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# DELAYS OF SHARED-SHORT LANES AT UNSIGNALIZED INTERSECTIONS

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# DELAYS OF SHARED-SHORT LANES AT UNSIGNALIZED INTERSECTIONS

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## ABSTRACT

At unsignalized intersections, both on the major street and on the minor street, there might be short turning lanes besides the through lanes following downstream from one single lane. We call this combined system a shared-short lane (SSL). Up to now only the capacity of these lanes at the stop line and the capacity of the diverging point, where the turning lane diverges from the through lane, can be calculated. For the total average delay of the involved individual movements, there is no applicable estimation procedure. As a special case, also the shared lane (SL), which is used by several movements without a separate turning lane, must be reconsidered. This paper presents a new model for the estimation of average delays of SSL with SL as a special case at unsignalized intersections. The model is based on the analogy to standard queuing systems. The results depend on the length of the short lane. The model is validated by simulation. The results demonstrate that the outcome of those models in current capacity manuals may be completely misleading with a risk to classify an intersection into a wrong Level of Service. Therefore, there is an urgent need to complete the relevant procedures in capacity manuals by more realistic estimation procedures for the total delay at an SSL or an SL. The methods in this paper – even if they are rather complex - are recommended to be incorporated into future versions of capacity manuals using some simplifications.


*Keywords*: Unsignalized intersection, Delay, Shared lane, Short lane

# INTRODUCTION

Highway capacity manuals like the HBS, (1) or the HCM, (2) offer methods for the calculation of capacities and delays at unsignalized intersections. These methods are results of a decade long series of investigations conducted by researchers, primarily like Harders (3), Siegloch (4), and Grossmann (5). But also other authors contributed a significant input (6, 7, 8, 9, *10*).

At an unsignalized intersection, more than one traffic movement can use a single traffic lane. This situation provides the special configuration of shared lanes (SL). Another common situation at unsignalized intersections is the so-called shared-short lane (SSL) where two traffic movements each use their own short traffic lane near the stop line, but they share a common traffic lane upstream from the short lane. In fact, an SL is a special case of an SSL with short lanes of length zero. Wu (*11*) developed a general methodology dealing with the capacity of those configurations. The proposed model there can also be used for calculating impedance effects of left turners from the major street (*12*). The idea dealing with the capacity of SSL can also be adopted to signalized intersections (*13, 14, 15*).

Once the capacity is given, the performance of traffic flow can be assessed using certain measures of effectiveness (MOE). For unsignalized intersections, average delays are used by guidelines worldwide as the MOE. For calculating delays, methodologies from queuing theory apply (cf. *16*). Although the queueing system at an unsignalized (priority-controlled) intersection is, in principle, a so-called M/G2/1 system (*17, 18, 19,* i.e. with a non-Markovian service time), the M/M/1 queue is used as a simplification, especially if a time-dependent solution of the queuing system is intended (*21*, *22*, *23*).

Although the capacity manuals provide methods for calculating capacities and delays for most of the geometric configurations, there are no methods for estimating delays at an SSL at unsignalized intersections. Actually, the queuing system at an SSL is a system consisting of two queues (cf. Figure 1), a short queue II at the stop line and a queue I upstream from the diverging point of the two movements. In the guidelines (1, 2) the delays can be calculated for the two queueing-systems separately - for system II, however, only under the assumption of two infinitely long lanes. There is no model to estimate the total average delays, which the two movements suffer when passing though the two subsequent queues. In addition, the service times at the stop-line are not calculated reasonably for both the SSL and SL situation. Therefore, the approach in the guidelines (1, 2) leads to a significant misjudgment of average delays and, thus, to an incorrect assessment of traffic quality, which is demonstrated in this paper. Therefore, a new model for delay estimation of SSL (with SL as a special case) at unsignalized intersections is needed.

Based on the capacity of SSL, which can be estimated based on previous investigations by the authors (*11*, *12*) and as an extension of standard queueing models, a delay model for SSL either for a minor or for a major approach is proposed in this paper. This model, then, is validated by simulation studies. The new model has the potential to close a gap in the current guidelines, e.g. HBS (1) and HCM (2).

The paper is organized as follows. First, an introduction to the problem is given and the motivation of the investigation is explained. Then, the new model for delay estimation at SSL is derived in detail. For the validation of the proposed model, a simulation study is conducted and some sensitivity analyses are depicted. For application of the proposed model, a simplified delay methodology in conformity with the HBS (1) and the HCM (2) is recommended. Finally, some conclusions are given in the light of the results.

## Nomenclature

| | | |
|---|---|---|
| $q$ | traffic volume | [veh/h] |
| $c$ | capacity | [veh/h] |
| $x$ | degree of saturation = $f(q, c)$ | [-] |
| $b$ | service time = $3600/c$ | [s] |
| $N$ | number of customers in the system | [veh] |
| $N_i$ | average number of customers on the waiting place $i$ | [veh] |
| $N_{i \text{ to } j}$ | average number of customers from the waiting place $i$ to $j$ | [veh] |
| $L$ | queue length in the queue | [veh] |
| $w$ | waiting time in the system = $b + d$ | [s] |
| $d$ | delay in the queue (waiting time in the queue) | [s] |
| $k$ | number of queue (waiting) places on a short lane | [veh] |
| $O_n$ | occupancy of the $n^{\text{th}}$ waiting place (= probability that the waiting place $n$ is occupied by a customer) | [-] |

## QUEUING SYSTEM WITH SSL AT UNSIGNALIZED INTERSECTIONS

At unsignalized intersections, there are often lanes for left-turn movements, which might be too short to accommodate the queue of turning vehicles during particular times. Such short lanes can exist both in the minor and major approaches (Figure 2a). Here we look only on situations where there is only one through lane. Together with the adjacent lane such a short lane forms one system since a queue, which spills back beyond the diverging point (where the two lanes diverge from one lane) on one of the two lanes blocks the access to the other lane. Thus, the capacities of the individual lanes cannot be completely utilized since they are estimated under the assumption of infinitely long lanes. Subsequently, the conventionally estimated average delay will not represent the delays experienced by drivers in reality. Actually, there are two sections on the approach, where the delay estimation has to account for, the shared-lane section (upstream from the diverging point, queue I in Figure 1) and the section with two parallel short lanes (queue II in Figure 1). The whole system is called an SSL.

### Capacities of SSL

According to Wu (*11*), the capacity of the SSL at a minor approach (Figure 2b) can be estimated by the following formula.

$$c_{L+T,\text{minor}} = \frac{q_{L+T}}{x_{L+T,\text{minor}}} = \frac{q_L + q_T}{x_{L+T,\text{minor}}} \quad (\text{subject to } c_{L+T,\text{major}} \leq \text{capacity of a single lane}) \qquad (1)$$

with the definition

$$x_{L+T,\text{minor}} = \left(x_L^{\,k+1} + x_T^{\,k+1}\right)^{\frac{1}{k+1}} = \left( (\frac{q_L}{c_L})^{k+1} + (\frac{q_T}{c_T})^{k+1} \right)^{\frac{1}{k+1}}$$

The indexes *L*, *T*, and *L+T* refer to the left-turn, though, and the shared movement left + through. The right-turn movement is not explicitly considered here. As an approximation, it can be included into the through movement. More precisely: eq. (1) calculates the capacity of the diverging point.

