

Total Capacity of Roundabouts - Analyzed by Conflict Technique

Ning Wu

Institute for Traffic Engineering and Management
Ruhr-University Bochum
Universitaetsstr. 150, D-44801 Bochum, GERMANY
E-Mail: ning.wu@rub.de

Werner Brilon (corresponding author)

Institute for Traffic Engineering and Management
Ruhr-University Bochum
Universitaetsstr. 150, D-44801 Bochum, GERMANY
E-Mail: werner.brilon@rub.de

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ABSTRACT

The conventional way of capacity analysis is based on gap-acceptance methods or on empirical regression or a combination of both. Influence of pedestrians is modeled by reduction factors with a questionable empirical background. Thus, the current methods are based on an incoherent mix of sophistications. Moreover, they do not account for the interaction between the different elements.

The paper presents a new model, which treats the whole intersection as one entity. Here all the conflicts where different streams (vehicles and non-motorized road users) intersect within the roundabout are identified. Each conflict point is treated as one queuing system with a simplified queuing mechanism. In addition, the interactions between the consecutive queuing systems at roundabouts are taken into account according to the theory of chains of queues where the distance (= storage area) between the conflict points becomes important.

The paper explains the sophistication of the model, presents the mathematical derivations of ready-to-use capacity equations plus parameter calibration by existing data, and demonstrates real-world application. The advantages of the technique are: all conflicts - both between vehicle streams and pedestrian conflicts at entries and exits - are treated by the same congruent methods. In addition, the interaction between consecutive arms of the roundabout is modeled. The method is also able to model limited priority (e.g. for pedestrians at crosswalks or zebra crossings) for all conflict points. Finally, the conflicts that are decisive for the performance of the whole intersection are identified. The method can be easily implemented into computer software.

INTRODUCTION

The capacity of roundabouts is a matter of investigations in many countries since the implementation of modern roundabouts in the 1980ies. The remarkable aspect is that each country has attempted to find its own solution. Up to some degree, this may be justified by different driver behaviors or divergent traffic rules. The differences between solutions in the various countries, however, have become rather small and may be mainly based on limited sample sizes or differing methods for analysis. Normally the classical approaches treat each entry to a roundabout like an isolated T-junction. Effects of mutual interaction between the various conflict points of the roundabouts remain disregarded.

Based on gap-acceptance theory, the maximum throughput (= capacity C) of one entry into the roundabout can be calculated by the Siegloch-formula (1):

$$C = \frac{3600}{t_f} e^{-\frac{q_C}{3600} \cdot t_0} \quad (1)$$

where

- C = capacity (veh/h),
- t_0 = $t_c - t_f / 2$
= so-called zero-gap (s),
- t_f = follow-up time (s),
- t_c = critical gap (s), and
- q_C = traffic volume on the circular lane in front of the entry (veh/h).

A consequent application would require reliable estimates for t_c and t_f (cf. 2). This formula is derived based on the assumption that the gaps on the circular lane are exponentially distributed (i.e. no bunching). This is rather unrealistic at roundabouts. Also effects of priority reversal or typical influences of the degree of saturation on t_c and t_f cannot be described.

Therefore, most researchers favor the so-called “empirical regression” approach for roundabout capacity estimation. Here the real-world traffic at a roundabout entry is measured over periods of steady queuing on the entry during specific time intervals, e.g. 1 minute. In case of a steady queue on the entry lane the observed flow volume is the capacity. This can be plotted over the observed conflict volume q_C on the circular lane. Then a regression line is applied to represent these results.

The earliest attempt following this idea was the well-known British investigation by Kimber (3). Here a linear regression equation for C as a function of q_C was applied where the parameters of the equation were modified according to geometric features of the intersection. Linear equations were also applied elsewhere, e.g. (4). Other authors found nonlinearities within their empirical results. Most of them used exponential regression functions. One such solution is to use t_c and t_f in eq. (1) as regression parameters (e.g. 5). Others (e.g. 6) transformed this into a simple exponential function. In this form, the capacity formulas of the new HCM (7) have become a standard for the US.

The linear and the exponential regression approach for estimating the capacity at roundabouts are rather pragmatic. One cannot be sure that the linear or exponential functions do also apply in areas of the C - q_C -diagram with few measurement points. Therefore, Wu (8)

proposed the following - more general - formula for the capacity of a single lane entry to a roundabout:

$$C = \frac{3600}{t_f} \cdot \left(1 - \frac{q_c \cdot \tau}{3600}\right) \cdot \exp\left(-\frac{q_c}{3600} \cdot (t_0 - \tau)\right) \quad (2)$$

$T1$ $T2$ $T3$

where

- C = capacity of the entry (veh/h),
- q_c = flow on circular lanes at the subject entry (veh/h), and
- τ = minimum headway between vehicles on the circular lane (s).

The specific advantage of this approach is that the minimum headway τ between successive vehicles on the circle is taken into account. Eq. (2) is the generalized form for the exponential and the linear capacity function. For example, this formula converges to an exponential function by $\tau = 0$ and to a linear function by $t_0 = \tau$. The equation can be understood as a multiplicative combination of the three terms $T1$, $T2$, and $T3$.

There are also reports, which found an influence of the exiting traffic on the entry capacity like Bovy et.al. (9) for single lane roundabouts in Switzerland, Wu (10) for double lane roundabouts with single lane exists, or Schmotz (11) for mini roundabouts which found its way into the German guideline HBS (12).

On the entries, there may be another type of conflict requiring capacity consideration. Besides the conflicting flow on the circular lane this is the pedestrian crossing with priority over the entering motorized vehicles, e.g. by a zebra crossing. The capacity reducing effect of the pedestrian crossing on the entry capacity has been studied by Stuwe (13). Her results, which were based on a rather limited sample size of German zebra crossings, have been implemented into the German guidelines HBS (12) and also into the HCM (7).

In addition, the exits from the roundabout can establish a bottleneck for vehicle traffic with limited capacity influenced by pedestrians crossing the exit. A solution to describe this capacity has been presented by Marlow and Maycock (14) in conjunction with Griffith's (15) capacity formula for vehicular traffic traversing a zebra crossing. Another more recent result for exit capacity at mini-roundabouts was presented by Schmotz (11).

One commonality of all the results published up to now is that they treat specific conflicts at the roundabout with differing and incoherent methods and that they are not regarding any interaction between adjacent conflict areas. Under this aspect, the solutions may be realistic as long as each element of the intersection is not operating near or above capacity. As soon as one of the conflict points is overloaded for a short period, queues may be formed. Due to limited storage spaces on the circular lane, these queues are impeding other conflict points with the consequence of reduced capacities there. Thus, a short overload at one point can easily become the starting point for an overload of the whole intersection – even if the analysis for all the single conflicts still gives a sufficient service quality. This kind of mutual interdependencies of different parts of the roundabout are not confined to the case of a complete overload. Instead, they occur already at a moderate increase of traffic demand. Due to this aspect, it is desirable to formulate algorithms that describe the capacity of the roundabout as a whole regarding capacities

of the various conflicts as well as the influence of limited internal storage areas.

This paper like the proceeding presentation (16) tries to point out a mathematical model, which is capable to estimate the capacity of the roundabout as a whole. The model represents the various conflict points by a homogeneous set of models including also a stochastic approach to cope with the interlocking between subsequent bottlenecks within the intersection.

