



Increasing the capacity of signalized intersections with left-turn waiting areas



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ABSTRACT

One of the most complex issues for the design of at-grade signalized intersections is accommodating left-turn (LT) movements, especially when approaches have insufficient available spatial resources. In this study, we mitigated this problem by reorganizing left-turning traffic flows within intersections through the use of a left-turn waiting area (LTWA). We proposed a series of design pattern left-turn waiting areas for different combinations of spatial and temporal treatments of left-turn movements: exclusive left-turn lanes with protected left-turn phasing, exclusive left-turn lanes with permitted left-turn phasing, and shared left-turn lanes with permitted left-turn phasing. Based on probability theory, an analytical procedure is developed for estimating the capacity of shared and short lanes. Explicit VISSIM simulations are conducted to validate the accuracy of the proposed capacity models, and the impact of design parameters for the proposed system on the left-turn capacity are studied. On the basis of the analyses, benefits of the proposed system are identified, and the domain of application where these benefits are most significant is identified. In addition, optimal LTWA design scheme and critical LT volumes of exclusive LT lane and protected LT phase with different LTWA schemes are presented from the operation efficiency perspective.

1. Introduction

Maximizing the potential capacity of existing roadways has become a priority in light of growing traffic demand and diminishing resources to develop more capacity (Li, 2011; Ma et al., 2014; Yin, 2008; Yu and Recker, 2006; Zhang et al., 2013). One of the most complex issues for increasing the capacity of at-grade signalized intersections is accommodating left-turn (LT) movements on all approaches efficiently. In some cases, LT movements may create a negative impact on the efficiency and even safety of traffic operations if not treated properly (Xuan et al., 2011).

At a typical four-legged signalized intersection under light LT demand conditions, the optimum signal timing plan normally involves two phases; left-turning vehicles (LVs) and through-going vehicles (TVs) share one phase during which left turning is permitted. If there is a sufficient number of LVs, a protected LT phase is typically introduced at signalized intersections to handle the flow (Oppenlander and Oppenlander, 1989; Zhang et al., 2006). This is considered the most effective way to provide safe traffic operations and handle large LT volumes with less delay (Al-Kaisy and Stewart, 2001). Several studies have suggested that potential criteria for selecting the protected LT phase fall into five categories: delay, traffic volume, crash/conflict experience, speed, and intersection geometry (Lin and Machemehl, 1983).

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Besides the signal phase and timing optimization, problems with LT traffic can also be addressed in the geometric design of intersections (Wong and Wong, 2003). According to Ousama’s research, exclusive LT lanes play an important role in the effective organization of intersections. Kikuchi and Oppenlander (Oppenlander and Oppenlander, 1989; Kikuchi et al., 1993) developed methodologies to determine recommended lengths of LT lanes at signalized intersections. They concluded that the LT length is related to the traffic volume, geometry, and intersection control type. The impact of the spatial distribution of LT lanes (inside and outside an approach) was evaluated by comparing turning speeds and saturation flow rates for LVs (Liu et al., 2011; Yang et al., 2012).

Previous works have focused on estimating the capacity of LT lanes. Lin provided logical explanations for the causal relationships between the capacity and saturation of a shared LT lane and governing variables such as the number of opposing lanes, opposing flow rate, proportion of LT traffic in opposing lanes, proportion of LT traffic (Lin, 1992), and the critical gap on the secondary road (Pollatschek et al., 2002). Al-Kaisy and Stewart (2001) discussed three scenarios involving exclusive LT and shared-lane operations and presented an argument for adding protected LT phases at signalized intersections that previously used permitted-only LT operation. Wu investigated the capacity of minor stream at an unsignalized intersection (Wu, 2001), and the capacity of shared short lanes at a signalized intersection in simulation studies (Tian and Wu, 2006). The approach capacity with a short LT lane was found to be specifically related to the length of the short lane, the ratio of turning vehicles, and the green times for both through-going and turning vehicles. Based on different combinations of the signal phase and lane distribution, HCM 2010 classifies LT traffic flow into six types to calculate the correction coefficient of the saturation flow rate (Transportation Research Board, 2010).

In large numbers, LVs contribute to oversaturation because they require separate green phase allocations, and these sub-phases reduce intersection capacity (Xuan et al., 2011; Zhao et al., 2015). These capacity problems are often avoided in practice by banning and rerouting the offending LVs (Zhao et al., 2014). These strategies are effective because they eliminate LVs and the need for LT lanes and phases, which increase intersection capacity. However, with these strategies, LVs need to make a detour in the road, which will increase the traffic flow in the network accordingly. Hence, the performance of these strategies depends on the geometric characteristics and demand patterns of the network. The implementation of these strategies usually involves construction and requires a substantial amount of space that may not be available.

Based on these challenges, this paper proposes a method to increase capacity by the proper design of the left-turn waiting area (LTWA) within an intersection. The implementation of this strategy does not require banning left turns or reconstructing the intersection. The proposed strategy focuses on making full use of the spatial resources of an intersection.

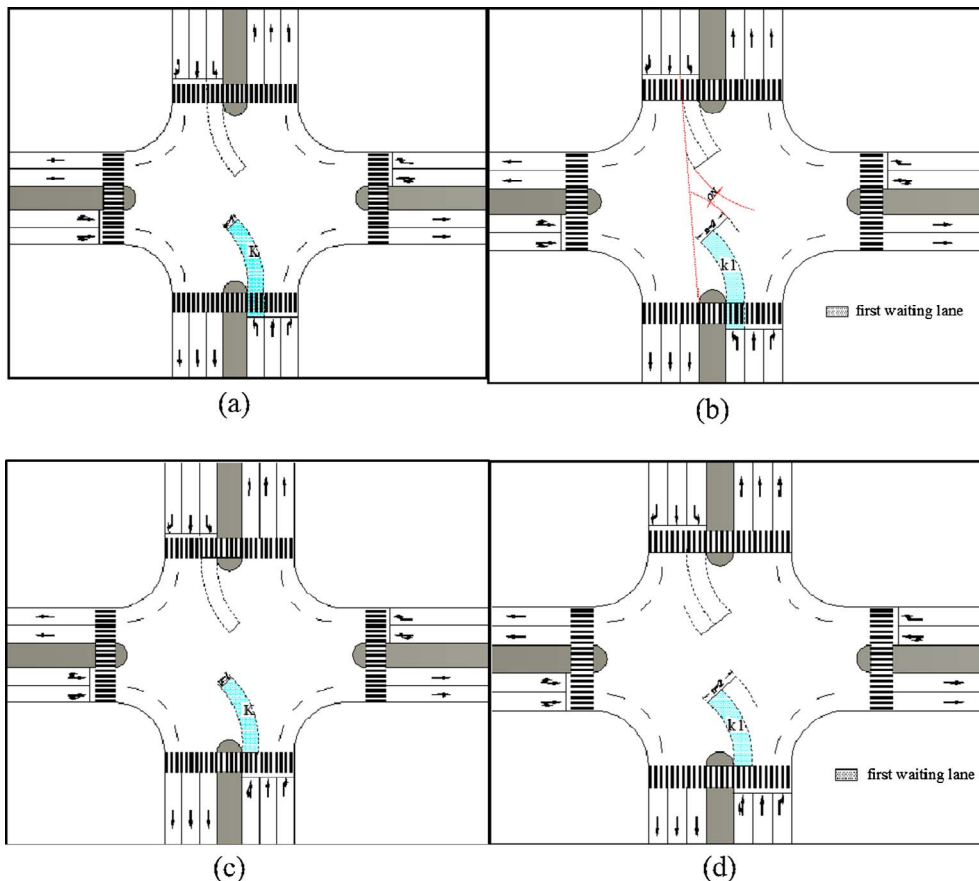


Fig. 1. Layouts of LT lanes and LTWAs.

The idea of an LTWA is not new. LTWAs have been used in several countries, including China, for many years. As shown in Fig. 1, this design allows LVs to enter the waiting areas after the through-going phase starts. Vehicles in the waiting areas are discharged during the following LT phase. When the LT phase ends, incoming vehicles are not allowed to enter the waiting areas until the start of the next through-going phase. In practice, this type of LTWA can be considered as an extension of an exclusive LT lane at a signalized intersection. The presence of an LTWA reduces the probability of spillback of LVs in adjacent through-going lanes during oversaturated conditions (Liu et al., 2011). Several studies have focused on the capacity of LT lanes with LTWAs in a protected/permitted LT phase (Messer and Fambro, 1977; Ni et al., 2006; Xu et al., 2008; Yang et al., 2012; Zhao et al., 2013; Zhou and Zhuang, 2012).

However, LT lanes and waiting lanes currently have a 1-to-1 relationship pattern, as shown in Fig. 1(a). Only the same number of lanes as those at the approaches can be used for discharging LVs. Hence, the capacity cannot be significantly increased (Liu et al., 2011). Moreover, LTWAs can only be used at intersections with a protected LT phase and exclusive LT lanes. This requirement limits their usage.