From Wu and Brilon (*12*) the capacity for an SSL at major approaches (Figure 2c) can be obtained as

$$c_{L+T,\text{major}} = \frac{q_{L+T}}{x_{L+T,\text{major}}} = \frac{q_L + q_T}{x_{L+T,\text{major}}} \text{ (subject to } c_{L+T,\text{major}} \le \text{capacity of a single lane)} \qquad (2)$$

with the definition

$$x_{L+T,\text{major}} = x_L \left(1 + \frac{x_T^{\,k+1}}{1 - x_T}\right)^{\frac{1}{k+1}} = \frac{q_L}{c_L}\left(1 + \frac{(\frac{q_T}{c_T})^{k+1}}{1 - \frac{q_T}{c_T}}\right)^{\frac{1}{k+1}}$$

Again, eq. (2) describes the capacity of the diverging point. Note, that the degrees of saturation of the SSL $x_{L+T}$ do not have the same expression for the minor and major approach (cf. *12*) due to different queuing processes. In case of $k = 0$, we have the SL situation.

Knowing the capacities of an SSL, the average delays, suffered upstream from the diverging point, can be estimated by extending existing queuing models.

## The Queuing System for SL

First, we consider the special situation with $k = 0$, that is the SL situation. As a simplification mentioned before, the queuing system for the single movements (*L* and *T*) at unsignalized intersections can be considered as an M/M/1 queuing system. Thus, due to the exponentially distributed service times, the variances of the service times for the left-turn and through movement are

$$\text{var}(b_L) = b_L^{\,2} = \left(\frac{3600}{c_L}\right)^2 \text{ and } \text{var}(b_T) = b_T^{\,2} = \left(\frac{3600}{c_T}\right)^2 \qquad (3)$$

The queuing system in the SL ($L+T$) is not an M/M/1 queuing system. Instead, it must be treated as an M/G/1 queuing system. The mean and the variance of the service time on the SL can be calculated from the means and the variances of the service times of both movements. The mean value of the service time of the SL ($L+T$) is

$$b_{L+T} = a_L b_L + a_T b_T \tag{4}$$

where $a_L$ and $a_T$ are the proportions of traffic volumes for left-turn and through movement on the SL.

The variance of the service time for the SL then is

$$\begin{aligned} \mathrm{var}(b_{L+T}) &= a_L(\mathrm{var}(b_L) + (b_L - b_{L+T})^2) + a_T(\mathrm{var}(b_T) + (b_T - b_{L+T})^2) \\ &= a_L({b_L}^2 + (b_L - b_{L+T})^2) + a_T({b_T}^2 + (b_T - b_{L+T})^2) \end{aligned} \tag{5}$$

According to the Pollaczek–Khinchine formula (see *16*) we get after some transformations for the delay averaged over all vehicles on the SL (delay = time in the queue without service time in the 1st position)

$$d_{L+T} = \frac{3600 L_{L+T}}{q_{L+T}} = \frac{3600 {x_{L+T}}^2}{q_{L+T}(1 - x_{L+T})} C_{0,L+T} \tag{6}$$

where

$$x_{L+T} = q_{L+T} / c_{L+T} \quad \text{and} \tag{7}$$

$$C_{0,L+T} = \frac{1}{2}\left(1 + \frac{\mathrm{var}(b_{L+T})}{{b_{L+T}}^2}\right) \text{ (as a parameter without dimension)} \tag{8}$$

Within the queue, both the left turners ($L$) and the through vehicles ($T$) experience equally this kind of delay. However, in the first position of the SL their service times are different ($b_L$ and $b_T$). Thus, the total average delays for the two movements ($L$ and $T$) including the relevant service time are

$$w_L = \frac{3600 x_L}{q_L} + \frac{3600 L_{L+T}}{q_{L+T}} = b_L + d_{L+T} = b_L + \frac{3600 {x_{L+T}}^2}{q_{L+T}(1 - x_{L+T})} C_{0,L+T} \tag{9}$$

and

$$w_T = \frac{3600 x_T}{q_T} + \frac{3600 L_{L+T}}{q_{L+T}} = b_T + d_{L+T} = b_T + \frac{3600 {x_{L+T}}^2}{q_{L+T}(1 - x_{L+T})} C_{0,L+T} \tag{10}$$

It is evident that the total delays for the left-turn and through movement are not identical due to different service times. The parameter $C_{0,\,L+T}$ has a value between 1.3 and 3.0 in real cases. Compared to an M/M/1 queue ($C_0 = 1$), the differences are significant.

In the current HCM (2), the total delay in the SL is calculated using the same formula for both the left-turn and through movement. The delay formulas there are derived based on an M/M/1 queue. That is

$$w_{L,HCM} = w_{T,HCM} = w_{L+T} = b_{L+T} + \frac{3600 {x_{L+T}}^2}{q_{L+T}(1 - x_{L+T})} \tag{11}$$

where

$$b_{L+T} = \frac{3000}{c_{L+T}} = a_L b_L + a_T b_T$$

So far, delays have been discussed based on stationary conditions, i.e. under a degree of saturation $x < 1$. The stationary solutions are, however, the basis for the time-dependent solutions which are offered in the guidelines. As an example, eq. 20-64 in the HCM (2) is based on eq. (11).

Under common situations, the use of this equation can lead to an under-estimation of average delay for the individual movements. Thus, using the procedures provided by the current guidelines (e.g. 1, 2) may lead to wrong conclusions: a left-turn vehicle would experience less total delay in an SL than in an exclusive left-turn lane (especially under low degree of saturation). This would mean that an SL could enhance the traffic quality of the single movements. This, of course, is incorrect. It can lead to a wrong assessment of traffic facilities and wrong planning decisions.

The extend of the error becomes clear if we compare the average delay calculated by eq. (9) and by eq. (11). The difference between these two results is plotted in Figure 3 over $x_L$ and $x_T$ for the left turners from the minor street. In this example the capacities for left-turn and through movement are set to $c_L = 150$ veh/h and $c_T = 600$ veh/h respectively and vice versa. It is evident that there is a large difference between the delays resulting from both equations and, consequently, also a large error in the time dependent equations, which are contained in the guidelines (e.g. 1, 2).

## The Queuing System for SSL

The general case of an SSL is illustrated in Figure 1. Here $k$ is the number of spaces for cars on the two short lanes. For $k > 0$, no methodologies to estimate average delays for the two movements exist in the current guidelines. Although the capacities for the diverging point can be calculated (eqs. (1) and (2)), the total average delays for the relevant movements cannot be estimated adequately. The German manual (HBS, 1) uses a pragmatic instruction to cope this problem: both the delays of the individual movements and the delay of the diverging point (cf. eqs. (1) and (2)) are examined separately; the larger delay is the decisive one to determine the level of service.

In the following, a new model is developed to estimate the average delay at an SSL. The queuing system at an SSL can be considered as a system of two interrelated queues: the shared-lane section upstream of the diverging point (queue I) and the section downstream from the diverging point (queue II) with two parallel short lanes of length $k$. The two queues are connected at the position $k + 1$. Approximatively, the two queues in section II can be treated as independent M/M/1 queues. The queue I upstream from the diverging point must be considered as a general queueing system of type M/G/1.

First, we look at the length of the queue in the combined "Two-Queue"-system II. It should be noted that the 1st place in both lines of queue II is treated as the service counter. The occupancy of the $n^{th}$ waiting place, that is the probability that the $n^{th}$ place is occupied by a vehicle, is called $O_n$. Thus, $O_n$ is the probability Pr(number of vehicles in the system ≥ $n$). The occupancies $O_n$ form an infinite sequence for each of the two movement $L$ and $T$:

$$(O_n)_{n=1}^{\infty} = (O_1, O_2, ..., O_n, ...) \quad (12)$$

The average number of vehicles on place $n$ is $O_n$. Thus, the average number of vehicles on all places, that is the average number $N$ of vehicles in the queuing system is

$$N = \sum_{n=1}^{\infty} O_n \quad (13)$$

Queue II is connected to queue I at place $k + 1$. Place $k + 1$ is also the first place of queue I and, thus, the service counter for queue I. The average number $N$ of queuing vehicles in the system can be treated by four components:

1. $N_1$ = average number of vehicles on the first place of queue II
2. $N_{2\text{ to }k}$ = average number of vehicles in the area of places 2 to $k$
3. $N_{k+1}$ = average number of vehicles in the counter of queue I on place $k + 1$, and
4. $N_{k+2\text{ to }\infty}$ = average number of vehicles on places beyond $k + 1$.