METHODOLOGY

Capacity of traffic streams with priority control

As a conflict, we treat the intersection of several streams that have to pass the same area within an intersection (cf. 17, 18). The vehicles involved into a conflict have to pass the area one after the other. The set of streams involved into the same conflict is called a conflict group. The simplest case is a conflict group of two streams (Figure 1a)). One of these streams (i) is assumed to have priority over the other (j).

It can be assumed that any major stream vehicle passing a conflict point (CP) will occupy the CP for a certain time. Only during the unoccupied time, the minor stream can pass the conflict point with a basic capacity C_0 . The basic capacity of a minor stream is the reciprocal of the follow-up time t_f . The conflict point can be occupied by a major stream vehicle in three different ways: it is occupied if there is a queue, if a platoon is passing the CP, or if a single vehicle is arriving. A minor stream vehicle can only pass the conflict point if the conflict point is neither occupied by a queue (spilling back from downstream) nor by a platoon nor by a single arriving vehicle in the major stream (19). The case of a spillback from downstream is treated in subsequent sections of this paper.

In case that no queue from downstream occurs in the major stream, the last two terms of eq. (2) describe exactly the portions of occupied time in the major stream. Thus, for a two-stream problem (Figure 1a)), the capacity of a minor stream j entering a major stream i can be calculated by a general model closely related to eq. (2):

$$C_j = T1 \cdot T2 \cdot T3 \quad (3)$$

where

$$\begin{aligned} C_j &= \text{capacity of the minor stream } j \text{ (veh/h),} \\ T1 &= C_{0,j} \\ &= 3600/t_{f,j} \\ &= \text{basic capacity of the minor stream } j \text{ in case of} \\ &\quad \text{no major stream } (i) \text{ vehicle is occupying the conflict point (veh/h),} \\ T2 &= 1 - \frac{q_{C,i} \cdot \tau_i}{3600} \\ &= \text{portion of time of no vehicle platooning (vehicles following} \\ &\quad \text{consecutively each other with a minimum headway } \tau_i) \\ &\quad \text{in the major stream } i, \text{ and} \\ T3 &= \exp\left(-\frac{q_{C,i}}{3600} \cdot (t_{0,ij} - \tau_i)\right) \end{aligned}$$

= portion of time of no impedance caused by a vehicle arriving from upstream in the major stream i .

As a simplification, we can assume $t_{0,ij} \approx \tau_i$. This can be considered as realistic in most cases for traffic behavior at a roundabout due to the usual estimates for t_c and t_f . Thus, eq. (3) yields as an approximation for the capacity C_j :

$$C_j = C_{0,j} \cdot \left(1 - \frac{q_{C,i} \cdot \tau_i}{3600}\right) = \frac{3600}{t_{f,j}} \cdot \left(1 - \frac{q_{C,i} \cdot \tau_i}{3600}\right) \quad (4)$$

where

$t_{f,j}$ = follow-up time of the minor stream j (s),
 $q_{C,i}$ = volume of the major stream near entry i
 (= circular stream at a roundabout) (veh/h), and
 τ_i = minimum time headway of the major stream i (s).

For more than one major stream on the circular roadway (Figure 1b), the portion of time T_2 of no vehicle platooning for different major streams i is assumed to be independent from each other. Thus, for a minor stream j with multiple major streams i we get

$$C_j = C_{0,j} \prod_{i \in I} \left(1 - \frac{q_{C,i} \cdot \tau_i}{3600}\right) \quad (5)$$

where I is the set of all major streams under consideration.

Here also the so-called "limited priority" (cf. 20) can be included. Limited priority means that the major stream vehicles do not use their right of way in each case. This effect can be taken into account by the probability b_{ij} . Eq. (5) then is expanded into (cf. 21).

$$C_j = C_{0,j} \prod_{i \in I} \left(1 - b_{ij} \frac{q_{C,i} \cdot \tau_i}{3600}\right) \quad (6)$$

where

b_{ij} = probability that a major stream vehicle goes first in case of a conflict (e.g.: $b_{ij}=1$: no i yielding to j , or in other words: the priority of i is always obeyed by j)

The values of b_{ij} can be defined according to a so-called conflict matrix.

The total capacity of an entry with more than one lane is the sum of capacities of all entry lanes (Figure 1c). That is,

$$C = \sum_{j \in J} C_j = \sum_{j \in J} \left(C_{0,j} \prod_{i \in I} \left(1 - b_{ij} \frac{q_{C,i} \cdot \tau_i}{3600} \right) \right) \quad (7)$$

where J is the set of all minor streams under consideration.

Multilane entry with limited length of lanes

If an entry lanes has only a limited length n_s , and both lanes emerge from a single upstream lane (Figure 1d)), the capacities of the single lanes at the entry cannot be fully utilized as calculated. According to the usual concept of unsignalized intersections, we have here a “short-share lane” problem. The capacity of shared lanes can be determined according to a formula first developed by Harders (22). This concept has been extended by Wu (8, 10) such that also additional lanes of limited length (short lane) can be taken into account. For the case of a single-lane approach with an additional short lane near the intersection, the capacity of the entry with short-share traffic lane can be calculated from (cf. 8, 10):

$$C_s = \frac{q_{2L} + q_{2R}}{n_s + 1 \sqrt{(x_{2L})^{n_s + 1} + (x_{2R})^{n_s + 1}}} = \frac{q_{2L} + q_{2R}}{n_s + 1 \sqrt{\left(\frac{q_{2L}}{C_{2L}}\right)^{n_s + 1} + \left(\frac{q_{2R}}{C_{2R}}\right)^{n_s + 1}}} \quad (8)$$

where

- C_s = capacity of the short-shared lane (veh/h),
- q_{2L} = volume for the entry lane 2L (veh/h),
- q_{2R} = volume for the entry lane 2R (veh/h),
- C_{2L} = capacity for the entry lane 2L (veh/h),
- C_{2R} = capacity for the entry lane 2R (veh/h),
- x_{2L} = degree of saturation of the entry lane 2L,
- x_{2R} = degree of saturation of the entry lane 2R, and
- n_s = number of vehicles which can queue up on the short lane.