This paper proposes a series of new LTWA design patterns for different combinations of spatial and temporal intersection characteristics, e.g., exclusive and shared LT lanes and permitted and protected LT phases. A set of capacity models is developed for each combination to capture the impact of LTWA layout parameters (e.g., the number of lanes and total storage capacity of waiting areas). Based on these models, the domain of application for setting LTWAs and optimal LTWA design are proposed. In addition, critical volumes of exclusive LT lane and protected LT phase with setting LTWAs are presented from the operation efficiency perspective. The remainder of this paper is organized as follows. Section 2 describes three typical spatial and temporal configuration scenarios. Section 3 presents capacity estimation models of LT traffic flow in three scenarios. Section 4 demonstrates six numerical examples and model validation, and presents the sensitivity analysis. Section 5 gives the conclusions and proposes future work.

2. Basic concept

Fig. 1 presents the basic idea of an LTWA. In terms of the temporal configuration, the LT movement is treated with two phase types: protected and permitted. In terms of spatial configuration, the LT movement is treated with two lane types: exclusive or shared. Note that exclusive lanes are needed if a protected LT phase is used. There are two types of corresponding relationships between LT lanes and LT waiting lanes: one-to-one (Fig. 1(a)) or one-to-multiple (Fig. 1(b)).

Fig. 1(a) and (b) presents simplified geometric features of an LTWA, which extends from an exclusive LT lane to the intersection. This is applied in exclusive LT lanes and is equivalent to advancing the stop line to the furthest edge of the LTWA. The only difference between LT lanes and waiting lanes is that the former is one-to-one, as shown in Fig. 1(a), while the latter is one-to-multiple, as shown in Fig. 1(b). Fig. 1(c) and (d) shows the geometric features when a shared LT lane is used. In these configurations, the effective storage capacity of the LTWA is less than that in Fig. 1(a) and (b). TVs that share lanes with LTs can proceed through the intersection when LVs remain in an LTWA and will be blocked if the number of LVs exceeds the effective storage capacity of the LTWA. Fig. 1(c) reflects the one-to-one relationship of LT lanes and LTWAs, while Fig. 1(d) depicts the one-to-multiple configuration. In actuality, in order to ensure safe motor vehicle operation, the layout of all types of LTWAs should meet two essential geometric conditions: On the one hand, the trajectory of bidirectional LT vehicles should keep a safe distance, $ON \geq 1.2 \text{ m}$ (SAC and AQSIO, 2009), as shown in Fig. 1(b). On the other hand, the setting of LTWA must not hinder the operation of opposing TVs.

In terms of combinations of LT phases (protected or permitted) and spatial features, three scenarios can be generalized, as given in Table 1.

3. Capacity analysis

This section presents an analysis on the operation mechanisms of traffic flow in exclusive or shared LT lanes for three scenarios. In general, there are two stop lines for LTs at intersections with LTWAs, as shown in Fig. 2: the approach stop line (SL I) and LTWA stop line (SL II). In order to make full use of the queuing space, SL II is located very close to the conflict point between LVs and opposing TVs. Therefore, the capacity of LT lanes is limited by the two stop lines. On this basis, quantitative models were developed to estimate the capacities of exclusive or shared LT lanes in three scenarios.

3.1. Assumptions

- 1) The arrivals of vehicles are independent and assumed to follow a Poisson distribution.

Table 1
Description of three scenarios.

Scenario	Phase type	Approach type	Corresponding relationship	Figure
Scenario 1	Protected	Exclusive LT	One-to-one	Fig. 1(a)
			One-to-multiple	Fig. 1(b)
Scenario 2	Permitted	Exclusive LT	One-to-one	Fig. 1(a)
			One-to-multiple	Fig. 1(b)
Scenario 3	Permitted	Shared LT	One-to-one	Fig. 1(c)
			One-to-multiple	Fig. 1(d)

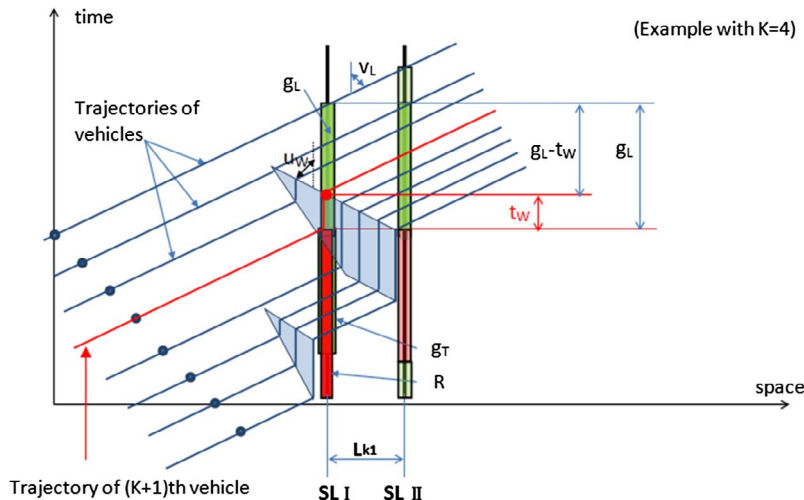


Fig. 2. Operation mechanism analysis for LVs in scenario 1.

- 2) Drivers are assumed to be both consistent and homogeneous, so the critical gap and follow-up time do not vary among LVs.
- 3) The locations of SL II and conflict points between LVs and opposite TVs are the same. Because the impact of opposing LVs is neglected, TVs pass SL I to go through the intersection.
- 4) Traffic demand and saturation are assumed to be consistent within a period of time.

3.2. Parameter description

To facilitate model presentation, Table 2 summarizes the key parameters used hereafter.

3.3. Capacity formulation

3.3.1. Scenario 1: protected LT phase and exclusive LT lane

This is the most common scenario in many cities in China. The LT traffic flow operation mechanism is illustrated below. LVs enter the waiting area during the through-going phase. Vehicles in the LTWA are discharged during the lagging LT phase. When the LT phase ends, the LVs are prohibited from entering the waiting areas until the next through-going phase is initiated.

The capacity of LT lanes is jointly determined by the capacities at SL I and SL II, as shown in Eqs. (1)–(5). The capacity at SL I can be calculated with Eq. (3) because all LVs can pass SL I during the green phase for LVs and TVs. The number of vehicles that pass SL II can be divided into two parts: the vehicles waiting at the LTWA, and vehicles entering the LTWA after the start of the LT phase. The first part is equal to the storage capacity of the LTWA K . The second part is calculated based on the operation state of left turn traffic flow; moreover, effective green time of LT phase is also influenced by associated shock waves in the LTWA and increased clearance time with last through-going phase.

As indicated in Fig. 2, at the beginning of the LT phase, LVs pass SL II and create a queuing shock wave that propagates backward to SL I. Then, vehicles waiting at SL I begin to discharge with the saturation flow rate, and the effective green time of the protected LT phase should subtract L_{k1}/u_W s. Meanwhile, as shown in Fig. 3, when LT phase begins, as discussed previously, LVs started from SL II at the intersection with LTWAs, instead of SL I at the intersection without LTWAs. This implies a longer clearance time is needed to guarantee the last through-going vehicle can pass through the collision point before the first left turn vehicle is in the lagging LT phase. Clearance time between through-going phase and lagging left-turn phase without and with LTWAs, is denoted by t_{clear} , t'_{clear} , respectively. They are calculated using Eqs. (1) and (2).

$$t_{clear} = 3.6 \left(\frac{L_{TO}}{V_{TO}} - \frac{L_L}{V_L} \right) \tag{1}$$

$$t'_{clear} = 3.6 \left(\frac{L_{TO}}{V_{TO}} - \frac{L_L - L_{k1}}{V_L} \right) = t_{clear} + 3.6 \frac{L_{k1}}{V_L} \tag{2}$$

Assuming that t_{clear} is greater than zero without LTWA, the clearance time should be increased $\left(3.6 \frac{L_{k1}}{V_L} \right)$ s. It is predicted that the cycle length and other phases' green time are unaffected; moreover, the effective green time of the protected LT phase should decrease $3.6 \frac{L_{k1}}{V_L}$ s.

Hence, the capacity at SL II can be calculated with Eq. (4). The capacity of this scenario is then the minimum of $Cap1$ and $Cap2$, as shown in Eq. (5).

Table 2
Notation.