Thus,

$$N = \sum_{n=1}^{\infty} O_n = N_1 + N_{2\text{ to }k} + N_{k+1} + N_{k+2\text{ to }\infty} \quad (14)$$

As an example, the left-turn movement $L$ from a minor approach is modeled (the through movement $T$ and the major approach can be modeled similarly). In queue II, the Markovian property applies (M/M/1) and, thus, the probability that $n$ and more than $n$ vehicles are in the system of queue II (including the vehicle in the 1st position) for both of the two partial queues is

$$O_n = \Pr(\text{no. of veh. in the system} \geq n) = x^n = \left(\frac{q}{c}\right)^n \tag{15}$$

where

$q$ = traffic volume of the partial queue (= $q_L$ or $q_T$)
$c$ = capacity of the partial queue (= $c_L$ or $c_T$)
$x$ = degree of saturation in the partial queue (= $x_L$ or $x_T$)

The occupancy of place $k + 1$ is the probability that the queue length in $L$ or in $T$ is larger than $k$ (cf. Wu, *11*). Thus,

$$O_{k+1} = O_{k+1,L+T} = O_{k+1,L} + O_{k+1,T} = \Pr(N_L > k) + \Pr(N_Z > k) = x_L{}^{k+1} + x_T{}^{k+1} = x_{L+T}{}^{k+1} \tag{16}$$

where (see also the definition in eq.(1))

$$x_{L+T} = \left(x_L{}^{k+1} + x_T{}^{k+1}\right)^{\frac{1}{k+1}} \tag{17}$$

The infinite sequence ($O_n$) of occupancies can be rewritten as

$$(O_n)_{n=1}^{\infty} = (x_L, x_L^2, ..., x_L^k, x_{L+T}^{k+1}, O_{k+2,L+T}, O_{k+2,L+T}, ...) \tag{18}$$

For queue II (M/M/1) with $n$ = 1 to $k$, the occupancy on the first place is the average number of vehicles in the service counter. That is,

$$N_{1,L} = O_{1,L} = x_L \tag{19}$$

The average number of vehicles occupying (waiting on) the places 2 through $k$ is

$$N_{2\text{ to }k,L} = \sum_{n=2}^{k} O_{n,L} = \sum_{n=2}^{k} x_L{}^n = x_L{}^2 \sum_{n=0}^{k-2} x_L{}^n = \frac{x_L{}^2}{1-x_L}(1-x_L{}^{k-1}) = (1-x_L{}^{k-1}) L_{\text{MM1},L} \tag{20}$$

The occupancies for queue I from place $k$ + 1 to infinite form an infinite sequence:

$$(O_{n,L+T})_{n=k+1}^{\infty} = (x_{L+T}^{k+1}, O_{k+2,L+T}, O_{k+2,L+T}, ...) = x_{L+T}^{k}(x_{L+T}, \frac{O_{k+2,L+T}}{x_{L+T}^{k}}, \frac{O_{k+2,L+T}}{x_{L+T}^{k}}, ...) \tag{21}$$

The sequence within the brackets on the right side is the sequence of occupancies of an M/G/1 queue with a degree of saturation $x_{L+T}$.

The average number of vehicles from places $k$ + 2 to infinite is

$$N_{k+2 \text{ to } \infty} = x_{L+T}^{k} L_{\mathrm{MG1,L+T}} = x_{L+T}^{k} \frac{x_{L+T}^{2}}{1-x_{L+T}} C_{0,L+T} \tag{22}$$

The average number of vehicles in the counter for queue I is $N_{k+1}$. That is (cf. eq.(16)),

$$N_{k+1} = x_{L+T}{}^{k+1} = O_{k+1,L+T} = O_{k+1,L} + O_{k+1,T} = x_{L}{}^{k+1} + x_{T}{}^{k+1} \tag{23}$$

Here, we account for only the left-turn vehicles. Thus,

$$N_{k+1,L} = O_{k+1,L} = x_{L}{}^{k+1} \tag{24}$$

Then, for the left-turn movement, the relevant total number of vehicles in the "Two-Queue" system then is

$$N = N_{1,L} + N_{2\,\text{to}\,k,L} + N_{k+1,L} + N_{k+2 \text{ to } \infty} = x_L + (1 - x_L{}^{k-1})\frac{x_L{}^2}{1-x_L} + x_L^{k+1} + x_{L+T}^{k} \frac{x_{L+T}^{2}}{1-x_{L+T}} C_{0,L+T} \tag{25}$$

Sorting the equation yields

$$N = x_L + (1 - x_L{}^{k})\frac{x_L{}^2}{1-x_L} + x_{L+T}^{k} \frac{x_{L+T}^{2}}{1-x_{L+T}} C_{0,L+T} = x_L + (1 - x_L{}^{k}) L_{\mathrm{MM1},L} + x_{L+T}^{k} L_{\mathrm{MG1},L+T} \tag{26}$$

That means, the delay in the queue of an SSL can be calculated as the superposition of an M/M/1 queue and an M/G/1 queue with the portions of times 1 - $x^{k}_{L}$ and $x^{k}_{L+T}$ respectively.

After some additional derivations, the service time of the queue II (M/G/1) for the SSL ($L$+$T$) can be expressed as

$$b_{L+T} = \frac{3600}{c_{L+T}} = a_{L,b} b_L + a_{T,b} b_T \tag{27}$$

with

$$a_{L,b} = \frac{a_L x_L{}^k}{x_{L+T}{}^k} = a_L \left( \frac{x_L}{x_{L+T}} \right)^k \quad \text{and} \quad a_{T,b} = \frac{a_T x_T{}^k}{x_{L+T}{}^k} = a_T \left( \frac{x_T}{x_{L+T}} \right)^k \tag{28}$$

The parameters $a_{L,b}$ and $a_{T,b}$ are proportions of vehicles, which are served on position $k + 1$ with the service times $b_L$ and $b_T$ respectively during the time portion of $x^k{}_{L+T}$. This formula for the service time $b_{L+T}$ can be interpreted also as follows: a left-turn vehicle must wait on place $k + 1$ when in front of him the place $k$ is occupied. Otherwise, it can move on without stopping. When queue $L$ at the stop line is served with the service time $b_L$, the total queue will move up by one place. The left-turn vehicle on place $k + 1$ can also move up to place $k$ after the time $b_L$. That is, the service time for the left-turn vehicle on place $k + 1$ is $b_L$ as well. The analog is true for the through vehicles. Thus, during the portion of time $x^k{}_{L+T}$ the proportions of vehicles on place $k + 1$ are served with the service times $b_L$ and $b_T$ are $a_{L,b}$ and $a_{T,b}$ respectively.