For $n_s = 0$, eq. (8) yields the well-known shared lane formula from Harders (cf. 22):

$$C_s = \frac{q_{2L} + q_{2R}}{x_{2L} + x_{2R}} = \frac{q_{2L} + q_{2R}}{\frac{q_{2L}}{C_{2L}} + \frac{q_{2R}}{C_{2R}}} \quad (9)$$

The two-stage queuing problem

Eq. (6) applies only for cases where the major streams are crossed by a minor stream at an isolated conflict point. In case of a pedestrian crossing at a roundabout, there are often several spaces between the circular major stream and the pedestrian crossing (Figure 1e). The minor stream at the entry can pass the pedestrian crossing and the circular major stream one-by-one and if necessary wait in between. The capacity of this two-stage queuing system with n_w waiting places is given by Brilon and Wu (23):

$$C_T = \begin{cases} \frac{1}{n_w + 1} \cdot (n_w \cdot C_b + C_{ab}) & \text{for } y = 1 \\ (1 - w_0) \cdot C_b + w_0 \cdot C_{ab} & \text{elsewhere} \end{cases} \quad (10)$$

where

$$\begin{aligned} C_T &= \text{total capacity of the combined queuing system (veh/h),} \\ w_0 &= \frac{y - 1}{y^{n_w + 1} - 1}, \\ n_w &= \text{possible waiting places between both stages,} \\ C_a &= C_{0,a} \cdot p_{0,a} \\ &= \text{capacity of the first stage } a \text{ (veh/h),} \\ C_b &= C_{0,b} \cdot p_{0,b} \\ &= \text{capacity of the second stage } b \text{ (veh/h),} \\ C_{ab} &= C_{0,ab} \cdot p_{0,a} \cdot p_{0,b} \\ &= \text{capacity of queuing system in case } n_w = 0 \text{ (veh/h),} \\ p_{0,a}, p_{0,b} &= \text{probabilities that the queuing stage } a \text{ or } b \\ &\quad \text{are not impeded by major vehicles, and} \\ y &= \frac{C_a - C_{ab}}{C_b - C_{ab}}. \end{aligned}$$

Different applications for the two-stage model at a roundabout are described in the subsequent sections of the paper. The calculation of C_a and C_b is explained for each case in these sections.

In Brilon and Wu (23), the capacity C_{ab} of the queuing system in case of $n_w = 0$ is only given for a special case with $C_{0,a} = C_{0,b} = C_{0,ab}$. For the two-stage problem at roundabouts regarding pedestrians, this assumption does not apply. However, also in the general case with $C_{0,a} \neq C_{0,b} \neq C_{0,ab}$ the capacity C_{ab} can be calculated. The capacity C_{ab} is a function of capacities of the two stages. That is,

$$C_{ab} = C_{0,ab} p_{0,a} p_{0,b} = C_{0,ab} \frac{C_a}{C_{0,a}} \frac{C_b}{C_{0,b}} = f_{ab} C_a C_b \quad (11)$$

where

$$f_{ab} = \frac{C_{0,ab}}{C_{0,a} \cdot C_{0,b}}$$

This leads to

$$y = \frac{C_a - C_{ab}}{C_b - C_{ab}} = \frac{C_a(1 - f_{ab}C_b)}{C_b(1 - f_{ab}C_a)} \quad (12)$$

The values of C_0 are actually the reciprocal of the follow-up time t_f . It is the time headway h at lane capacity C_{ln} plus the lost time Δt experienced by an approaching vehicle needed for deceleration and orientation maneuver. That is,

$$\Delta t = t_f - h = \frac{3600}{C_0} - \frac{3600}{C_{ln}} \quad (13)$$

where Δt is given in seconds.

For crossing stage a and b in one step, the deceleration and orientation time Δt is the sum of Δt in both stages. That is,

$$\Delta t_{ab} = \Delta t_a + \Delta t_b = \frac{3600}{C_{0,a}} + \frac{3600}{C_{0,b}} - \frac{2 \cdot 3600}{C_{ln}} \quad (14)$$

Thus,

$$C_{0,ab} = \frac{3600}{h + (\Delta t_a + \Delta t_b)} = \frac{1}{\frac{1}{C_{0,a}} + \frac{1}{C_{0,b}} - \frac{1}{C_{ln}}} \quad (15)$$

In the following, a time headway of $h = 2.2$ s is used at roundabouts. Thus, the lane capacity C_{ln} is $3600/2.2\text{s} = 1640$ veh/h.

Eq. (10) was derived for a queuing system with Markovian arrivals and departures. In reality, those presumptions are rarely satisfied. For accounting for the stochastic property of a queuing system, a factor C_n can be applied to the parameter n_w . Thus we get as an approximation to the stochastic property of the queuing system instead of n_w a parameter $n_w^* = C_n n_w$ in eq. (10). For a queuing system with Markovian arrivals and departures is $C_n = 1$. The factor C_n is normally larger than 1. For example, one can use $C_n = 1.68$ for a queuing system with Markovian arrivals and deterministic service times (24). The value of C_n is subject to calibration.

In general, the total capacity of a two-stage queuing system then can be expressed by four significant parameters:

$$C_T = f(C_a, C_b, C_{0,ab}, n_w^*) \quad (16)$$

The formulation (eq. (10)) of the capacity of a two-stage queuing system is very complex. As a simplification, the following formulation can be used instead of eq. (10):

$$C_T = \min \left\{ \begin{array}{l} C_a \cdot \left(1 - (1 - f_{ab} \cdot C_b)^{n_w^* + 1} \right) \\ C_b \cdot \left(1 - (1 - f_{ab} \cdot C_a)^{n_w^* + 1} \right) \end{array} \right\} = f(C_a, C_b, C_{0,ab}, n_w^*) \quad (17)$$

For practical applications, this approximation provides a rather good fit.

CONFLICT GROUPS AT A ROUNDABOUT

All capacities of the individual conflicts or conflict groups at a roundabout can be estimated using the equations presented in the previous section. The total set of conflict points at a roundabout with 4 arms and 12 movements is illustrated in Figure 2a) and b). For a particular double lane entry yielding to a double lane circular roadway, the traffic streams and the involved conflict points are depicted in Figure 2c). The traffic lanes are named left (L) and right (R) entry lane, outer (O), inner (I) circular lane, and pedestrian crosswalk (PE).

The circular volume q_C , the entry volume q_E , and the exit volume q_A at each subject approach k can be easily calculated from the O-D-matrix of traffic demands at the intersection.

For double lane roundabouts, one can assume that the volume on the left entry lane (L) corresponds to the left-turn volume at the entry. Furthermore, it can be assumed that nearly none (0%) of the left-turn vehicle from the upstream arm will use the inner circular lane (I) at a very low circular volume and nearly all of them will use the inner circular lane at high circular volume (up to 1600 veh/h). In between, the volumes can be obtained by interpolation. These assumptions are realistic because the inner circular lane and the left entry lane are mostly used by the corresponding left-turn vehicles. That is,

$$q_{C,I,k} = q_{L,k-1} \cdot \frac{q_C}{1600} = q_{k-1,k+2} \cdot \frac{q_C}{1600} \quad \text{and} \quad q_{L,k} = q_{k,k+3} \quad (18)$$

For example, for an entry with two lanes (L and R) shown in Figure 2c) with one outer (O), one inner (I) circular lane, and one pedestrian crosswalk (PE), the capacity of the left and the right entry are

$$C_L = C_{0,L} \cdot \left(1 - b_{IL} \cdot \frac{q_{c,I} \cdot \tau_I}{3600} \right) \cdot \left(1 - b_{OL} \cdot \frac{q_{c,O} \cdot \tau_O}{3600} \right) \cdot \left(1 - b_{PEL} \cdot \frac{q_{c,PE} \cdot \tau_{PE}}{3600} \right) \quad (19)$$

$$C_R = C_{0,R} \cdot \left(1 - b_{OR} \cdot \frac{q_{c,O} \cdot \tau_O}{3600} \right) \cdot \left(1 - b_{PER} \cdot \frac{q_{c,PE} \cdot \tau_{PE}}{3600} \right) \quad (20)$$