LTWA designing parameters	
L	Number of LT lanes, 1, 2, ..., n
K	Capacity of LTWA (pcu)
k_1	Capacity of the first waiting lane, which is the one closest to the road centerline in the LTWA (pcu)
k	Average capacity of LTWA lanes other than the first waiting lane (pcu)
n	Number of lanes in the LTWA
L_{k1}	Length of first waiting lane (m)
α	Reduction factor of other waiting lanes in the LTWA to the first waiting lane. This is determined by the geometric conditions of the intersection. In order to simplify the calculation between α and capacity and reflect their relationship clearly, $k = \alpha k_1$ and $K = k_1 + (n-1)k$
β	Proportionality factor of the length and capacity of the first waiting lane in the LTWA. In other words, the reciprocal of the jam density $\beta = L_{k1}/k_1$ (m/pcu). This varies with the driving characteristics at different intersections and should be determined by field measurements according to the formulation from the literature (Oppenlander and Oppenlander, 1989)
Vehicle operation characteristic parameters	
q_L	Volume of LT traffic flow of entrance lane (pcu/h)
q_T	Volume of through traffic flow of entrance lane (pcu/h)
q_{TO}	Volume of opposing through traffic flow (pcu/h)
λ_L	Arrival rate of LT traffic flow (pcu/s)
λ_T	Arrival rate of through traffic flow (pcu/s)
λ_{TO}	Arrival rate of opposing through traffic flow (pcu/s)
s_L	Saturation flow rate of LT traffic flow (pcu/s)
s_T	Saturation flow rate of through traffic flow (pcu/s)
s_{TO}	Saturation flow rate of opposing through traffic flow (pcu/s)
s_{TL}	Saturation flow rate of left through-going traffic flow (pcu/s)
v_L	Average travel speed of LT traffic flow (km/h)
v_T	Average travel speed of through-going traffic flow (km/h)
v_{TO}	Average travel speed of opposing through-going traffic flow (km/h)
h_t	Minimum time headway of LT traffic flow (s)
t_c	Critical gap of LT traffic flow (s)
t_f	Following time headway of LT traffic flow (s)
Cap	Capacity of exclusive LT or shared lane (pcu/h)
Signal scheme parameters	
C	Cycle length (s)
g_L	Effective green time of LT phase (assuming that the effective green time is equal to the green signal-showing time) (s)
g_T	Effective green time of through-going phase (s)
g_{TL}	Effective green time of left through-going phase (s)
I	Amber time of adjacent phase (s)
Parameters involved in three specific scenarios	
Scenario 1	
L_L	Distance of left turn vehicles travel from stop line in LTWAs to collision points with LTWAs, as shown in Fig. 3
L_{TO}	Distance of through vehicles travel from stop line to collision points with LTWAs, as shown in Fig. 3
g'_L	Effective green time of LT phase with LTWA in scenario 1, which is equal to $g_L - \epsilon L_{k1}/v_L$ (s)
u_w	Wave velocity of starting wave (km/h)
t_w	Time needed for starting wave to spread from furthest edge of LTWA to stop line (s)
$Cap1$	Maximum number of LVs passing through stop line per hour (pcu/h)
$Cap2$	Maximum number of LVs passing through conflict spot per hour (pcu/h)
Scenarios 2 and 3	
P_L	Proportion of LVs in shared LT lane
P_T	Proportion of TVs in shared LT lane
S_{TO}	Distance from stop line to collision point of opposing through-going traffic flow (m)
S_L	Distance from stop line to collision point of LT traffic flow (m)
Q_L	Capacity of minor traffic flow (LT traffic flow) (pcu/s)
t_i	Duration of stage i (explained in Sections 3.3.2 and 3.3.3) (s)
p_i	Probability of i TVs passing SL I in a certain stage
N_{Li}	Maximum or expected number of LVs passing through "collision point" in stage i (pcu)
N_{Ti}	Maximum or expected number of TVs passing through SL II in stage i (pcu)
L_{dl}	Number of LT arrival vehicles at beginning of stage 2 (pcu)
L_{dT}	Number of through-going arrival vehicles at beginning of stage 2 (pcu)
L'_{dl}	Number of arriving LVs at beginning of stage 3 (pcu)
L'_{dT}	Number of arriving TVs at beginning of stage 3 (pcu)
Cap_L	Capacity of LT traffic flow in shared lane (pcu/h)

$$Cap1 = \frac{3600}{C} \frac{(g_T + I + g'_L)}{h_t} \tag{3}$$

$$Cap2 = \frac{3600}{C} \left(K + \frac{g'_L - t_w}{h_t} \right) = \frac{3600}{C} \left(K + \frac{g_L - 3.6L_{k1}/u_w - 3.6L_{k1}v_L}{h_t} \right) \tag{4}$$

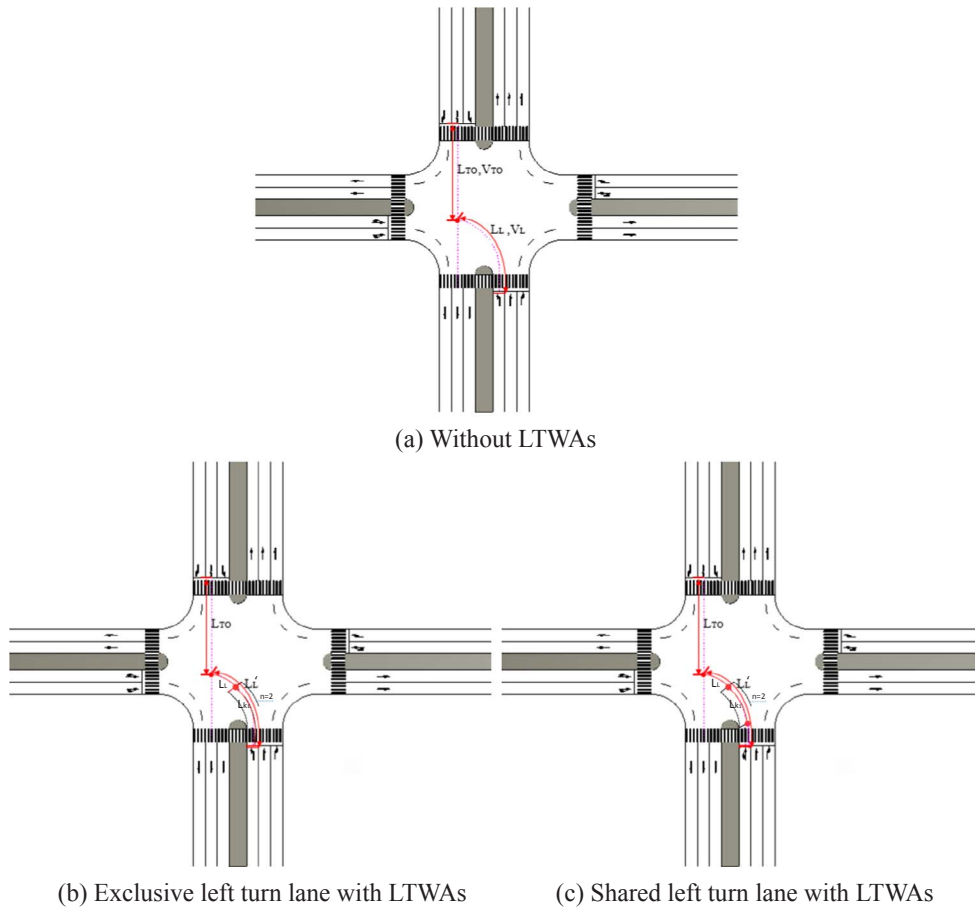


Fig. 3. Trajectories of LVs and opposing TVs.

$$Cap = \min\{Cap1, Cap2\} \tag{5}$$

3.3.2. Scenario 2: permitted LT phase and exclusive LT lane

In Scenario 2, LVs cannot enter the LTWA until the permitted LT phase is initiated. Along the same line, the capacity of an exclusive LT lane must be assessed according to two aspects: the maximum number of vehicles passing SL I and whether or not vehicles can proceed through SL II. It is assumed that opposing TVs have higher priority than LVs, which means that LVs only pass through the intersection by utilizing an appropriate critical gap. During left turn green time, LVs can stop in LTWAs to seek appropriate critical gaps. When the permitted LT phase ends, vehicles in the LTWA must leave the area. According to traffic rules, no vehicle is allowed to stay in the area. Therefore, the operation mechanism for LVs to pass through the intersection can be divided into four stages.

I. Stage 1: Green initial

There are two possible scenarios in stage 1. If drivers obey traffic rule I (i.e., opposing TVs first pass the collision point), the expected number of LVs passing through the collision point (SL II) in stage 1, N_{L1} , is zero. However, if drivers follow traffic rule II (i.e., vehicles that arrive first at the collision point are first to leave), then the number of LVs is determined by the geometric conditions and vehicle velocity. Generally speaking, the distance from SL I to the collision point (SL II) of LVs is closer than that for opposing TVs, so LVs may move through the collision point before the first opposing TVs do. Therefore, if drivers follow traffic rule II, which is very popular in China, t_1 and N_{L1} can be obtained with the following formulas:

$$t_1 = S_{TO}/v_{TO} \tag{6}$$

$$N_{L1} = \left\lfloor \left(\frac{S_{TO}}{v_{TO}} - \frac{S_L}{v_L} \right) / t_f \right\rfloor \tag{7}$$

Eq. (6) represents the time for the first opposing TV to move from SL I to SL II. In Eq. (7), the number of LVs passing SL II in stage 1

is calculated based on a ratio of two variables: the time lag between the first LV and the opposing TV, $\left(\frac{s_{TO}}{v_{TO}} - \frac{s_L}{v_L}\right)$, and the following headway time of LVs, t_f .