With some additional derivation, the two parameters for a major approach can be calculated as follows (cf. *12*).

$$b_{L+T} = \frac{3600}{c_{L+T}} = a_{L,b} b_L + a_{T,b} b_T \tag{29}$$

with

$$a_{L,b} = \frac{a_L x_L{}^k}{\left( x_L{}^{k+1} + \frac{(x_L x_T)^{k+1}}{1 - x_T} \right)^{\frac{k}{k+1}}}, \quad a_{T,b} = \frac{a_T \frac{x_L (x_L x_T)^k}{1 - x_T}}{\left( x_L{}^{k+1} + \frac{(x_L x_T)^{k+1}}{1 - x_T} \right)^{\frac{k}{k+1}}} \tag{30}$$

The proportion of vehicles which, during the time portion $x^k{}_{L+T}$, can pass the place $k + 1$ without stopping is then

$$\left( 1 - a_{L,b} - a_{T,b} \right) \tag{31}$$

Therefore, the variance of the service time during the time portion $x^k{}_{L+T}$ is

$$Var(b_{L+T}) = \left( b_L{}^2 + \left( b_L - b_{L+T} \right)^2 \right) a_{L,b} + \left( b_T{}^2 + \left( b_T - b_{L+T} \right)^2 \right) a_{T,b} + b_{L+T}{}^2 \left( 1 - a_{L,b} - a_{T,b} \right) \tag{32}$$

Then, the parameter $C_{0,\, L+T}$ for the M/G/1 queue at the diverging of SSL is

$$C_{0,L+T} = \frac{1}{2}\left(1 + \frac{Var(b_{L+T})}{b_{L+T}^{\ 2}}\right) \tag{33}$$

Then, both at a minor and major approach the total delay of a left-turn vehicle in the "Two-Queue" system as an SSL is

$$w_L = 3600\left(\frac{x_L + (1 - x_L^{\ k})L_{\mathrm{MM1},L}}{q_L} + \frac{x_{L+T}^k L_{\mathrm{MG1},L+T}}{q_{L+T}}\right) = b_L + (1 - x_L^{\ k})d_{\mathrm{MM1},L} + x_{L+T}^{\ \ k} d_{\mathrm{MG1},L+T} \tag{34}$$

The total delay of a through vehicle in the "Two-Queue" system is different for a minor approach and for a major approach. For a minor approach, the total delay of a through vehicle is

$$w_{T,\mathrm{minor}} = 3600\left(\frac{x_T + (1 - x_T^{\ k})L_{\mathrm{MM1},T}}{q_T} + \frac{x_{L+T}^k L_{\mathrm{MG1},L+T}}{q_{L+T}}\right) = b_T + (1 - x_T^{\ k})d_{\mathrm{MM1},T} + x_{L+T}^{\ \ k} d_{\mathrm{MG1},L+T} \tag{35}$$

For a major through vehicle, there is no delay in queue II and the service time occurs only by queuing in queue I on the position $k + 1$. Thus, the total delay of a major through vehicle is (cf. eq. (21) and eq.(25))

$$w_{T,\mathrm{major}} = x_{L+T}^{\ \ k} b_T + x_{L+T}^{\ \ k} d_{\mathrm{MG1},L+T} = x_{L+T}^{\ \ k}\left(b_T + d_{\mathrm{MG1},L+T}\right) \tag{36}$$

Note the derivation here is only valid for the special case that queue I is a consequence of queue II of type M/M/1. For a more general case, for example the "two-stage queuing", the derivation can be extended as well.

## VALIDATION OF THE PROPOSED MODEL BY SIMULATION STUDIES

To examine the proposed model, different combinations of SSL have been simulated. For the simulations a T-Junction was used in order to avoid too many interferences within the traffic movements. Using this configuration, the SSL both on the major street and on the minor street can be simulated properly. The traffic volumes used in the simulation are mentioned in Figure 4 together with the number $k$ of spaces on the relevant short lane.

As a software tool for simulation, KNOSIMO (*25*, 5) has been applied. KNOSIMO has been calibrated and verified by several studies (e.g. 5) and is widely applied in practice in Germany. It is an event-oriented microscopic simulation tool. It imitates events like arrivals, queue move-up, and departures as well as driver's decisions according to available gaps. It can be set to a combination of parameters and methods, which coincide, completely with the basics of the gap acceptance theory, which is the fundament of guideline procedures (1, 2). It can consider more realistic characteristics of traffic operation at unsignalized intersections than the methodologies in the capacity manuals. For example, realistic distributions for arriving headways, critical gaps, and follow-up times can be accounted for. However, for reasons of comparability, the model has been confined to the assumptions made by the guidelines (1, 2), i.e. Markovian property (exponentially distributed headways) as well as constant critical gaps and follow-up times. The model runs very fast so that a huge sample size (and in consequence reliable average delays) can be produced within short time.

Because the capacities of the individual movements *L* and *T* are needed for the calculation of the SSL-capacity and its delay, capacities have to be estimated beforehand. A simulation will not produce a result for capacities, only for delays or queue lengths. For validation of the proposed model, the capacities were defined by two different scenarios:

1. The capacities *c* of the single movements are defined as functions of the simulated delays. Assuming an M/M/1 queue for the individual movements at the stop line, the delay under steady-state condition is $w = 3600/(c - q)$, and thus, $c = 3600/w + q$. The total delay *w* results directly from the simulation. For this step, the *L*- and *T*-movements have been assigned to separate infinitely long lanes. In this case, the delay for the major through movement cannot be simulated because no queue can be observed at the stop line. The capacity *c* of the major through movement is then defined as the reciprocal of the minimum headway, which is ca. 1.6s in the simulation. Thus, the capacity *c* of the major through movement is set to = 2200 veh/h (= 3600/1.6s).
2. The capacities *c* of the two individual movements are calculated according to the procedures in the HBS (1). Also in this case, to keep the comparability with the simulation, the minor movement capacity *c* is calculated with a realistic minimum headway of 1.2s in the major movements. In this case, a modified capacity formula is used accounting for the minimum headway. For the major through movement, the capacity *c* is set to 2200 veh/h as well instead of the default value in the HBS ($c$ = 1800 veh/h).

**Procedures in the Germany Highway Capacity Manual (HBS) for SSL**

To demonstrate the importance of a new model, the calculated left-turn average total delays *w* according to the delay procedures from the HBS (1) and HCM (2) are compared to the simulated total delays in Figure 5a. The HBS-procedures examine the delays of individual movements (*L* and *T*) and the delay of the diverging point ($L + T$) separately. The larger delay is the decisive one to determine the level of service, which is applied here. The delays, which occur on the shared-lane section (corresponding to the HCM-procedures), are compared in Figure 5b. Traffic volumes for these example comparisons can be obtained from Figure 4 (upper part of  Figure 5 see Figure 4b, lower part see Figure 4a). Of course, the average delay depends on the number of spaces *k* on the short lane. The simulation results are compared with total delays *w* from the HBS-procedures and with delays *w*, which occur on the shared-lane section only (HCM-procedures). It can be seen, that the differences are significant for both comparisons. This shows that the current methods are completely misleading and that, thus, a new model is needed for an accurate estimation of the total average delays at an SSL.

## Results of the Validation

The total average delays obtained from the proposed model are presented in Table 1 (columns Model with accurate $C_{0,L+T}$) for an example together with the simulation results for both capacity scenarios (Model = eq.(34)) and in Figure 6. Here we see that the values for the model and for the simulation results nearly coincide. The differences between the simulated and the calculated delays are extremely small, and they can be considered as irrelevant for use in practice. The capacities for both capacity scenarios are comparable, that is, the capacities from the highway capacity manuals are confirmed by the simulation as well.

## Simplification for Use in Practice

The parameters $a_{L,b}$ and $a_{T,b}$ depend on the length $k$ of the sort lane section $k$. The formulas (eq. (28) and eq.(30)) hereof are complex. For use in practice these parameters can be simplified by using the values for $k$ = 0. That is:

$$a_{L,b} \approx a_L \quad \text{and} \quad a_{T,b} \approx a_T \tag{37}$$

For a major approach the two parameters must be calculates as follows.

$$a_{L,b} \approx a_L, \quad a_{T,b} \approx \frac{x_L a_T}{1 - x_T} \tag{38}$$

This simplification can only be used for calculation the parameter $C_{0,\,L+T}$ (eq. (8)). For calculation the mean service time of the diverging point, $b_{L+T}$=3600/$c_{L+T}$ where $c_{L+T}$ from eq. (1) or eq. (2), remains.