At roundabouts, there is usually a space with n_E storage places ($n_E \geq 1$) between the outer circular lane and the pedestrian crosswalk. A minor stream vehicle can cross the pedestrian crosswalk first and wait in between. In this case, the capacity of a minor stream must be

calculated for crossing the pedestrian crosswalk (C_a) and for entering the major stream (C_b). Thus,

$$C_{L,a} = C_{0,PEL} \cdot \left(1 - b_{PEL} \cdot \frac{q_{C,PE} \cdot \tau_{PE}}{3600} \right) \quad (21)$$

$$C_{L,b} = C_{0,L} \cdot \left(1 - b_{IL} \cdot \frac{q_{C,I} \cdot \tau_I}{3600} \right) \cdot \left(1 - b_{OL} \cdot \frac{q_{C,O} \cdot \tau_O}{3600} \right) \quad (22)$$

The combined capacity of this two-stage problem can be obtained from eq. (10) or eq. (17):

$$C_{L,T} = f(C_{L,a}, C_{L,b}, C_{0,L,ab}, n_E) \quad (23)$$

Similarly, for the right entry lane we have

$$C_{R,a} = C_{0,PER} \cdot \left(1 - b_{PER} \cdot \frac{q_{c,PE} \cdot \tau_{PE}}{3600} \right) \quad (24)$$

$$C_{R,b} = C_{0,R} \cdot \left(1 - b_{OR} \cdot \frac{q_{c,O} \cdot \tau_O}{3600} \right) \quad (25)$$

$$C_{R,T} = f(C_{R,a}, C_{R,b}, C_{0,R,ab}, n_E) \quad (26)$$

where f is the functionality given by eq. (10) or eq. (17).

Obviously, for a multi-lane roundabout, the volume distribution between q_I and q_O on the circular lanes and the volume distribution between q_L and q_R on the entry lanes must be calculated in advance. The distributions of those volumes can be estimated according to the turning movements at the intersection. For a single lane roundabout, for all equations the indices I and L are no longer applicable and terms containing these indices just can be neglected.

Taking into account the volumes of both left (q_L) and right (q_R) entry lanes and the length of the double-lane area (n_d) upstream from the pedestrian crossing, the total capacity of the double-lane entry ($C_{E,d}$) can then be calculated using eq. (8).

$$C_{E,d} = \frac{q_L + q_R}{\sqrt[n_d+1]{\left(\frac{q_L}{C_{L,T}}\right)^{n_d+1} + \left(\frac{q_R}{C_{R,T}}\right)^{n_d+1}}} \quad (27)$$

Setting $n_d = 0$, this equation yields the shared lane capacity ($C_{E,s}$) of a single lane entry opposing two circular lanes at a roundabout:

$$C_{E,s} = \frac{q_L + q_R}{\frac{q_L}{C_{L,T}} + \frac{q_R}{C_{R,T}}} \quad (28)$$

In this case two minor streams on one entry lane are actually calculated (cf. Figure 2d). One of them (q_L) crosses the outer circular lane and proceeds into the inner circular lane. And another (q_R) continues directly into the inner circular lane.

Note, at a single lane entry the approach arm has normally a flare area to enable the turning movement of the right-turn vehicle entering the roundabout. Thus, there is actually a double lane area able to accommodate one vehicle. Under real world conditions, this flare area is not used by all vehicles but only by a portion a_f of them. Thus the capacity of the single lane entry yielding to a double lane circular roadway is

$$\begin{aligned} C_{E,singlelane} &= a_f C_{E,d} \cdot (n_d = 1) + (1 - a_f) \cdot C_{E,s} \\ &= a_f \frac{q_L + q_R}{\sqrt[2]{\left(\frac{q_L}{C_{L,T}}\right)^2 + \left(\frac{q_R}{C_{R,T}}\right)^2}} + (1 - a_f) \frac{q_L + q_R}{\frac{q_L}{C_{L,T}} + \frac{q_R}{C_{R,T}}} \end{aligned} \quad (29)$$

Again, the value of a_f is subject to calibration depending on the geometric layout of the entry area. With $a_f = 0$ one is on the safer side of calculation.

Setting the traffic volume on the inner circular lane $q_I = 0$, all formulas mentioned here can be used for a roundabout with a single lane circular roadway.

As a summary, all parameters mentioned above can be defined using a conflict matrix. Table 1 shows parameters for the model calibrated to represent German roundabouts adjusted to methods from the HBS (12). The basic capacities of the minor stream $C_{0,j}$ and the minimum headway τ_l of the corresponding major stream mentioned above are given for traffic streams under consideration (veh, ped, and two-stage). For the calibration of double lane roundabouts, it was assumed that nearly no (0%) circular vehicles are using the inner circular lane at very low circular volume and 30% at high circular volume (1600 veh/h). For a double lane entry 30% of the total entry volume is assumed to use the left entry lane. These assumptions are realistic because the inner circular lane and the left entry lane are mostly used by the corresponding left-turn vehicles and due to the fact that in Germany the left lanes at a two-lane roundabout are used by the drivers rather reluctantly. That means:

$$\frac{q_{C,I}}{q_C} = \frac{0.3 \cdot q_C}{1600} \quad \text{or} \quad q_{C,I} = \frac{0.3 \cdot q_C^2}{1600} \quad (30)$$

and

$$q_L = 0.3 \cdot q_E \quad (31)$$

In Figure 3a) and b) a comparison of capacities obtained from the model compared to the HBS (12) formulas is depicted. It is obvious that the simplified model matches the HBS data

very well.

Figure 3c) – h) shows a comparison of the model to the HBS method regarding the pedestrian impedance factor at roundabout entries. For single lane roundabouts the model results represent the HBS methodology quite well (Figure 3c) +d)). For double lane roundabouts, the model results cannot represent the HBS method for the whole range of circular volumes. The reason of those deviations must be found in the HBS model, because the HBS model is a regression based on very limited data without any theoretical background. However, in the common range of circular volumes the results of HBS data can also be represented by the new model properly.

CAPACITY OF AN EXIT AT A ROUNDABOUT

An exit at a roundabout can also be considered as a two-stage queuing system (cf. Figure 2e)). The combined capacity of this two-stage problem can be obtained from eq. (10) or eq. (17):

$$C_{A,T} = f(C_{A,a}, C_{A,b}, C_{0,A,ab}, n_A) \quad (32)$$

where

$$\begin{aligned} C_{A,a} &= C_{\text{In}} \\ &= \text{capacity of the exit lane at the edge of the outer circular lane,} \\ C_{A,b} &= \text{capacity of the pedestrian crossing at the exit, and} \\ C_{0,A,ab} &= \text{the basic capacity of the two-stage problem at the exit.} \end{aligned}$$

The capacity $C_{A,b}$ can be calculated as

$$C_{A,b} = C_{A,PA} = C_{0,RAR} \cdot \left(1 - b_{PAR} \cdot \frac{q_{a,PA} \cdot \tau_{PA}}{3600} \right) \quad (33)$$

The upstream outer circular lane at an exit can be treated as a shared lane consisting of the exit lane (q_A) and the downstream outer circular lane ($q_{O,D}$). Thus, the capacity of the upstream outer circular lane at an exit is

$$C_{O,U} = \frac{q_A + q_{O,D}}{\frac{q_A}{C_{A,T}} + \frac{q_{O,D}}{C_{O,D}}} \quad (34)$$

In accordance, the parameters for an exit are given in Table 1 line e). The lane capacity of the downstream outer circular lane $C_{O,D}$ is set to the lane capacity C_{In} (1640 veh/h).