II. Stage 2: Opposing through-going traffic flow passes through the collision point (SL II) at the saturation flow rate

This stage starts as the first opposing TV arrives at SL II and ends when the opposing through-going queue is cleared. Because opposing TVs that arrive at the stop line during the red phase need to leave SL II at the saturation flow rate, LVs have to wait during this period in the LTWA, and $N_{L2} = 0$. The duration of this stage t_2 is expressed by Eq. (9):

$$N_{L2} = 0 \tag{8}$$

$$t_2 = \frac{\lambda_{TO}(C - g_T)}{s_{TO} - \lambda_{TO}} \tag{9}$$

III. Stage 3: LVs pass the collision point (SL II) utilizing the accepted gap

This stage starts as the opposing through-going movement begins to pass SL II at an unsaturated flow rate and ends when the green phase is ended. The opposing through-going traffic flow is assumed to be the major flow, and the LT traffic flow is assumed to be the minor one. The critical gap t_c and following headway time t_f are affected by factors such as the number of opposing through-going lanes and proportion of heavy traffic. These can be calculated according to the Highway Capacity Manual 2010. Based on critical gap theory, the capacity of the minor traffic flow Q_L (LT traffic flow in this stage) can be obtained by using Eq. (8), and t_3 and N_{L3} are as given in Eq. (11):

$$Q_L = \lambda_{TO} \frac{e^{-\lambda_{TO} t_c}}{1 - e^{-\lambda_{TO} t_f}} \tag{10}$$

$$t_3 = g_{TL} - t_1 - t_2 N_{L3} = \min\{n t_3 Q_L, t_3 s_L\} \tag{11}$$

In Eq. (9), the duration of the stage is the remaining permitted LT phase time $(g_{TL} - t_1 - t_2)$, and $n t_3 Q_L$ is the maximum number of vehicles that pass SL II considering the multiple waiting lane parameter n . $t_3 s_L$ is the maximum number of vehicles passing SL I in stage 3. Therefore, N_{L3} takes the smaller value between $n t_3 Q_L$ and $t_3 s_L$.

IV. Stage 4: LTWA is cleared during inter-green interval

When the permitted LT phase ends, there may be some vehicles left in the LTWA. These leave the area and pass SL II during the inter-green interval. The inter-green interval starts from the end of the current green light to the start of next green light, aiming to separate conflicting traffic flows at signalized intersection. The duration of inter-green interval, along with amber and red intervals, is a key design variable in providing efficient and safe operation at intersections. Amber signal light is set to deal with dilemma zone when signal changes from green light to red light. All-red time is to make sure vehicles which have passed stop lines during amber pass through collision points safely before the first vehicle in the following phase arrives at the point. There are various signal timing design methods to calculating it in present study, and the main influencing factors include clearance/entering distance, speeds, approach slopes, the length of vehicles, and crossing time. One method is shown in Eqs. (1) and (2) in the paper. To simplify calculation in case studies, amber time and all-red time are considered to be 3 and 0 seconds in the paper. In practice, inter-green interval should be determined by specific conditions. The maximum number of LVs passing SL II N_{L4} is the storage capacity of the LTWA K .

In conclusion, for the scenario of an exclusive LT lane with a permitted LT phase, the maximum number N_L of LVs passing SL II and the capacity of the exclusive LT lane Cap are as given in Eqs. (12) and (13):

$$N_L = \min\{(N_{L1} + N_{L2} + N_{L3} + N_{L4}), g_{TL} s_L\} \tag{12}$$

$$Cap = \frac{3600}{T} N_L \tag{13}$$

In Eq. (12), the maximum number of LVs passing the intersection is the sum of N_{Li} and is restricted by the saturation volume of LVs $g_{TL} s_L$.

3.3.3. Scenario 3: permitted LT phase and shared LT lane

In the case of a shared LT lane with a permitted LT phase, the operation of the LT traffic flow is similar to that in scenario 2 and is classified into four stages. The only difference is that when LVs spill out of the waiting area, vehicles in the shared LT lane will be blocked. Thus, the movement process of the through-going traffic flow in a shared LT lane is particularly complex and the number of TVs passing the intersection is related to the queue length of LVs, the capacity of the LTWA, and the arrival rates of the LVs and TVs. The computational models for the capacity of a shared LT lane can be developed based on the following four stages.

Stage 1: Green initial

In stage 1, t_1 and N_{L1} can be calculated as given in Eqs. (6) and (7), and the maximum number of TVs in the shared LT lane can be

estimated as the product of the duration t_l and saturation of the TVs s_T .

Stage 2: Opposite through-going traffic flow passes the collision point (SL II) at the saturation flow rate

The duration and number of LVs passing SL II of stage 2 can be calculated by using Eqs. (8) and (9). As described above, if the number of arriving LVs is more than K , the $(K + 1)$ th LV blocks the TVs in the shared lane. As a result, it is necessary to compare the parameters L_{dL} and K and calculate the expected number of TVs according to binominal distribution theory (Tian and Wu, 2006). The probability that i TVs passing through SL I and the expected number of TVs are denoted by p_i and E_T , respectively. These are given by Eqs. (14)–(19):

$$\begin{cases} L_{dL} = (C-g)\lambda_L + \lambda_L t_1 - N_{L1} \\ L_{dT} = (C-g)\lambda_T + \lambda_T t_1 - N_{T1} \end{cases} \quad (14)$$

In Eq. (14), the number of LT arrival vehicles at the beginning of stage 2 L_{dL} is divided into three parts. $(C-g)\lambda_L$ represents the number of LVs arriving during the red light, $\lambda_L t_1$ is the average number of LVs arriving during stage 1, and N_{L1} is the number of LVs departing during stage 1. The number of through-going arrival vehicles at the beginning of stage 2 L_{dT} is calculated similarly to L_{dL} . According to the relationship between L_{dL} and the storage capacity of the LTWA K , the capacity at stage 2 can be calculated with the following equations.

1) $L_{dL} \geq (K + 1)$

When stage 2 starts, the number of arriving LVs is more than the capacity of the LTWA K . The probability p_i that i TVs pass through the intersection before the $(K + 1)$ th LV is described in Eq. (15):

$$p_i = C_{i+K}^i P_T^i P_L^K P_L = C_{i+K}^i P_T^i P_L^{K+1} (i = 0, 1, \dots, L_{dT}) \quad (15)$$

$$E_T = \sum_{i=0}^{L_{dT}} p_i i = \sum_{i=0}^{L_{dT}} i C_{i+K}^i P_T^i P_L^{K+1} \quad (16)$$

Eq. (16) is used to calculate the expected number of TVs. Owing to $L_{dL} \geq (K + 1)$, the maximum number of departing TV is L_{dT} .

2) $L_{dL} = K$

When stage 2 starts, the number of arriving LVs is equal to the capacity of the LTWA K :

$$p_i = C_{i+K}^i P_T^i P_L^{K+1} (i = 0, 1, \dots, L_{dT} + t_2 s_T) \quad (17)$$

$$E_T = \sum_{i=0}^{L_{dT}+t_2 s_T} p_i i \quad (18)$$

Eqs. (17) and (18) are explained similar to Eqs. (15) and (16). The only difference is the maximum number of departing TVs. When the arriving LVs K , enter the waiting area, TVs can still pass SL I. The maximum number of departing TVs is $(L_{dT} + t_2 s_T)$.

3) $L_{dL} < K$

When stage 2 starts, the number of arriving LVs is less than the capacity of the LTWA K . Moreover, the number of arriving LVs at the end of stage 2 may be more or less than K . Thus, the calculation of p_i and E_T is divided into two situations:

$$\begin{aligned} (1) \quad L_{dL} + t_2 \lambda_L \geq (K + 1), \quad p_i &= C_{i+K}^i P_T^i P_L^K P_L, E_T = \sum_{i=0}^{L_{dT}+t_2 s_T} p_i i \\ (2) \quad L_{dL} + t_2 \lambda_L \leq K, \quad p_i &= C_{i+(L_{dL}+t_2 \lambda_L)}^i P_T^i P_L^{(L_{dL}+t_2 \lambda_L)}, E_T = \sum_{i=0}^{L_{dT}+t_2 s_T} p_i i \end{aligned}$$

Therefore, the expected number of TVs passing through the intersection in stage 2 N_{T2} can be summarized as

$$N_{T2} = \begin{cases} \sum_{i=0}^{L_{dT}} i C_{i+K}^i P_T^i P_L^{K+1} (L_{dL} \geq (K + 1)) \\ \sum_{i=0}^{L_{dT}+t_2 s_T} i C_{i+K}^i P_T^i P_L^{K+1} (L_{dL} = K) \\ \sum_{i=0}^{L_{dT}+t_2 s_T} i C_{i+K}^i P_T^i P_L^{K+1} (L_{dL} < K, L_{dL} + t_2 \lambda_L \geq (K + 1)) \\ \sum_{i=0}^{L_{dT}+t_2 s_T} i C_{i+(L_{dL}+t_2 \lambda_L)}^i P_T^i P_L^{(L_{dL}+t_2 \lambda_L)} (L_{dL} < K, L_{dL} + t_2 \lambda_L \leq K) \end{cases} \quad (19)$$