The results for the total delays using these simplified values to calculate $C_{0,L+T}$ (eq. (8)) are depicted in Table 1 (columns Model with simpl. $C_{0,L+T}$) together with results for accurate values of $C_{0,L+T}$. The total model delays using the accurate and simplified parameter $C_{0,L+T}$ are quite similar. However, the model delays using the simplified $C_{0,L+T}$ are sometimes not exactly monotony falling with regards to the number of waiting places $k$ (cf. bold numbers in Table 1). This is theoretically incorrect. Nevertheless, the deviations are marginal, and they can be neglected for use in practice.

# POSSIBLE APPLICATIONS OF THE PROPOSED MODEL

The proposed model can be easily incorporated into the current highway capacity manuals. However, the delay models used in the manuals are time-dependent models accounting for temporary oversaturation. Thus, the proposed model must be adopted to the time-dependent delay model.

The time-dependent delay formula for a movement $m$ has a general expression as follows (cf. *2*, eq. 20-64).

$$w_m = b_m + d_m + (5) = \frac{3600}{c_m} + 900T\left[\left(\frac{q_m}{c_m} - 1\right) + \sqrt{\left(\frac{q_m}{c_m} - 1\right)^2 + \frac{\frac{8q_m}{c_m}}{c_m T} C_0}\right] + (5) \quad (39)$$

where $T$ is duration of the time interval under consideration, e.g. 15 minutes for the HCM (*2*), and $m$ is an index for one movement. $C_0$ is a parameter taking account for the stochastic property of the queuing system (eq.(8)). For an M/M/1 queue $C_0 = 1$. The last term of the equation (5 s), is a constant value representing the geometric delay. It can be omitted if only the queuing delays are considered (cf. 1).

For an SSL, the queuing system in the short lane section (queue II) is an M/M/1 with $C_0 = 1$ and the queuing system in the shared-lane section (queue I) is M/G/1 with

$$C_{0,SH} = \frac{1}{2}\left(1 + \frac{Var(b_{SH})}{b_{SH}^{\ 2}}\right) \quad (40)$$

where $SH$ is the index for the shared-lane part of the SSL-System.

Thus, the total delay for movement $m$ ($L$ and $T$) within an SSL at a minor approach is

$$w_{m,\text{minor}} = b_m + d_m + d_{SH} + (5) \quad (41)$$

with

$$d_m = 900T\left[(x_m - 1) + \sqrt{(x_m - 1)^2 + \frac{8x_m}{c_m T}}\right] C_2,$$

$$d_{SH} = 900T\left[(x_{SH} - 1) + \sqrt{(x_{SH} - 1)^2 + \frac{8x_{SH}}{c_{SH} T} C_{0,SH}}\right] C_1$$

and

$$b_m = \frac{3600}{c_m},\ C_2 = 1 - x_m^{\ k},\ C_1 = x_{SH}^{\ k},\ x_m = \frac{q_m}{c_m},\ \text{and}\ x_{SH} = \frac{q_{SH}}{c_{SH}}.$$

The values of $x_m$ and $x_{SH}$ is subject to less than or equal to 1. Furthermore, because the input of queue II can never be larger the than the output of queue I, the value of $q_m$ is subsect to less than or equal to $a_m \cdot c_{SH}$.

For a minor SSL with two movements $L$ and $T$ with the approximation $a_{L,b} \approx a_L$ and $a_{T,b} \approx a_T$ we have

$$C_{0,SH} \approx \frac{1}{2}\left(1 + \frac{\left(b_L{}^2 + \left(b_L - b_{SH}\right)^2\right)a_L + \left(b_T{}^2 + \left(b_T - b_{SH}\right)^2\right)a_T}{b_{SH}{}^2}\right) \tag{42}$$

with $b_{SH} = 3600/c_{SH}$ where $c_{SH}$ from eq. (1).

In many cases in realty there are actually three movements $L^*$, $T^*$ and $R^*$ but only two possible traffic lanes, for example at a flared minor street. Thus, the three movements should be combined into two movements. Depending on the configuration we can use a combined movement $L = L^*+T^*$ or $T = T^*+R^*$. Note, for $k > 0$, the derivation is not exact; it is only a pragmatic approximation.

For a major SSL, only a solution with two movements $L$ and $T$ is possible. A combined movement $T = T^*+R^*$ always applies. That is, with

$$a_{L,b} \approx a_L \text{ and } a_{T,b} \approx a_T \frac{x_L}{1-x_T}, \tag{43}$$

we have

$$C_{0,SH} \approx \frac{1}{2}(1 + \frac{\left(b_L{}^2 + \left(b_L - b_{SH}\right)^2\right)a_L + \left(b_T{}^2 + \left(b_T - b_{SH}\right)^2\right)a_T \frac{x_L}{1-x_T} + b_{SH}{}^2\left(1 - a_L - a_T \frac{x_L}{1-x_T}\right)}{b_{SH}{}^2}) \tag{44}$$

with $b_{SH} = 3600/c_{SH}$ where $c_{SH}$ from eq. (2).

Again, the value of $q_L$ or $q_T$ for calculating $x_L$ or $x_T$ is subsect to less than or equal to $a_L \cdot c_{SH}$ or $a_T \cdot c_{SH}$. This will ensure that the value of $x_L$ or $x_T$ is always less than or equal to 1.

The formulation of total delays for a major through and for a major left-turn vehicle are different and they are

$$w_{L,\text{major}} = b_L + d_L + d_{SH} + (5) \text{ and } w_{T,\text{major}} = b_T C_1 + d_{SH} + (5) \quad \text{(cf. eq. (36))} \tag{45}$$

In case of SSL with $k = 0$, we have again the SL situation and only the shared-lane section exists. Thus, $C_2 = 0$, $C_1 = 1$, and $a_{m,b} = a_m$. Then, the total delay of the movement $m$ at both a minor and a major approach is

$$w_m = b_m + d_{SH} + (5) = b_m + 900T\left[\left(x_{SH}-1\right)+\sqrt{\left(x_{SH}-1\right)^2+\frac{8x_{SH}}{c_{SH}T}C_{0,SH}}\right]_1 + (5) \qquad (46)$$

In case of a minor approach all three movements $L^*$, $T^*$ and $R^*$ can be considered directly. That is,

$$C_{0,SH} = \frac{1}{2}(1+\frac{\left(b_{L^*}{}^2+\left(b_{L^*}-b_{SH}\right)^2\right)a_{L^*}+\left(b_{T^*}{}^2+\left(b_{T^*}-b_{SH}\right)^2\right)a_{T^*}+\left(b_{R^*}{}^2+\left(b_{R^*}-b_{SH}\right)^2\right)a_{R^*}}{b_{SH}{}^2}) \qquad (47)$$

For a major SSL the solution with $k = 0$ (SL-situation) with two movements is

$$C_{0,SH} = \frac{1}{2}(1+\frac{\left(b_L{}^2+\left(b_L-b_{SH}\right)^2\right)a_L+\left(b_T{}^2+\left(b_T-b_{SH}\right)^2\right)a_T\frac{x_L}{1-x_T}+b_{SH}{}^2\left(1-a_L-a_T\frac{x_L}{1-x_T}\right)}{b_{SH}{}^2}) \qquad (48)$$

## CONCLUSION

In this paper, the queuing problem of SSL at an unsignalized intersection is modeled based on standard queuing models. The derivations lead to equations for the estimation of total average delay for each of the movements on the SSL. The proposed model was validated by simulation studies.