REDUCTION OF ENTRY CAPACITY AT A ROUNDABOUT DUE TO QUEUING AT THE DOWNSTREAM EXIT

For both the left and right entry lane, the capacity can be impeded by queuing vehicles on the outer circular lane caused by the downstream exit. Assuming that the downstream exit obeys an M/G/1 queuing system, the impedance factor $f_{imp,EA}$ can approximately be calculated as following.

$$f_{imp,EA} = p_{0,EA,n_{EA}} = 1 - x_{O,EA}^{C_n \cdot n_{EA} + 1} \quad (35)$$

where

$$\begin{aligned} p_{0,EA,n_{EA}} &= \text{probability that the entry is not impeded by queuing} \\ &\quad \text{from the downstream exit,} \\ &= \Pr(Q_{O,EA} \leq n_{EA}) \\ &= \text{probability that the space between the downstream exit} \\ &\quad \text{and the subject entry is not totally occupied by the downstream queue,} \\ x_{O,EA} &= q_{O,EA}/C_{O,EA} \\ &= \text{degree of saturation of the downstream outer circular lane,} \\ q_{O,EA} &= \text{volume of the downstream outer circular lane (veh/h),} \\ C_{O,EA} &= \text{capacity of the downstream outer circular lane (veh/h),} \\ n_{EA} &= \text{storage places between the entry and the downstream} \\ &\quad \text{exit on the outer circular lane (veh), and} \\ C_n &= \text{Factor accounting for the stochastic property of the} \\ &\quad \text{queuing system.} \end{aligned}$$

In this equation, the service time of the queuing system is considered as less stochastic using a factor $C_n = 1.68$ applied to the parameter n_{EA} assuming the service time of the queuing system on a circular lane is nearly deterministic (24). In Table 1 line f), the parameters for calculating the impedance caused by the downstream queue are given. The value of $n_{EA} = 3$ is assumed for a midsize single lane roundabout with an outer diameter $D = 35\text{m}$. For a real world roundabout, the value of n_{EA} can be obtained from the given geometry.

Considering the impedance of queuing caused by the downstream exit, the capacity of the second stage of the left lane is

$$C_{L,b}^* = \prod p_{0,q,i} \cdot C_{L,b} = p_{0,EA,n_{EA}} \cdot C_{L,b} = f_{imp,EA} \cdot C_{L,b} \quad (36)$$

with $C_{L,b}$ from eq. (22) and $f_{imp,EA}$ from eq. (35).

The capacity of the first stage remains unchanged:

$$C_{L,a}^* = C_{L,a}$$

with $C_{L,a}$ from eq. (21) and the functionality f from eq. (10) or eq. (17),

$$C_{L,T}^* = f(C_{L,a}^*, C_{L,b}^*, C_{0,L,ab}, n_E) \quad (37)$$

Similarly, for the right entry lane we get

$$C_{R,b}^* = p_{0,EA,n_{EA}} \cdot C_{R,b} = f_{imp,EA} \cdot C_{R,b} \quad (38)$$

$$C_{R,a}^* = C_{R,a} \quad (39)$$

with $C_{R,a}$ from eq. (24) and $C_{R,b}$ from eq. (25).

Thus, with functionality f from eq. (10) or eq. (17),

$$C_{R,T}^* = f(C_{R,a}^*, C_{R,b}^*, C_{0,R,ab}, n_E) \quad (40)$$

with all parameters defined previously.

For applications in practice, the following steps of an algorithm with the corresponding equations are summarized as a guide for the calculation procedure at an entry-exit constellation:

- 1 Estimation of the demand volumes q_C , q_E and q_A at the subject entry and exit from the O-D-matrix of traffic demand at the intersection
- 2 Estimation of the distribution of demand volumes by lanes (where applicable) both for circular lanes q_I and q_O and entry lanes q_L and q_R according to applicable assumptions
- 3 Downstream exit:
 - 3.1 Estimation of the first stage capacity C_a of the exit lane at the edge of the outer circular lane (e.g. $C_a = 1400$ veh/h)
 - 3.2 Estimation of the second stage capacity C_b against the pedestrian stream (C_b from eq. (33))
 - 3.3 Estimation of the two-stage capacity $C_{A,T}$ at the exit lane ($C_{A,T}$ from eq. (32), C_a from point 3 and C_b from point 3.2)
 - 3.4 Estimation of the shared lane capacity $C_{O,U}$ of the diverge point at the exit (eq. (34)) consisting of the downstream circular lane ($C_{O,D} = C_{ln}$) and the exit lane ($C_{T,A}$ from point 3.3)
 - 3.5 Estimation of the impedance factor $f_{imp,EA}$ (eq. (35)) to the upstream entry lanes due to queues from the downstream circular lane ($C_{O,U}$ from point 3.4)
- 4 Upstream entry (entry under consideration):
 - 4.1 Estimation of the first stage capacities C_a on entry lanes against the pedestrian stream (eqs. (21) and (24))
 - 4.2 Estimation of the second stage capacities C_b on entry lanes against the circular major stream (eqs. (22) and (25))
 - 4.3 Estimation of the second stage capacities C_a^* and C_b^* impeded by the downstream impedance factor (eqs. (36) and (38), $f_{imp,EA}$ from section 3.5)
 - 4.4 Estimation the two-stage capacities $C_{L,T}^*$ and $C_{R,T}^*$ at the entry lanes (eqs. (37) and (40))
 - 4.5 Estimation of the shared lane capacity $C_{E,d}$ (if applicable, eq. (27), with $C_{L,T} = C_{L,T}^*$ and $C_{R,T} = C_{R,T}^*$ from point 4.4)

EVALUATION OF THE PROPOSED MODEL

To evaluate the proposed model, several examples have been constructed for a single lane roundabout using the calibrated parameters in the previous sections. One example for an entry – exit constellation where the blockage effect due to queue spillback from the downstream exit is demonstrated in Wu and Brilon (16).

In the second example, we consider a whole single-lane roundabout with all the entries and exits in a consecutive way. Here the queues from an entry will impede the upstream exit and a downstream exit may impede the upstream entry. This interrelation happens between all of the four arms of the roundabout. This circular interference requires an iterative calculation.

For the traffic demand, we consider that the roundabout is connecting a major (arms 1 and 3, cf. Figure 2 a)) and a minor (arms 2 and 4) road. The flow split of the major and the minor volume is 60% to 40% of the total intersection volume. The volumes of the turning movements are defined as 20%/60%/20% at a major street arm and 30%/40%/30% at a minor street arm (right/through/left).

We assume that one vehicle ($n_{AE} = 1$) can be stored on the circle between the exit and the entry (at the same arm) and three vehicles can be stored between an entry and the subsequent exit ($n_{AE} = 3$).

The roundabout can be preloaded by any smaller total traffic volume (e.g. 1000 veh/h). Then the volume can be stepwise increased and the degrees of saturation of all the queuing systems on the roundabout can be evaluated iteratively counterclockwise at each step. The iteration is ended when the iteration will no longer lead to a stable solution. That would mean: with an increase of demand, the capacity is going to be reduced due to spillback effects on the circle.