Stage 3: LVs pass the collision point (SL II) utilizing the accepted gap

The calculations for t_3 , Q_L , and N_{L3} are the same as those in scenario 2 and can use Eqs. (10) and (11). N'_{L2} and N'_{T2} represent the

average number of LVs and TVs, respectively, leaving the intersection during stage 2. If the number of arriving LVs at the end of stage 3 is more than $(nt_3Q_L + K)$, the $(nt_3Q_L + K + 1)$ th LV blocks the TVs in the shared lane; otherwise, there is no possibility of the arriving TVs passing SL I at the end of stage 3.

$$\begin{cases} L'_{dL} = L_{dL} + \lambda_L t_2 - N'_{L2} = L_{dL} + \lambda_L t_2 - 0 \\ L'_{dT} = L_{dT} + \lambda_T t_2 - N'_{T2} = L_{dT} + \lambda_T t_2 - \sum_{i=0}^{L_{dT} + t_2 \lambda_T} p_i i \end{cases} \quad (20)$$

In Eq. (20), the number of arriving LVs and TVs at the initial part of stage 3 L'_{dL} and L'_{dT} are calculated just like L_{dL} and L_{dT} . This depends on three factors: the number of arriving vehicles at the initial part of stage 2, the number of arriving vehicles during stage 2, and the number of departing vehicles during stage 2.

1) $L'_{dL} + t_3 \lambda_L \geq (nt_3Q_L + K + 1)$

When stage 3 ends, the number of arriving LVs is more than $(nt_3Q_L + K)$. Under these conditions, the arriving LVs block some TVs from progressing through SL I. Then, p_i (the probability of i TVs passing SL I in stage 3) and E_T are calculated as follows:

$$p_i = C_{i+(nt_3Q_L+K)}^i P_T^i P_L^{(nt_3Q_L+K)} P_L(i = 1, 2, \dots, L'_{dT} + t_3s_T) \quad (21)$$

$$E_T = \sum_{i=0}^{L'_{dT} + t_3s_T} p_i i \quad (22)$$

In Eq. (21), $(nt_3Q_L + K)$ represents the number of LVs passing SL I in the stage, and the maximum number of TVs passing SL I is $(L'_{dT} + t_3s_T)$.

2) $L'_{dL} + t_3 \lambda_L \leq (nt_3Q_L + K)$

When stage 3 ends, the number of arriving LVs is $(L'_{dT} + t_3s_T)$. In this case, all of the arriving LVs stay in the LTWA and do not block TVs. Compared to Eq. (19), $(nt_3Q_L + K)$ should be replaced with $(L'_{dL} + t_3\lambda_L)$, as given in Eqs. (23) and (24):

$$p_i = C_{i+(L'_{dL}+t_3\lambda_L)}^i P_T^i P_L^{(L'_{dL}+t_3\lambda_L)} (i = 1, 2, \dots, L'_{dT} + t_3s_T) \quad (23)$$

$$E_T = \sum_{i=0}^{L'_{dT} + t_3s_T} p_i i \quad (24)$$

Therefore, the expected number of TVs passing through SL I in stage 3 N_{T3} is summarized as follows:

$$N_{T3} = \begin{cases} E_T = \sum_{i=0}^{L'_{dT} + t_3s_T} i C_{i+(nt_3Q_L+K)}^i P_T^i P_L^{(nt_3Q_L+K+1)} (L'_{dL} + t_3\lambda_L \geq (nt_3Q_L + K + 1)) \\ E_T = \sum_{i=0}^{L'_{dT} + t_3s_T} i C_{i+(L'_{dL}+t_3\lambda_L)}^i P_T^i P_L^{(L'_{dL}+t_3\lambda_L)} (L'_{dL} + t_3\lambda_L \leq (nt_3Q_L + K)) \end{cases} \quad (25)$$

Stage 4: LTWA is cleared during the inter-green interval.

Similar to stage 4 in situation 2, the maximum number of LVs passing SL II N_{L4} is K , and $N_{T4} = 0$. In summary, in scenario 3, the capacity of the shared LT lane and the LT traffic flow are given as follows:

$$Cap = \min \left\{ (N_{L1} + N_{L2} + N_{L3} + N_{L4} + N_{T1} + N_{T2} + N_{T3} + N_{T4}), 3600 \frac{g_{TL}}{C} s_{TL} \right\} \quad (26)$$

$$Cap_L = Cap P_L \quad (27)$$

4. Model validation and analysis

4.1. Case description

Fig. 4 shows the lane distributions and signal schemes for the three scenarios. To validate the accuracy and illustrate the applicability of the proposed models, the capacity of left turn and through traffic flows considering setting LTWAs from the intersection south approach is studied, and six numerical cases of three scenarios are designed in the paper.

- Scenario 1: exclusive LT lane with protected LT phases, as shown in Fig. 4(a) and (c).
- Scenario 2: exclusive LT lane with permitted LT phase, as shown in Fig. 4(a) and (d).
- Scenario 3: shared LT lane with permitted LT phase, as shown in Fig. 4(b) and (d).

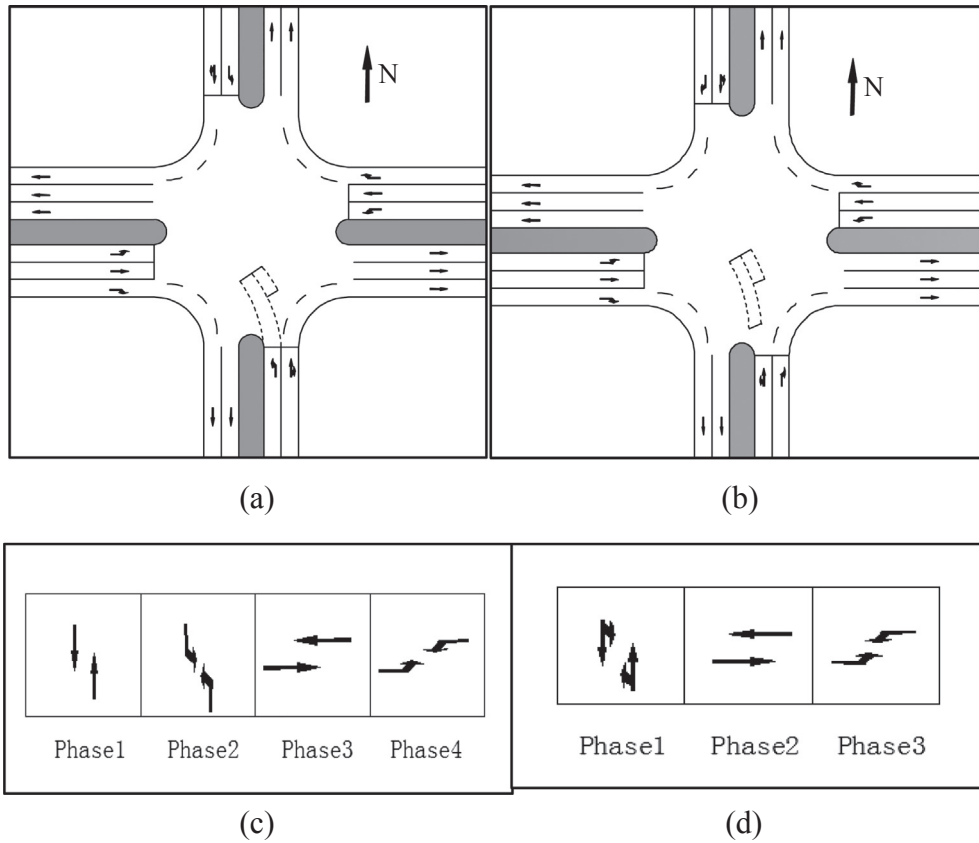


Fig. 4. Lane layouts and signal phases.

The traffic demands were the same in the three scenarios, and the signal-timing scheme was calculated based on the principle that the phases have the same key traffic flow saturation. α and β were 0.5 and 8.5 in these cases. Table 3 presents the parameters for the cases.

4.2. Model validation

In order to test the accuracy of the proposed model, simulation models were built for each scenario listed above with the VISSIM software package. Parameters in numerical cases were obtained based on the observation of real signalized intersections in Shanghai to guarantee their rationality, and VISSIM simulation model parameters should be adjusted accordingly. In the VISSIM models, the average speeds of the LVs and TVs were 20 and 25 km/h, respectively. The critical gap and following headway were 5.5 and 2.5 s,

Table 3
Parameters used in six cases.