Detailed analysis shows, that the delay estimation procedure for SL - as a special case of SSL, i.e. without a turning lane - in the current highway capacity manuals (HBS, 1; HCM, 2) is very inaccurate and it can lead to significant deviations in traffic performance assessment and, thus, to wrong LOS classification. For a general SSL with a short turning lane the procedure in the current highway capacity manuals for the delay estimation is inaccurate as well.

The proposed model is theoretically well supported but very complex. For use in practice a simplified solution is given. This solution can be easily incorporated into the current highway capacity manuals. A HCM- and HBS-conform calculation procedure is proposed as a recommendation.

It should finally be noted, that the derivations in this paper are only applicable for situations with only one lane on the approach to the intersection. For multilane situations, one movement (e.g. the left turner $L$) would not block the diverging point since in case of a long $L$-queue the other movement (e.g. the through traffic) can switch over to the other lane.

As a next step, the proposed model can be further generalized and be extended to the so-called two-stage priority and to multilane situations.

**AUTHOR CONTRIBUTION -STATEMENT**

Both authors have cooperated in conception and design of the research, in analysis and in interpretation of results, and manuscript preparation. Both authors have approved the final version of the paper.

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## List of Tables and Figures

**TABLE 1 Results for Total Average Delay *w* [s] Obtained from Simulation Compared to Values Calculated by the Model**

| | Minor street | | | | | | | | Major street | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Model with accur. $C_{0,L+T}$ | | | | Model with simpl. $C_{0,L+T}$ | | | | Model with accur. $C_{0,L+T}$ | | | | Model with simpl. $C_{0,L+T}$ | | | |
| | left L | | through T | | left L | | through T | | left L | | through T | | left L | | through T | |
| | *c* simulated ($c = 3600/w_{simu} + q$) | | | | | | | | | | | | | | | |
| | $c$ = 187 [veh/h] | | $c$ = 537 [veh/h] | | $c$ = 442 [veh/h] | | $c$ = 2200 [veh/h] | | $c$ = 187 [veh/h] | | $c$ = 537 [veh/h] | | $c$ = 442 [veh/h] | | $c$ = 2200 [veh/h] | |
| *k* | S | M | S | M | S | M | S | M | S | M | S | M | S | M | S | M |
| 0 | 82.4 | 85.1 | 69.2 | 72.5 | 82.4 | 85.1 | 69.2 | 72.5 | 34.9 | 35 | 26.8 | 28.2 | 34.9 | 34 | 26.8 | 25.8 |
| 1 | 44 | 44.9 | 23.8 | 23.9 | 44 | 45.4 | 23.8 | 24.4 | 19.8 | 20.1 | 7.6 | 7.4 | 19.8 | 19.8 | 7.6 | 7 |
| 2 | 42.3 | 42.2 | 16.4 | 16.2 | 42.3 | 42.3 | 16.4 | 16.3 | 19.4 | 19.5 | 4.3 | 3.9 | 19.4 | **19.2** | 4.3 | 3.6 |
| 3 | 41.8 | 41.7 | 12.9 | 12.8 | 41.8 | 41.7 | 12.9 | 12.8 | 19.4 | 19.4 | 2.5 | 2.2 | 19.4 | **19.3** | 2.5 | 2 |
| 4 | 41.5 | 41.5 | 11.2 | 11.2 | 41.5 | 41.5 | 11.2 | 11.1 | 19.4 | 19.4 | 0.8 | 0.7 | 19.4 | **19.3** | 0.8 | 0.7 |
| 5 | 41.5 | 41.5 | 10.2 | 10.3 | 41.5 | 41.5 | 10.2 | 10.3 | 19.4 | 19.4 | 0 | 0 | 19.4 | **19.4** | 0 | 0 |
| 6 | 41.5 | 41.5 | 9.7 | 9.8 | 41.5 | 41.5 | 9.7 | 9.8 | 19.4 | 19.4 | 0 | 0 | 19.4 | 19.4 | 0 | 0 |
| 7 | 41.5 | 41.5 | 9.5 | 9.6 | 41.5 | 41.5 | 9.5 | 9.6 | 19.4 | 19.4 | 0 | 0 | 19.4 | 19.4 | 0 | 0 |
| 10 | 41.5 | 41.5 | 9.3 | 9.3 | 41.5 | 41.5 | 9.3 | 9.3 | 19.4 | 19.4 | 0 | 0 | 19.4 | 19.4 | 0 | 0 |
| 20 | 41.5 | 41.5 | 9.3 | 9.3 | 41.5 | 41.5 | 9.3 | 9.3 | 19.4 | 19.4 | 0 | 0 | 19.4 | 19.4 | 0 | 0 |
| $R^2$ | 0.998 | | | | 0.999 | | | | 1.000 | | | | 0.999 | | | |
| *SD* | 1.00 | | | | 1.04 | | | | 0.35 | | | | 0.40 | | | |
| | *c* calculated from the HBS (*1*) | | | | | | | | | | | | | | | |
| | $c$ = 187 [veh/h] | | $c$ = 558 [veh/h] | | $c$ = 438 [veh/h] | | $c$ = 2200 [veh/h] | | $c$ = 187 [veh/h] | | $c$ = 558 [veh/h] | | $c$ = 438 [veh/h] | | $c$ = 2200 [veh/h] | |
| *k* | S | M | S | M | S | M | S | M | S | M | S | M | S | M | S | M |
| 0 | 82.4 | 80.9 | 69.2 | 68 | 82.4 | 80.9 | 69.2 | 68 | 34.9 | 36.5 | 26.8 | 29.7 | 34.9 | 35.5 | 26.8 | 27.3 |
| 1 | 44 | 44.6 | 23.8 | 23.1 | 44 | 45 | 23.8 | 23.6 | 19.8 | 20.5 | 7.6 | 7.7 | 19.8 | 20.3 | 7.6 | 7.2 |
| 2 | 42.3 | 42.1 | 16.4 | 15.6 | 42.3 | 42.2 | 16.4 | 15.7 | 19.4 | 19.9 | 4.3 | 4.1 | 19.4 | **19.6** | 4.3 | 3.8 |
| 3 | 41.8 | 41.7 | 12.9 | 12.3 | 41.8 | 41.7 | 12.9 | 12.3 | 19.4 | 19.8 | 2.5 | 2.3 | 19.4 | **19.7** | 2.5 | 2.2 |
| 4 | 41.5 | 41.6 | 11.2 | 10.7 | 41.5 | 41.5 | 11.2 | 10.6 | 19.4 | 19.8 | 0.8 | 0.8 | 19.4 | **19.7** | 0.8 | 0.7 |
| 5 | 41.5 | 41.5 | 10.2 | 9.8 | 41.5 | 41.5 | 10.2 | 9.8 | 19.4 | 19.8 | 0 | 0.1 | 19.4 | **19.8** | 0 | 0.1 |
| 6 | 41.5 | 41.5 | 9.7 | 9.3 | 41.5 | 41.5 | 9.7 | 9.3 | 19.4 | 19.8 | 0 | 0 | 19.4 | 19.8 | 0 | 0 |
| 7 | 41.5 | 41.5 | 9.5 | 9.1 | 41.5 | 41.5 | 9.5 | 9.1 | 19.4 | 19.8 | 0 | 0 | 19.4 | 19.8 | 0 | 0 |
| 10 | 41.5 | 41.5 | 9.3 | 8.8 | 41.5 | 41.5 | 9.3 | 8.8 | 19.4 | 19.8 | 0 | 0 | 19.4 | 19.8 | 0 | 0 |
| 20 | 41.5 | 41.5 | 9.3 | 8.8 | 41.5 | 41.5 | 9.3 | 8.8 | 19.4 | 19.8 | 0 | 0 | 19.4 | 35.5 | 0 | 0 |
| $R^2$ | 0.999 | | | | 1.000 | | | | 0.998 | | | | 1.000 | | | |
| *SD* | 0.60 | | | | 0.60 | | | | 0.82 | | | | 0.36 | | | |