In the example this point is reached at a total demand volume of 1918 veh/h (=critical total volume). The conditions at this point are characterized by table 2. Beyond this volume, the roundabout is expected to become gridlocked and a breakdown will occur with no chance of recovery as long as the demand volumes remain constant. During the iteration the gridlock effect becomes obvious by a steady increase of the x -values and a steady reduction of the capacities (down to zero beyond the critical volume) during the run of the iteration. At the critical volume, the maximum degree of saturation on the circular lane in the example is only 0.77. The corresponding maximum degree of saturation at the approach entries and exits is 0.71 (without regard to impedance on circular lane). The total capacity of 1918 veh/h contrasts to 2720 veh/h which would result as the total capacity from the HBS methods without regard to queue interference effects.

Studying further examples it becomes obvious that the split of demand between major and minor streets as well as the proportions of the turning movements have an influence on the maximum total intersection capacity (for details see Wu and Brilon, 16). It was found that a misbalanced split of demands would cause a reduction of the total intersection capacity.

Analogously, the total intersection capacities of double lane roundabouts can be estimated as well.

CONCLUSION

This paper presents a model for capacity analysis of roundabouts with a completely new sophistication. It is based on the analysis of conflicts within the roundabout. The conflict points between the traffic streams of different types (cars and pedestrians) are considered by a homogenous model. The interaction between consecutive conflict points can be modeled according to the impedance probabilities. Together with this, the distance between the consecutive conflict points is modeled properly.

Based on the proposed model, the capacity of the total roundabout can be analyzed more precisely. Especially, the interaction between different traffic streams and consecutive conflict points at roundabouts can be accurately taken into account according to the model. Using the proposed model, the whole roundabout can be treated as one entity and the total intersection capacity can be obtained according to given traffic volumes for the movements at the intersection. As a result, the capacities of all the conflict points together with their degrees of saturation are obtained.

As one important result, it becomes obvious that the interference of potential queuing processes between conflicts on the circular lanes cannot be neglected, as it is the case for all the conventional roundabout capacity calculation methods. The current practice may lead to a significant overestimation of the total intersection capacity. Above a degree of saturation of $x = 0.7$ (obtained by conventional capacity estimation) a risk of queuing gridlock on the circle may occur.

The correct application of the derived equations may be rather complex. It is, however, not too problematic to implement them into a computer program.

To transfer the model to other countries a recalibration of the decisive parameters may be useful. The model has to be modified for multilane roundabouts where the exiting traffic is interfering with traffic from the upstream entry.

Further research may be directed on the influence of the b_{ij} (degrees of priority observation) between vehicles and in the vehicle/pedestrian conflicts. Additional empirical research should also analyze how these b_{ij} are depending on the saturation of the intersection. In addition, an empirical verification of the whole model will be a task for future research.

AUTHOR CONTRIBUTION STATEMENT

The authors confirm contribution to the paper as follows: study conception and design, analysis and interpretation of results, and draft manuscript preparation performed by both authors. Both authors reviewed the results and approved the final version of the manuscript.

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FIGURE 2 Conflict points at roundabouts.

FIGURE 3 Comparison of the model calibration to the HBS 2015 (12) data for the basic entry capacity (a) & b) and for the impedance factor for pedestrians (c) to h) , dots= HBS, lines=new model.

TABLE 1 Conflict matrix for parameters of a roundabout entry for different configurations

Major lane i \ Minor lane j		Circular inner (I)	Circular outer (O)	Ped (PE)	$C_{0,j}$ (veh/h)			Places btw. two stages n_E (veh)
		Stage b		Stage a	Veh Stage b	Ped Stage a	2-Stage ab	
a) mini roundabout entry with single lane entry and single circular lane (1/1 mini)								
	τ_i (s)	2.6		3.0				
Entry lane	b_{ij}	0.9		0.9	1080	1500	1020	1
b) roundabout entry with a single lane entry and a single circular lane (1/1)								
	τ_i (s)	1.8+14.5/D [*])		2.8				
Entry lane	b_{ij}	0.9		0.9	1200	1550	1150	1
c) roundabout entry with one entry lane and two circular lanes (1/2)								
	τ_i (s)	2.3	2.3	2.3				
Entry left (L)	b_{ij}	0.9	0.9	0.9	1270	1330	1080	1
Entry right (R)	b_{ij}	0.9	0.9	0.9	1420	1420	1250	1
d) roundabout entry with two entry lanes and two circular lanes (2/2)								
	τ_i (s)	2.4	2.4	2.6				
Entry left (L)	b_{ij}	0.9	0.9	0.9	1010	1010	730	1
Entry right (R)	b_{ij}	0.9	0.9	0.9	1100	1100	830	1
e) parameters of a roundabout exit with one exit lane								
	τ_i (s)	-		2.9				
Exit lane (A)	b_{ij}	1		0.9	1400	1550	1330	1
Upstream outer circular lane as a share lane (O,U)		Exit lane (A)						$C_{A,T}$
		Downstream outer circular (O,D)						1640
f) parameters for an entry-exit constellation								
Major lane i \ Minor lane j		Downstream outer lane at the entry with $C_j = C_{0,j}$	Upstream outer lane at the exit with $C_j = C_{O,U}$ from Eq. 34	$C_{0,j}$ (veh/h)			Places btw. two stages $C_n n_{EA}$ (veh)	
		Stage a	Stage b	Stage a	Stage b	2-Stage ab		
	τ_i (s)	-	-	Stage a	Stage b	2-Stage ab		
Outer circular lane btw. an entry and an exit (EA)	b_{ij}	1	1	1640	1640	1640	1.68*3	

TABLE 2 Capacity of the conflict points on the circular lane considering all arms with circular queuing interference at a total volume of 1918 veh/h

Cross section at arm <i>k</i>		On circular lane						At corresponding arm		
		<i>q</i>	<i>C</i> *	downstream section		<i>C</i>	<i>x</i>	<i>q</i>	<i>C</i>	<i>x</i>
				<i>n</i> ₊₁	<i>f</i> _{imp,+1}					
1	Exit	959	1347	<i>n</i> _{AE} = 1	0.96	1296	0.74	575	1382	0.42
	Entry	384	1640	<i>n</i> _{EA} = 3	0.79	1298	0.30	575	811	0.71
2	Exit	959	1377	<i>n</i> _{AE} = 1	0.90	1243	0.77	384	1382	0.28
	Entry	575	1640	<i>n</i> _{EA} = 3	0.84	1373	0.42	384	936	0.41
3	Exit	959	1347	<i>n</i> _{AE} = 1	0.96	1296	0.74	575	1382	0.42
	Entry	384	1640	<i>n</i> _{EA} = 3	0.79	1298	0.30	575	811	0.71
4	Exit	959	1377	<i>n</i> _{AE} = 1	0.90	1243	0.77	384	1382	0.28
	Entry	575	1640	<i>n</i> _{EA} = 3	0.84	1373	0.42	384	936	0.41
1	Exit	959	1347	<i>n</i> _{AE} = 1	0.96	1296	0.74			

max *x* 0.77 0.71

- q* = traffic volume at the considered cross section on the circular lane (veh/h)
- n*₊₁ = storage places on the subsequent downstream section of the circular lane (veh)
- = *n*_{EA} (storage places between the entry and the downstream exit on the circular lane)
- = *n*_{AE} (storage places between the exit and the entry on the circular lane)
- C** = capacity of the considered conflict point without impedance of downstream queuing (veh/h)
- C* = *C***f*_{imp,+1}
- = capacity of the considered conflict point with impedance by downstream queuing (veh/h)
- f*_{imp,+1} = impedance factor of downstream queuing on the subsequent section of the circular lane (cf. eq. (47))
- (e.g.: for the cross section Exit 3 is *C**=1347, *f*_{imp,+1} = 1-0.3^(1.68×1+1) = 0.96, *C*=1347*×0.96=1296)