Scenario	Scenario 1	Scenario 2	Scenario 3
Figure	Fig. 4(a) and (c)	Fig. 4(a) and (d)	Fig. 4(b) and (d)
Signal scheme	$C = 120/90$ s $g_T = 35/20$ s $g_L = 25/24$ s $I = 3$ s	$C = 120/80$ s $g_{TL} = 60/40$ s $I = 3$ s	$C = 120/80$ s $g_{TL} = 60/40$ s $I = 3$ s
Traffic volume	$q_L = 277$ pcu/h $q_T = 416$ pcu/h $s_L = 1420$ pcu/h $s_T = 1520$ pcu/h	$q_L = 277$ pcu/h $q_T = 416$ pcu/h $q_{TO} = 500$ pcu/h $s_L = 1300$ pcu/h $s_T = 1800$ pcu/h $s_{TO} = 1800$ pcu/h $P_L = 0.4; P_T = 0.6$	$q_L = 277$ pcu/h $q_T = 416$ pcu/h $q_{TO} = 500$ pcu/h $s_{TL} = 1300$ pcu/h $s_{TO} = 1800$ pcu/h $P_L = 0.4; P_T = 0.6$
Vehicle movement	$v_L = 20$ km/h; $v_T = v_{TO} = 25$ km/h; $t_C = 5.5$ s; $t_f = 2.5$ s		
Geometric conditions	$S_L = 27$ m; $S_{TO} = 45$ m		
Ratio factors	$\alpha = 0.5; \beta = 8.5$		

Table 4
Results of models, simulations, and hypothesis tests of three scenarios.

Scenario 1	n	1			2			
C = 120	k_j	0	1	2	3	1	2	3
	VISSIM	290	293	297	297	301	320	334
	Model	295	300	309	313	309	325	340
	Relative Error	1.7%	0.7%	4.0%	5.4%	3.0%	1.6%	1.8%
$T = \sum_{i=1}^7 r_i = 62, c_1 < T < c_2$								
Accept the null hypothesis; no significant difference								
C = 90	VISSIM	398	390	392	372	400	410	416
	Model	378	378	378	377	398	418	437
	Relative error	-5.0%	-3.1%	-3.6%	1.3%	-0.5%	2.0%	5.0%
	$T = \sum_{i=1}^7 r_i = 55.5, c_1 < T < c_2$							
Accept the null hypothesis; no significant difference								
Scenario 2	n	1			2			
C = 120	k_j	0	1	2	3	1	2	3
	VISSIM	265	300	345	388	491	516	532
	Model	270	300	330	360	450	480	540
	Relative error	1.9%	0.0%	-4.3%	-7.2%	-8.4%	-7.0%	1.5%
$T = \sum_{i=1}^7 r_i = 53, c_1 < T < c_2$ Accept the null hypothesis; no significant difference								
C = 80	VISSIM	276	344	382	432	523	528	599
	Model	270	315	360	405	495	540	630
	Relative Error	-2.2%	-8.4%	-5.8%	-6.3%	-5.4%	2.3%	5.2%
	$T = \sum_{i=1}^7 r_i = 54, c_1 < T < c_2$ Accept the null hypothesis; no significant difference							
Scenario 3	n	1			2			
C = 120	k_j	0	1	2	3	1	2	3
	VISSIM	154	221	232	239	222	230	230
	Model	168	204	228	252	222	252	252
	Relative Error	9.1%	-7.7%	-1.7%	5.4%	0.0%	9.6%	9.6%
$T = \sum_{i=1}^7 r_i = 48.5, c_1 < T < c_2$ Accept the null hypothesis; no significant difference								
C = 80	VISSIM	170	246	262	262	236	243	245
	Model	180	234	252	252	252	252	252
	Relative Error	5.9%	-4.9%	-3.8%	-3.8%	6.8%	3.7%	2.9%
	$T = \sum_{i=1}^7 r_i = 46, c_1 < T < c_2$ Accept the null hypothesis; no significant difference							

respectively. The signal scheme, saturations of different lanes, proportion of LVs and TVs, and geometric conditions were guaranteed to be the same in the simulations and capacity estimation models. Ten simulations from different random seeds were conducted and the final results were the average values over ten simulations.

The difference between the capacity estimation models and simulations was considered by using the rank sum test, which is a nonparametric hypothesis test method. The null hypothesis H_0 means that there is no difference between two samples. In the rank sum test, the test statistic T is the rank sum of two samples. Assuming a confidence level of 0.05, the sample size and critical values were 7 and $c_1 = 37$ and $c_2 = 68$. The test results are given in Table 4.

In scenario 1, for cases of different signal cycles, there was no significant difference in the model and simulation results when the confidence level was 0.05. The range of the relative error was [-5.0%, 5.4%], and the average relative error of the samples was 1.02%. The existing relative error was mainly caused by the parameter β (i.e., proportionality factor of the length and capacity of the first LT waiting lane), which was given a default value in simulations and could not be accurately calibrated to 8.5. Furthermore, the VISSIM simulation results were random, which led to some deviations from the capacity in the estimated models. For cases of different signal cycles in scenario 2, there was no significant difference when the confidence level is 0.05. The range of the relative error was [-8.4%, 5.2%], and the average relative error of the samples was -3.15%. For this model, the calculation of LVs passing SL II in stage 3 was simplified by assuming that the critical gap obeyed a uniform distribution rather than a Poisson distribution in the simulation. This may have resulted in some relative errors in scenario 2. For cases of different signal cycles in scenario 3, there was no significant difference when the confidence level was 0.05. The range of the relative error was [-7.7%, 9.6%], and the average relative error of the samples was 2.22%. For the model, the probability that TVs were blocked by LVs was described by using a binomial distribution probability model (Tian and Wu, 2006; Zhou and Zhuang, 2012) to obtain the expected capacity of LVs in the shared LT lane. In contrast, the simulation results were more stochastic, which may have produced some relative errors.

In conclusion, the rank sum test results showed no significant difference (at a confidence level of 0.05) for the six cases and that the range of the relative error was [-10%, 10%]. Thus, the three capacity estimation models were confirmed to have high accuracy.

4.3. Sensitivity analysis

As stated previously, several critical design factors affect the results of capacity calculation models; these include the number of lanes in the LTWA, n , and capacity of the first waiting lane in the LTWA k_j . Sensitivity analyses are conducted on the parameters n

and k_1 to provide operational guidelines for designing LTWAs with certain combinations of lane distributions and signal phases at signalized intersections. In the numerical cases, n ranges from 1 to 4, k_1 ranges from 1 to 12, and the signal schemes are described in Table 3. The cycle length for all three scenarios is 120 s. $cap0$ denotes the initial capacity of LVs without an LTWA, while $cap(\min)$ and $cap(\max)$ denote the minimum and maximum capacities of LVs with n and k_1 of 1–4 and 1–12, respectively.

4.3.1. Scope of application of LTWAs layout in scenario 1/2/3

As stated previously in 3.3.1, compared with the capacity of exclusive LT lane without LTWAs, the capacity of left turn traffic flow with LTWAs is the result of mutual balance between storage capacity of LTWAs and the decrease of effective green time of LT phase. According to Eq. (3), $3.6\beta s_L(1/v_L + 1/u_W) < (K/k_1) = 1 + \alpha(n-1)$, setting LTWAs can improve the capacity of the LT lane, which increases with parameters n and k_1 increasing. When $3.6\beta s_L(1/v_L + 1/u_W) = 1 + \alpha(n-1)$, the capacity of the LT lane is not affected by the LTWA. When $3.6\beta s_L(1/v_L + 1/u_W) > 1 + \alpha(n-1)$, setting LTWAs reduces the capacity of the LT lane, which decreases with increasing k_1 . In order to facilitate applications in practice, this paper discusses the adaptable range of setting LTWAs in different combinations (v_L, s_L), as illustrated in Fig. 5.

Fig. 5 shows the scope of application of setting LTWAs in scenario 1 under different β (6 m/pcu, 8 m/pcu, 10 m/pcu) when average speed of left turn vehicles v_L lies within [15 km/h, 45 km/h] and saturation flow rate of left turn vehicles s_L belongs to [1400pcu/h, 1800pcu/h]. The green region indicates that the capacity of left turn traffic flow will increase with setting LTWAs (n , the number of lanes in LTWAs, ≥ 1), and the red region means the capacity will increase only when $n \geq 2$. If the factor n is limited no more than 1, the red region also represents it shouldn't set LTWAs. Moreover, with the factor β increasing, the green area decreases and red area increases, which means the requirement of setting LTWAs gets much stricter. The reference values of v_L and s_L can be adopted as Fig. 5 shown.

In scenario 2 (an exclusive LT lane with a permitted LT phase) and scenario 3 (a shared LT lane with a permitted LT phase), the layout of the LTWA can improve the capacity of exclusive and shared LT lane. From the perspective of traffic operation benefit, LTWAs are recommended in scenario 2 and scenario 3.