$R^2$ = coefficient of determination, *SD* = standard deviation, S = simulation, M = model

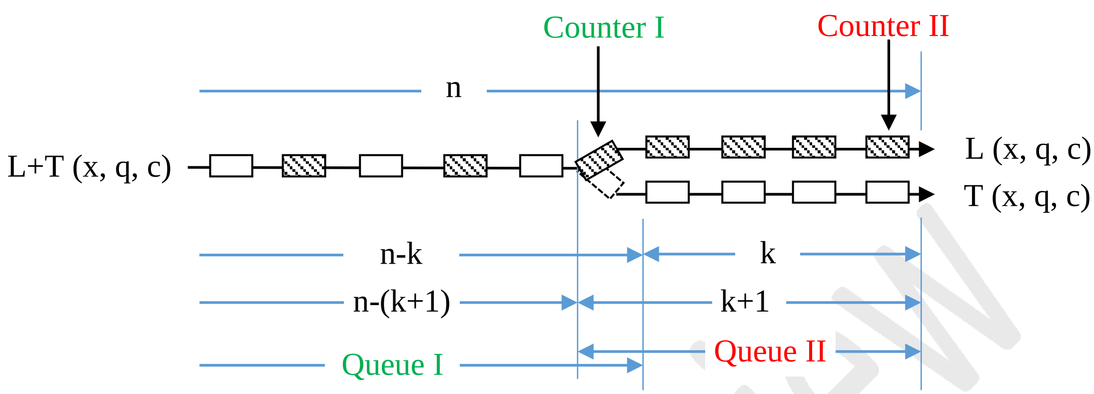



**FIGURE 1 "Two-Queue" system at a SSL, Queue I: M/G/1, Queue II: M/M/1**

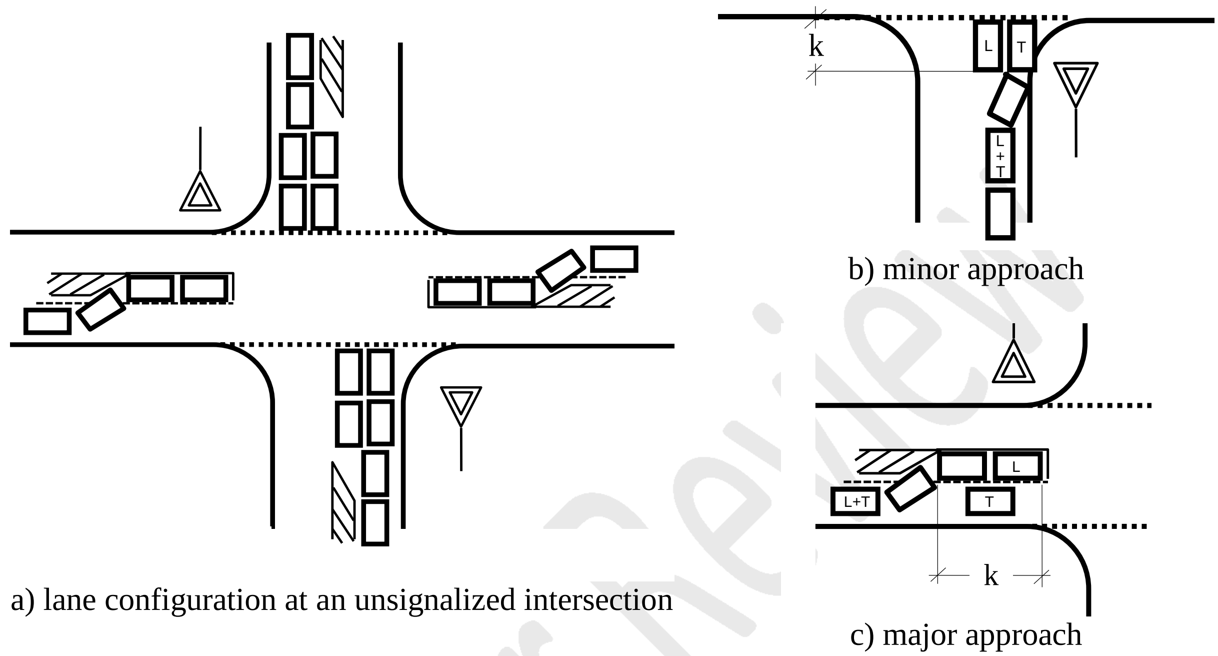
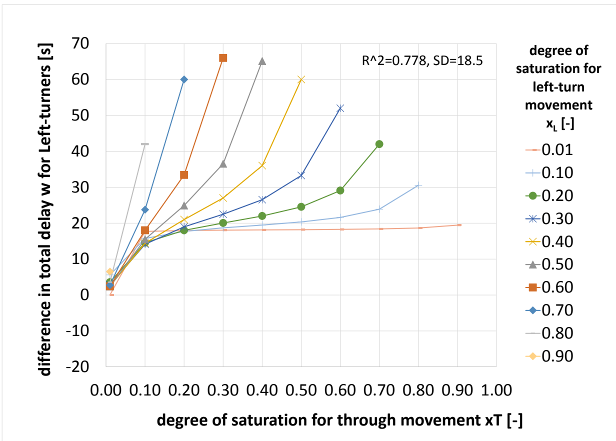
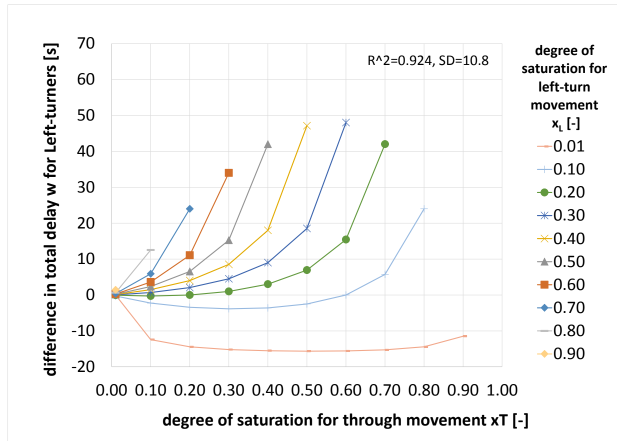