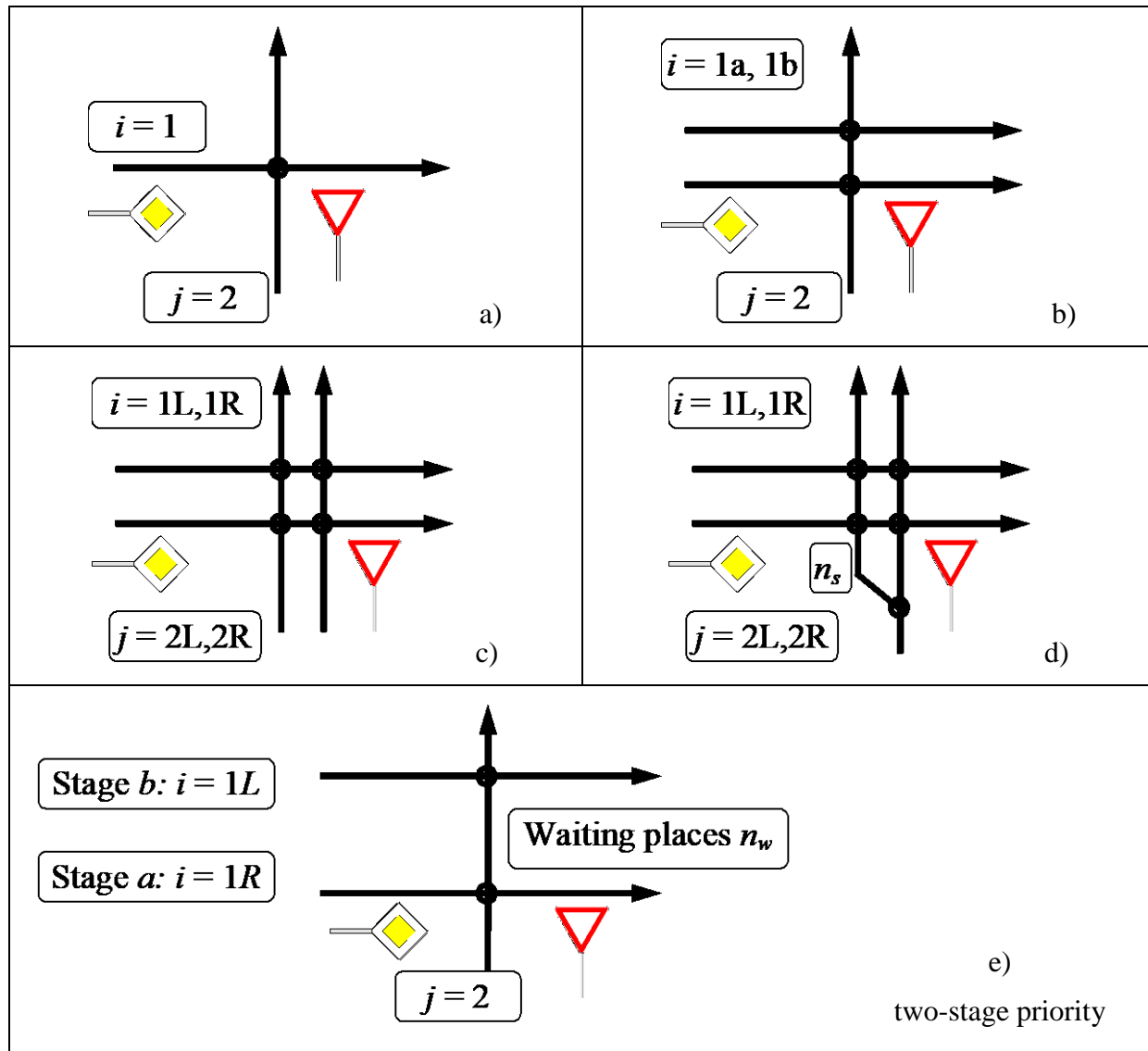
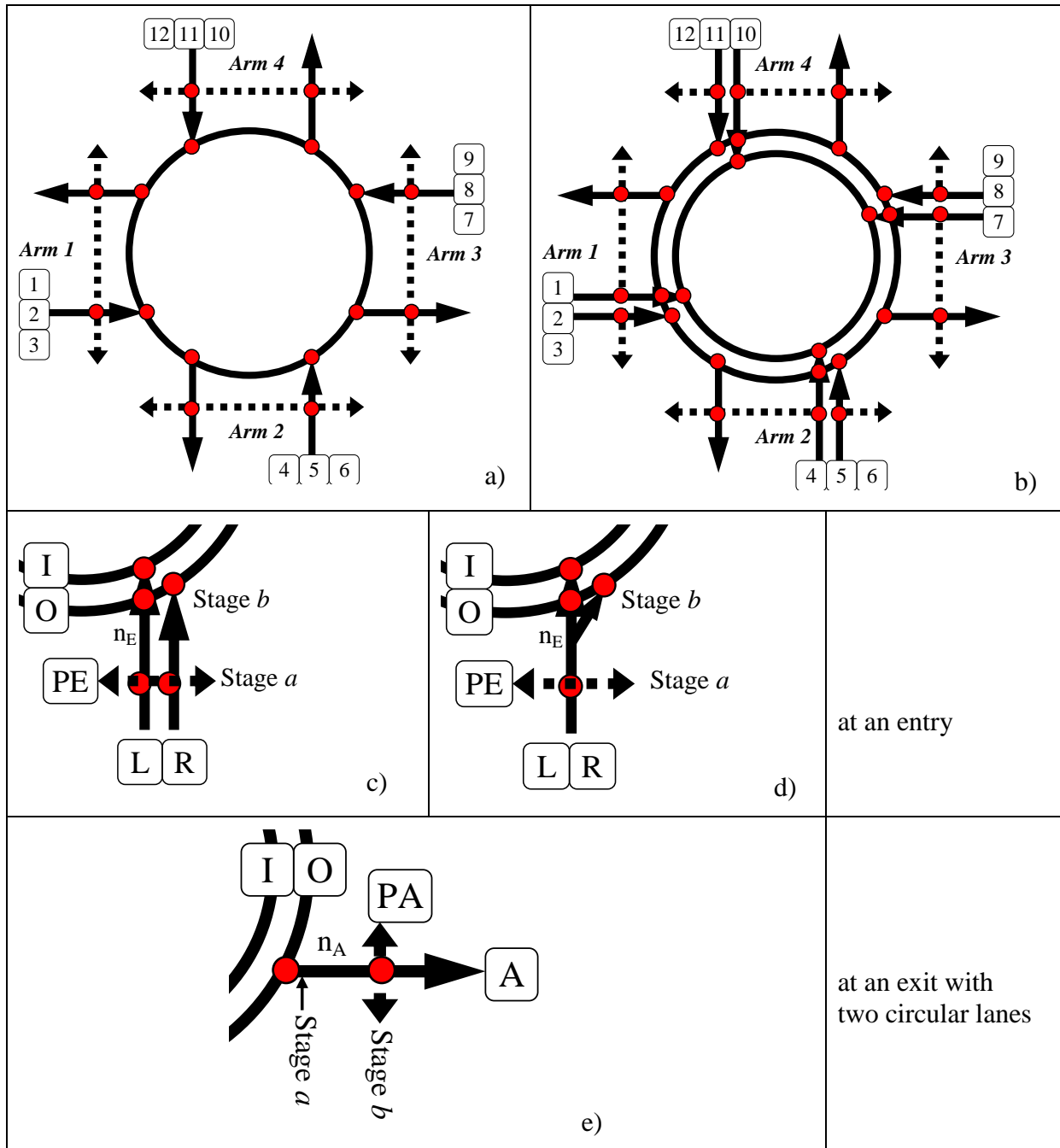
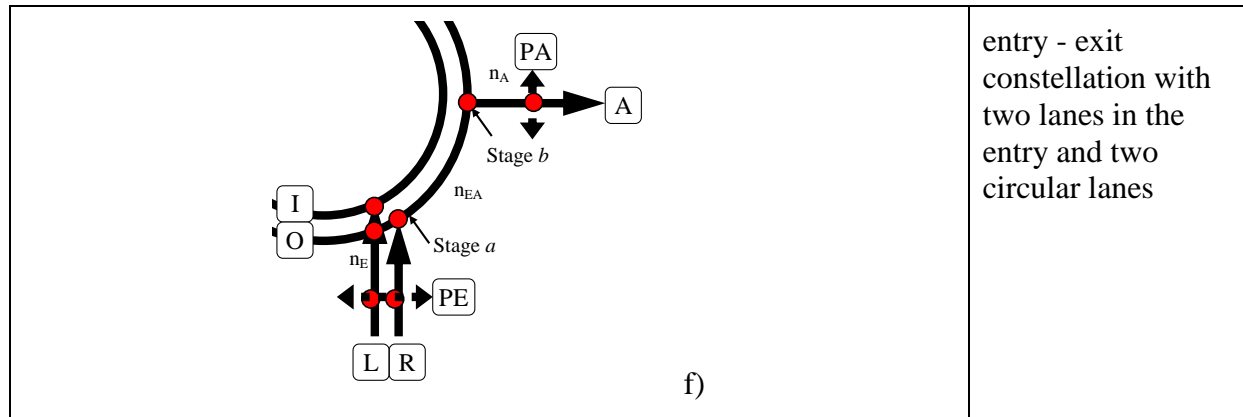


