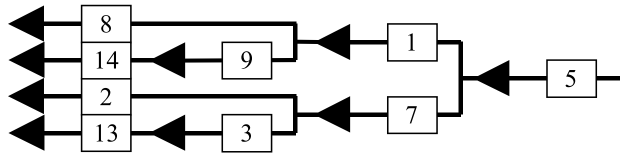
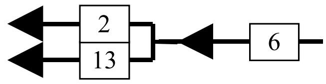
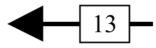
number of stream	Rank 1 / Rank 2 / Rank 3 / Rank 4 / Rank 5	number of the major streams
1		8+14 +9
2		
3		13
4		2+8+13 +3+12 +1+7 +11
5		2+8+13+14 +9 +1+7
6		2+13
13		
7 through 12 and 14	symmetric to 1 through 6 and 13	

Fig. 19 - Scheme for calculation of the Queuing-free states in the major streams

Accordingly, the probability of Queuing-free state for the stream 4 at this standard crossroad reads

$$p_{0,S,\text{all streams vs }4}{}^{*} = \left(\frac{1}{1+\dfrac{1-\left(\dfrac{1}{1+\dfrac{1-p_{0,S,1}}{p_{0,S,1}}+\dfrac{1-p_{0,S,9}}{p_{0,S,9}}}\right)\cdot\left(\dfrac{1}{1+\dfrac{1-p_{0,S,3}}{p_{0,S,3}}+\dfrac{1-p_{0,S,7}}{p_{0,S,7}}}\right)}{\left(\dfrac{1}{1+\dfrac{1-p_{0,S,1}}{p_{0,S,1}}+\dfrac{1-p_{0,S,9}}{p_{0,S,9}}}\right)\cdot\left(\dfrac{1}{1+\dfrac{1-p_{0,S,3}}{p_{0,S,3}}+\dfrac{1-p_{0,S,7}}{p_{0,S,7}}}\right)}+\dfrac{1-p_{0,S,11}}{p_{0,S,11}}}\right)\cdot p_{0,S,12} \tag{91}$$

With

$$p_{0,B,\text{all streams vs } 4}{}^{*} = \prod_{\substack{i=1,2,\\7,8,\\11,12,13}} (1 - q_i \cdot \tau_i) \tag{92}$$

and

$$p_{0,F,\text{all streams vs } 4}{}^{*} = \exp(-\sum_{\substack{i=1,2,\\7,8,\\11,12,13}} q_i \cdot (t_{0,i} - \tau_i)) \tag{93}$$

one obtains

$$C_4 = C_s \cdot p_{0,s,\text{all streams vs } 4}{}^{*} \cdot p_{0,B,\text{all streams vs } 4}{}^{*} \cdot p_{0,F,\text{all streams vs } 4}{}^{*}$$

$$= 1/t_{f,4} \cdot \text{eq.}(91) \cdot \text{eq.}(92) \cdot \text{eq.}(93)$$

Here, also the Queuing-free states in streams 3 and 9 caused by crossing pedestrians is considered. The critical gap t_g (or the zero-gaps t_0) against pedestrians is calculated as the walking time of pedestrians crossing the walkway.

For the case, that no queue exists in streams 3 and 9 (no impedance by pedestrians) the equation (91) is reduced into

$$p_{0,S,\text{alle streams vs } 4}{}^{*} = \left(\frac{1}{1 + \frac{1 - p_{0,S,1} \cdot p_{0,S,7}}{p_{0,S,1} \cdot p_{0,S,7}} + \frac{1 - p_{0,S,11}}{p_{0,S,11}}}\right) \cdot p_{0,S,12} \tag{94}$$

The eq.(94) is verified by intensive simulation studies (cf. Wu, 1998). It is incorporated into the German HCM 2000.

The capacity of stream 4 with continuous departure and const. parameters t_0 and τ then reads as a complete formula

$$C_{\text{stream } 4} = \left(\left(\frac{1}{1 + \frac{1 - p_{0,S,1} \cdot p_{0,S,7}}{p_{0,S,1} \cdot p_{0,S,7}} + \frac{1 - p_{0,S,11}}{p_{0,S,11}}}\right) \cdot p_{0,S,12} \cdot \prod_{\substack{i=1,2,\\7,8,\\11,12}} (1 - q_i \cdot \tau_i) \right) \cdot \frac{1}{t_f} \cdot \exp(-\sum_{\substack{i=1,2,\\7,8,\\11,12}} q_i \cdot (t_{0,i} - \tau_i)) \tag{95}$$

If no minimum gap τ within the major streams is considered (i.e., $\tau_i = 0$) the equation turns into

$$C_{\text{stream } 4} = \left(\frac{1}{1 + \frac{1 - p_{0,S,1} \cdot p_{0,S,7}}{p_{0,S,1} \cdot p_{0,S,7}} + \frac{1 - p_{0,S,11}}{p_{0,S,11}}}\right) \cdot p_{0,S,12} \cdot \frac{1}{t_f} \cdot \exp(-\sum_{\substack{i=1,2,\\7,8,\\11,12}} q_i \cdot t_{0,i}) \tag{96}$$

6 SUMMARIES

A new procedure for the determination of capacity at unsignalized (priority-controlled) intersections is presented here. This new procedure can be applied for the conditions

- arbitrary many major streams with different critical gaps t_g (or zero-gaps t_0), and minimum gaps τ
- arbitrary high rank of the minor stream
- arbitrary parallel and serial configurations of the major streams
- arbitrary distribution of the gaps t in the major streams for const. t_g (or zero-gaps t_0), t_f, and τ
- arbitrary distribution of critical gaps t_g (or zero-gaps t_0), move-up times t_f and minimum gaps τ if the corresponding Laplace transform of the distribution are given, for shift-negative. exponentially distributed gaps t in the major streams and Erlang-distributed t_g (or zero-gaps t_0), and τ, explicit capacity functions are given
- arbitrary queuing and bunching saturations in the major streams

For a Queuing system with one major stream and one minor stream, he most universal formula for capacity determination are given by eqs.(26) and (27). According to the equations, the effect of distributed critical gaps, t_g, distributed move-up times, t_f, and distributed minimum gaps, τ, can be easily analysed. One can recognize that for inconsistent and consistent driver behavior the effect of distributed t_g, t_f, and τ are quite different. The effect of distributed t_g, t_f, and τ mutually neutralize themselves for consistent and inconsistent behavior. Therefore, one can neglect the distribution of the critical gaps t_g, the move-up times t_f, and the minimum gaps τ for practical uses because in the real world the driver behavior cannot be found exactly.

The new procedure provides a generalized form of all the usual formulae for calculating the capacity at unsignalized intersections. The usual formula are reproduced by the new procedure if the corresponding parameters are set.

The new procedure takes into consideration most of the parameters at unsignalized intersections. For practical uses, the procedure should be calibrated and validated with measurements or simulations. The initial calibrations and validation for roundabouts (Wu, 1997b) and for streams of fourth rank at priority-controlled intersections (Wu, 1998) already shows enormous potential and the amazing accuracy of the new procedure.

The most useful formulae in this paper are given by eqs.(85) and (86). In combination with eqs.(73), (74), (75) for calculating C_s and eqs.(42), (43), (44) for calculation $p_{0,S}$, $p_{0,B}$, and $p_{0,F}$ one can obtain the capacity for all streams at unsignalized (priority-controlled) intersections under all possible configurations in the real world.

The new procedure provides at first time a conclusive theory to deal with a unsignalized intersection with more than four priority ranks. This is in terms of considering pedestrians a very important new development (cf. eq.(91)).

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