**FIGURE 2 Possible queues and lane configurations at approaches of unsignalized intersections**

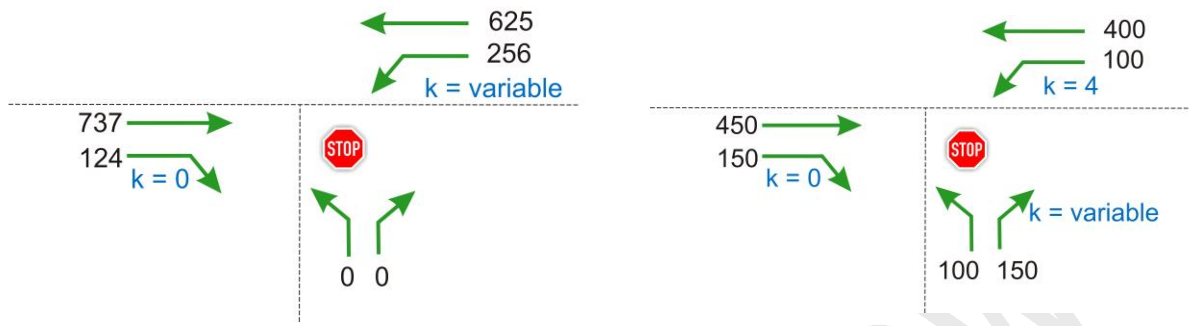



a) example for total delay: $c_L$ = 150 veh/h, $c_T$ = 600 veh/h



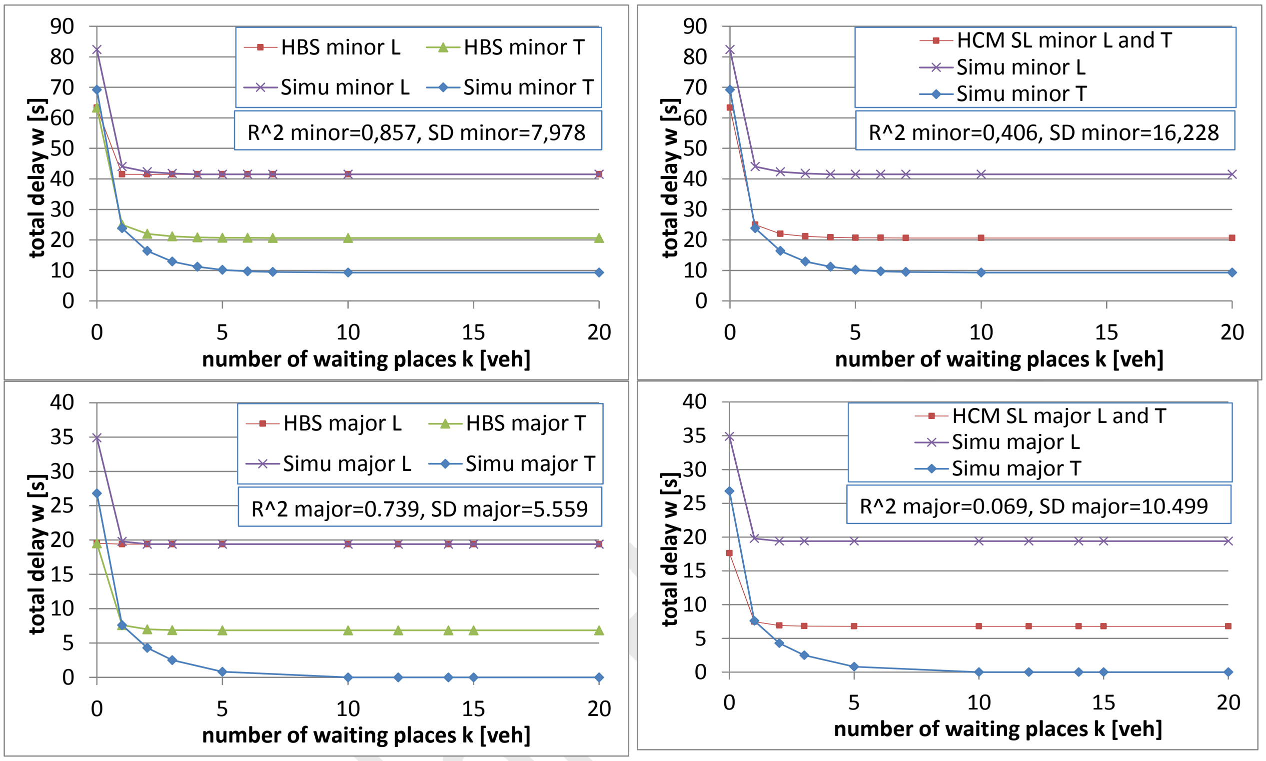

b) example for total delay: $c_L$ = 600 veh/h, $c_T$ = 150 veh/h

**FIGURE 3 Difference of total delays *w* calculated for left-turn movement on a shared lane (SL) (eq. (11) minus eq. (9))**




a) simulation of a major approach b) simulation of a minor approach

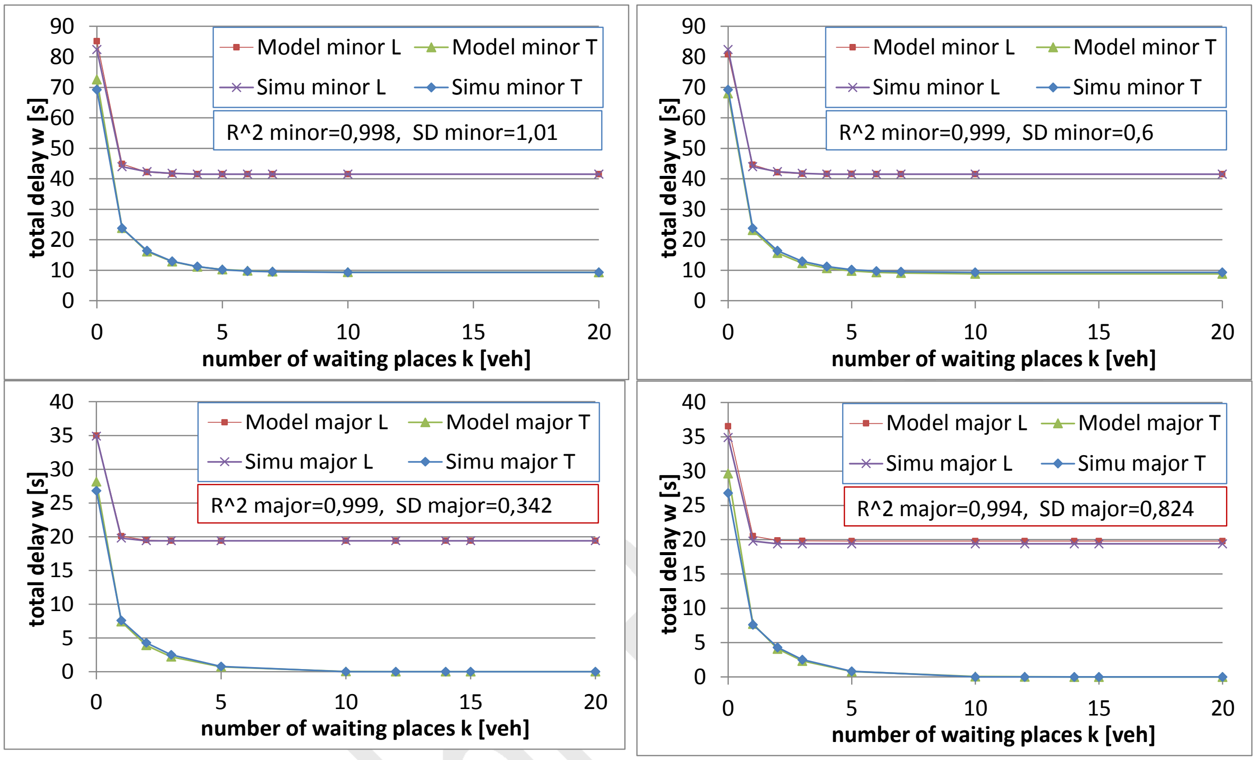
**FIGURE 4 Traffic volumes for the simulation study**




a) comparison of the simulated $w$ to the result from the HBS-procedure (here $w$=max($w_L$; $w_{L+T}$) or $w$=max($w_T$; $w_{L+T}$))

b) comparison of the simulated delay to the result from the HCM-procedure - (in both cases delay on the shared-lane section only $w = w_{L+T}$

**FIGURE 5 Simulation results compared to the results from the HBS- and HCM-procedure. Note: in case of delays on the shared-lane section only (HCM), the curves for HCM *L* and HCM *T* are identical and they are overlapping. $R^2$ = coefficient of determination, *SD* = standard deviation**




a) $c$ simulated ($c = 3600/w_{simu} + q$) b) $c$ calculated from the HBS (1)

**FIGURE 6 Simulation results, total delay $w$ of SSL vs. number of waiting places $k$. $R^2$ = coefficient of determination, $SD$ = standard deviation**