FIGURE 1 Conflict points at unsignalized intersections.





entry - exit
constellation with
two lanes in the
entry and two
circular lanes

FIGURE 2 Conflict points at roundabouts.

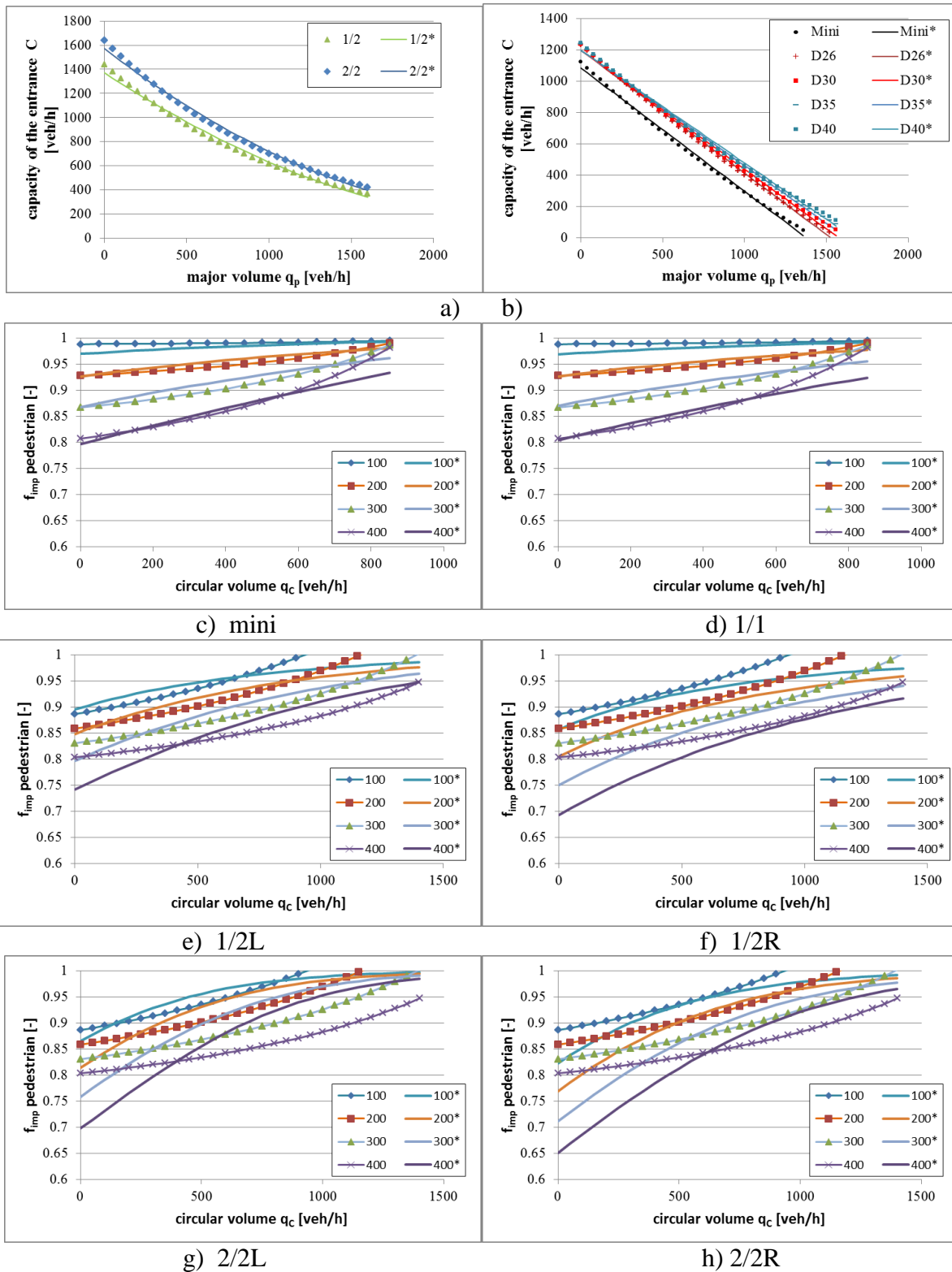


FIGURE 3 Comparison of the model calibration to the HBS 2015 (12) data for the basic entry capacity (a) & b) and for the impedance factor for pedestrians (c) to h) , dots= HBS, lines=new model.