4.3.2. Capacity analysis in scenario 1/2/3

As shown in Fig. 6(a1)–(a3), setting an LTWA can improve the capacity of exclusive or shared LT lanes with a permitted LT phase (scenarios 2 and 3). For scenario 1 (with exclusive LT lane and protected LT phase) in the case when $n = 1$, $3.6\beta s_L(1/v_L + 1/u_W) \approx 0.80 < 1 = 1 + \alpha n - 1$, the capacity of LT traffic flow can be improved by setting LTWAs, as shown in Fig. 6(a1). As shown in Fig. 6(a2) and (a3), the capacity curves with an LTWA always lie above the initial capacity curves without an LTWA. In three scenarios, increasing the capacity of the first waiting lane k_1 and number of lanes in the LTWA n can help improve the capacity of LVs. The maximum capacity in the three considered scenarios was limited by the saturation flow rate of left turns, the proportion of LVs and TVs, and the volume of opposing TVs. Compared with scenario 1, scenarios 2 and 3 could obtain the maximum capacities with increasing k_1 more easily when $n \geq 2$.

In order to review the performance of the proposed model under different situations, a new parameter, termed the capacity ratio, was defined:

$$\text{capacity ratio} = \frac{cap - cap0}{cap} \tag{28}$$

Fig. 6(b1)–(b3) shows the capacity ratio for different scenarios. The counter lines in these figures verified that both the storage capacity of the first waiting lane k_1 and number of lanes n affect the LV capacity, and a capacity ratio value corresponds to multiple combinations [n, k_1]. However, the LTWA had different degrees of impact in the three scenarios, and the maximum capacity ratios were 1.4, 1.2, and 0.3, respectively for the three scenarios. In the three scenarios, the curves became more severe along the Y-axis (n). This shows that a greater value of n allowed the maximum capacity of k_1 to be obtained more easily. In the cases corresponding to the maximum capacity, the combinations for [n, k_1] were {[2,5][4,4]}, and {[2,6][3,5][4,4]} for scenarios 2 and 3, respectively.

Figs. 7 and 8 compare and plot the capacities of an exclusive or shared left turn lane, with and without LTWA, in the three typical scenarios with the same signal timing algorithm, cycle length, traffic volume, and geometric conditions.

In scenario 1 (an exclusive LT lane with a protected LT phase), $cap0$, $cap(\min)$, and $cap(\max)$ were equal to 295, 309, and 756, respectively. In scenario 2 (an exclusive LT lane with a permitted LT phase), $cap0$, $cap(\min)$, and $cap(\max)$ were 300, 300, and 690, respectively. In scenario 3 (a shared LT lane with a permitted LT phase), $cap0$, $cap(\min)$, and $cap(\max)$ were 284, 311, and 686, respectively. There existed an overlap of [300, 686] among LV capacity sets in three scenarios. That is, the differences among operation efficiency in three scenarios can be reduced by setting LTWAs. Determining whether the spatial configuration and signal scheme without LTWAs is suitable to the condition with LTWAs requires further study.

In scenario 2 (the exclusive LT lane with a permitted LT phase), setting an LTWA can improve the capacity of LT traffic flow and may surpass the capacity in scenario 1 (an exclusive LT lane with a protected LT phase). Due to the decrease in phases and lost time, operation effectiveness of intersection in scenario 2 will be better than that in scenario 1. Therefore, setting LTWAs is likely to change the critical volume for designing protected LT phase, and various design schemes of LTWA may corresponding to different critical volumes. Similarly, the critical volume of exclusive LT lane with LTWAs may be adjusted. Considering that capacity of LVs cannot estimate the operation of approach exactly, the ‘‘average control delay of approach’’ is chosen as the critical criterion, and different critical volumes with LTWAs are concluded in Table 5. Critical volumes of exclusive left turn lane are obtained by comparing the average control delay of approach in scenario 2 and scenario 3, and critical volumes of protected left turn phase are determined based on delay comparison between scenario 1 and scenario 2.

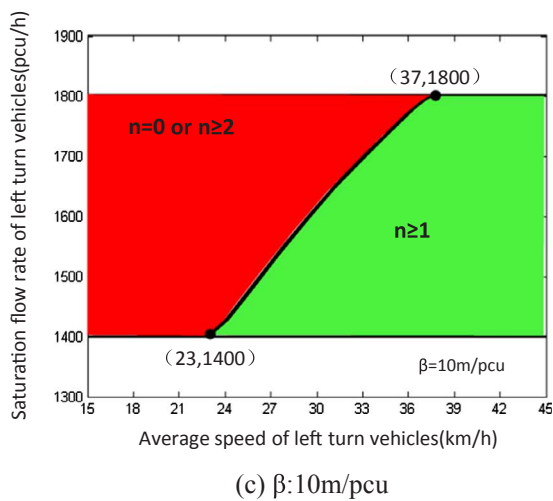
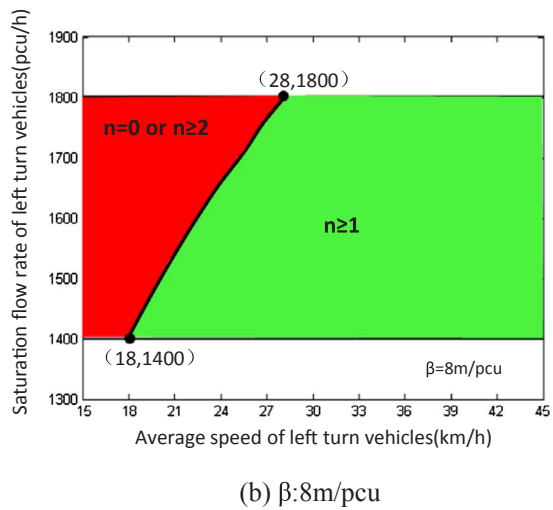
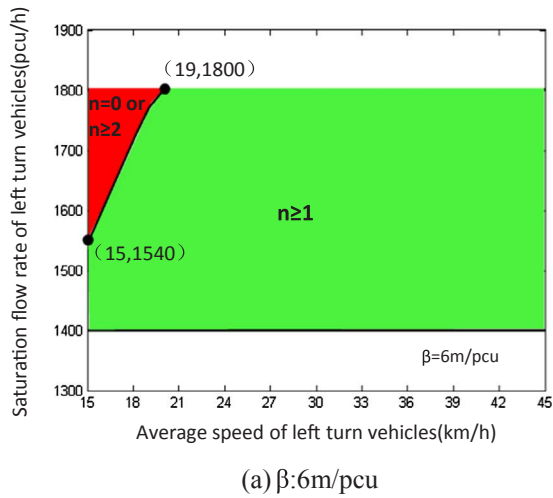
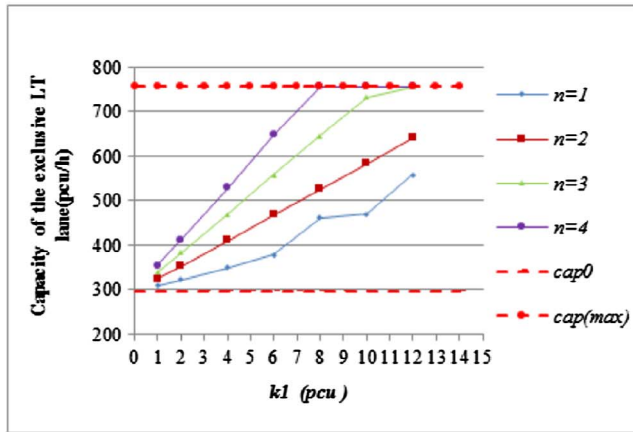
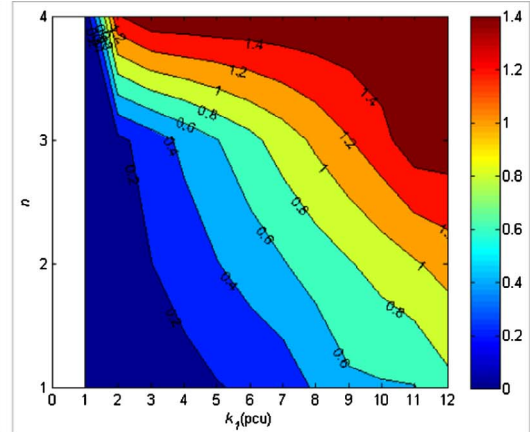


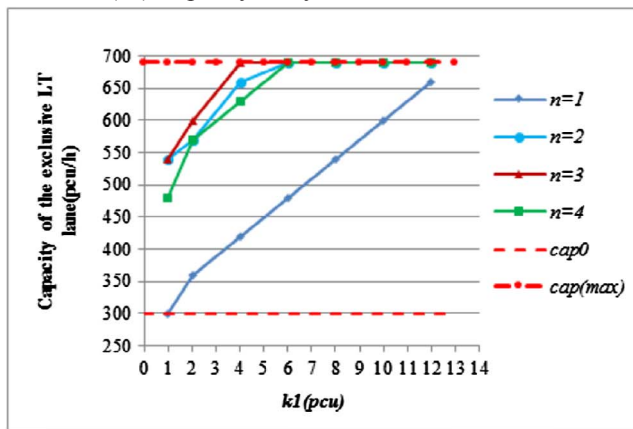
Fig. 5. Scope of application of LTWAs layout in scenario 1.



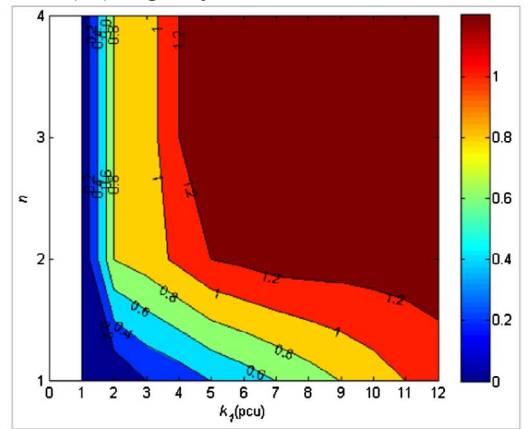
(a1) Capacity analysis for scenario 1.



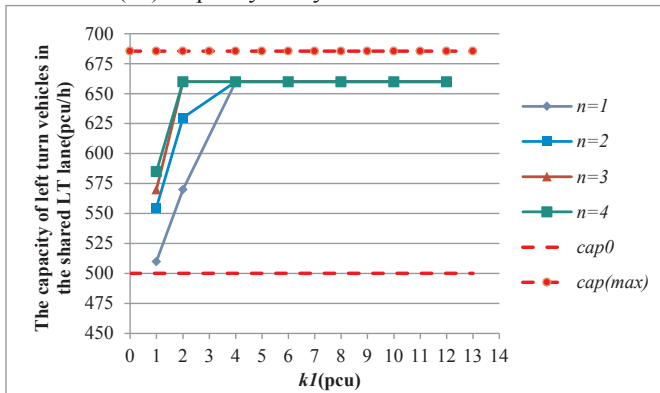
(b1) Capacity ratio for scenario 1.



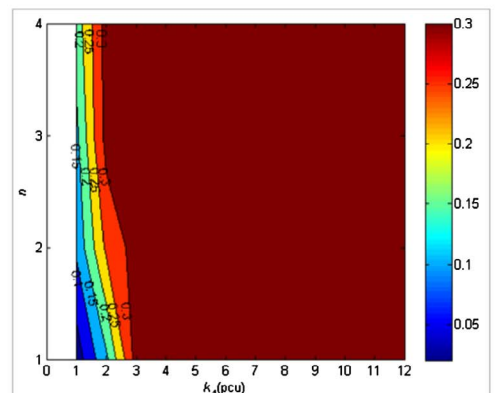
(a2) Capacity analysis for scenario 2.



(b2) Capacity ratio for scenario 2.



(a3) Capacity analysis for scenario 3.



(b3) Capacity ratio for scenario 3.

Fig. 6. Capacity analysis for left turn traffic flow in three scenarios.

Compared with HCM 2010 conclusions (100 vph for exclusive LT lane and 240 vph for protected LT phase), setting LTWAs can increase critical volumes of exclusive LT lane and protected LT phase with increasing n or k_1 , which may change the best lane layout and signal scheme in intersections compared with no LTWAs.

5. Conclusion

Probability theory and analysis of the LV operation mechanism are used to propose capacity estimation models for LT traffic flow in exclusive or shared LT lanes in three typical scenarios. Numerical examples and simulation results are presented to demonstrate the

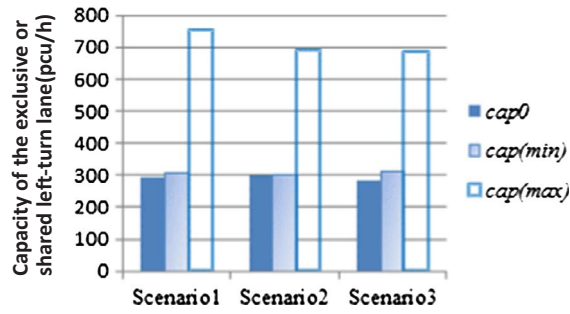


Fig. 7. $cap0$, $cap(min)$, and $cap(max)$ in three scenarios.

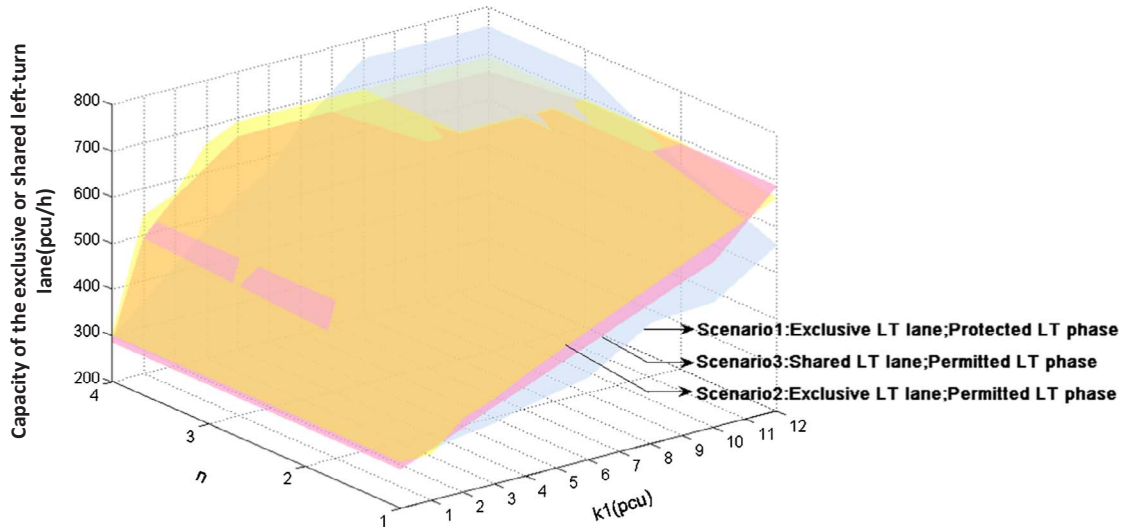


Fig. 8. Variations in capacity of left lane with variable k_1 and n in three scenarios.

Table 5
Critical volumes of exclusive LT lane and protected LT phase with LTWAs.

q_{T0} (pcu/h)	500		300	
	Exclusive LT lane	Protected LT phase	Exclusive LT lane	Protected LT phase
(1,0)	65	224	65	222
(1,2)	94	284	94	280
(1,4)	104	342	108	458
(1,6)	100	402	158	422
(2,2)	94	298	96	298
(2,4)	107	406	130	410
(2,6)	158	406	186	422
(3,2)	94	298	106	416
(3,4)	158	406	186	422
(3,6)	180	406	188	432

effectiveness of the proposed capacity estimation models.

This paper discusses the impact of LTWAs on operation of signalized intersection from the perspective of motor vehicle efficiency (capacity and delay), and draws some conclusions about how to set LTWAs.

The first issue is whether an approach needs LTWAs. For scenario 1 (exclusive LT lane with a protected LT phase), setting an LTWA can improve the LT traffic flow capacity only when the parameters satisfy the inequality $1 + \alpha n - 1 > 3.6\beta s_L(1/v_L + 1/u_W)$ (as shown in Fig. 5). When β is 10 m/pcu, setting LTWAs is recommended if the average speed of LVs exceeds 37 km/h and not recommended if that's less than 23 km/h. Else, determining whether an approach is applicable for LTWAs should be based on the maximum n (number of lanes in LTWAs) and the inequality. For scenario 2/3 (an exclusive LT lane with a permitted LT phase or a shared LT lane with a permitted LT phase), the layout of the LTWA can improve the LT traffic flow capacity and decrease delay in any scenario. Thus, setting LTWAs in appropriate conditions is recommended.

The second issue is how to set LTWAs to achieve maximum operation efficiency. Capacity will increase with increasing n (number of lanes in the LTWA) or k_1 (capacity of the first waiting lane). When K (Capacity of LTWA) is kept constant, the more n , the better operation efficiency of LVs. Moreover, the capacity of LVs does not correlate positively with K , and it is influenced by LTWA design parameters (n, k_1). In practical applications, engineers can choose an appropriate LTWA design scheme based on accurate service level using capacity ratio contour plot, as shown in Fig. 6(b1)–(b3). Finally, this paper chooses average control delay of approach as critical criterion, and concludes different critical left turn volumes of exclusive LT lanes and protected LT phase with different LTWAs design schemes, which is instructive to left turn traffic design.

This paper presents the preliminary evaluation results for the proposed model under bidirectional balanced LT traffic demand. More extensive numerical experiments or field tests will be conducted in the future to assess the effectiveness of the proposed model with various traffic demand patterns and intersection geometry configurations. Moreover, future work will explore the best LTWAs design scheme from multiple perspectives, such as traffic safety and motor vehicles' energy consumption. In addition, research into the LTWA capacity can be combined with a dynamic lane assignment approach, which may greatly improve traffic efficiency at signalized intersections.

Acknowledgments

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